register allocation

a piece of code:

A ← ....
B ← ... A ...
...... B ....
C ← ....
...... A ....
D ← ....
...... D ....
...... C ....

variables must be stored in registers, and must stay there until their last access

R₁ ← ....
R₂ ← ... R₁ ...
...... R₂ ....
R₂ ← ....
...... R₁ ....
R₁ ← ....
...... R₁ ....
...... R₂ ....

life times
combinatorial problems

- **existence problems**: given a graph, is there a path containing each edge exactly once.

- **decision problems**: given a piece of code and integer K, can all variables be stored in K registers.

- **optimization problems**: given a piece of code, what is the minimum number of registers that can store all variables.

- **optimal object generation**: given a piece of code, allocate all variables to a minimum number of registers.
a piece of code:

A ← .......
B ← .......A...
......B.....
C ← .......
......A.....
D ← .......
......D.....
......C.....

variables must be stored in registers, and must stay there until their last access

R₁ ← .......
R₂ ← .......R₁...
......R₂.....
R₂ ← .......
......R₁.....
R₁ ← .......
......R₁.....
......R₂.....

life times
two variables may be stored in the same register, if their life times do not overlap

or

the last access to one variable is before the first access of the other variable

A ← ........ A<D
B ← .... A ... B<C
...... B .... B<D
C ← ........
...... A ....
D ← ........ D<D
......D......
...... C......

A partial order

input:
a partial order

output:
a minimum number of linearly ordered subsets of the given partial order
optimal track assignment

metal strips cannot be placed in the same track when their contacts do interleave
constraint

two metal strips may be placed in the same track, if their intervals do not overlap

or

two metal strips cannot be placed in the same track when their intervals do overlap

interval graph

partition the vertices of the graph such that vertices in the same block are not connected and the number of blocks is minimal

graph coloring problem
intractability

COMPUTERS AND INTRACTABILITY
A Guide to the Theory of NP-Completeness

Michael R. Garey / David S. Johnson
[GT4] GRAPH K-COLORABILITY (CHROMATIC NUMBER)

INSTANCE: Graph $G = (V, E)$, positive integer $K \leq |V|$.

QUESTION: Is $G$ $K$-colorable, i.e., does there exist a function $f: V \rightarrow \{1, 2, \ldots, K\}$ such that $f(u) \neq f(v)$ whenever $\{u, v\} \in E$?

Reference: [Karp, 1972]. Transformation from 3SAT.

Comment: Solvable in polynomial time for $K = 2$, but remains NP-complete for all fixed $K \geq 3$ and, for $K = 3$, for planar graphs having no vertex degree exceeding 4 [Garey, Johnson, and Stockmeyer, 1976]. Also remains NP-complete for $K = 3$ if $G$ is an intersection graph for straight line segments in the plane [Ehrlich, Even, and Tarjan, 1976]. For arbitrary $K$, the problem is NP-complete for circle graphs and circular arc graphs (even given their representation as families of arcs), although for circular arc graphs the problem is solvable in polynomial time for any fixed $K$ (given their representation) [Garey, Johnson, Miller, and Papadimitriou, 1978]. The general problem can be solved in polynomial time for comparability graphs [Even, Pnueli, and Lempel, 1972], for chordal graphs [Gavril, 1972], for $(3,1)$ graphs [Walsh and Burkhard, 1977], and for graphs having no vertex degree exceeding 3 [Brooks, 1941].
the greedy extension paradigm

U: set of objects,  
eg. variables, metal strips
C: list of "free" objects,  
eg. registerless variables, trackless strips
S: (partial) solution

C ← U
S ← ∅

while admissible(C) ≠ ∅ do

x ← select (admissible (C))
C ← C \ {x}
S ← S ∪ {x}

if complete (S) then return S

complete: check.
eg. all variables assigned  
all metal strips placed
admissible: free and consistent
select: find "best" object
eg. "earliest" variable,  "closest" strip"

when "select" is based on locally optimal choices, we speak of a "greedy" algorithm.
observation

- track length is a constant $t$
- sum of interval lengths is a constant $\sum I$

if $n$ tracks are used,
then the "unused" track length is $n \cdot t -- \sum I$

minimizing "unused track length" is the same as minimizing the number of tracks

sort intervals by "left edge"

while there are free intervals do
  open a new track
  while not all free intervals tried do
    choose the "next" one (by left edge)
    if consistent then assign to current track
the left edge algorithm
\[ y'' + 3xy' + 3y = 0 \]

\[ 0 \leq x \leq a \]

\[ x(0) = x \]

\[ y(0) = y \]

\[ y'(0) = u \]

---

**data flow graphs**

**forward euler**

```plaintext

diffeq{
  read(x,y,u,dx,a);
  repeat{
    x1 = x + dx;
    u1 = u - 3*x*u*dx - 3*y*dx;
    y1 = y + u*dx;
    c = x1 < a;
    x = x1; u = u1; y = y1;
  }
  until (c);
  write (y);
}
```
data flow graphs

\[ x_1 := x + dx \]
\[ u_1 := u - 3 \times x \times u \times dx - 3 \times y \times dx \]
\[ y_1 := y + u \times dx \]
\[ c := x_1 < a \]
data flow graphs
data flow graphs
"small" modifications

iterative instead of straight-line processes

suggestion:
open up the ring and try greedy extension

what property misses the conflict graph of iterative processes that the conflict graph of straight-line processes have?
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