Sensors and Actuators
Introduction to sensors

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BRIDGE CIRCUITS

(Chapter 5.9-5.12)
Interface circuits

- resistance of linear resistive sensor: \( R(x) = R_0(1+x) \)
  - range of \( x \) depends on type of sensor

- requirements on signal conditioners for resistive sensors
  - electric voltage or current must be applied
  - supply and output voltage/current are limited by self-heating

- current excitation
  - maximal self-heating when \( R \) maximal
  - maximal sensitivity when \( R \) maximal

- voltage excitation
  - when does maximal self-heating error occur?
  - when is sensitivity maximal?
  - when is non-linearity error minimized?
Voltage divider – self-heating error

- sensor driven by voltage source
  - sensor: $R$
  - load resistance: $R_r$

- when does maximal self-heating error occur?
  - power consumption by sensor
    \[
    P = \left( \frac{V_r}{R + R_r} \right)^2 R
    \]

- maximal power consumption occurs when
  \[
  \frac{dP}{dR} = 2 \frac{V_r}{R + R_r} \left( \frac{-V_r}{(R + R_r)^2} R + \left( \frac{V_r}{R + R_r} \right)^2 \right) = 0 \Rightarrow R_r = R
  \]

- self-heating error is maximal when $R_r = R$
- power consumption is then equal to
  \[
  P = \left( \frac{V_r}{R_r + R_r} \right)^2 R_r = \frac{V_r^2}{4R_r}
  \]
Voltage divider – self-heating error

example – dimension voltage divider for temperature measurement

- measure temperature from 0°C to 100°C
- PT 100 sensor \((R_0=100\,\Omega\) and \(\alpha=0.00389\,\Omega/\Omega/K\) at 0°C)
- maximal power dissipation in sensor is 1mW
- voltage source \(V_r = 5V\)
- **what resistance \(R_r\) must be used for this voltage divider?**
- power dissipation in sensor

\[
\left( \frac{V_r}{R + R_r} \right)^2 R < 1mW
\]

- maximal dissipation when \(R=R_r\)

\[
\left( \frac{V_r}{R_r + R_r} \right)^2 R_r < 1mW \Rightarrow R_r > \frac{V_r^2}{2 \cdot 0.001W} = \frac{(5V)^2}{2 \cdot 0.001W} = 6.25k\Omega
\]

- sensor range is from 100Ω to 139Ω
- always \(R < R_r\), thus power dissipation always below limit
Voltage divider – linearity

- measure fractional change in resistance $x$
  - sensor: $R = R_0(1+x)$
  - load resistance: $R_r = R_0k$

- output voltage of the circuit
  $$v_o = \frac{R}{R_r + R}V_r = \frac{R_0(1+x)}{R_0k + R_0(1+x)}V_r = \frac{1+x}{k+1+x}V_r$$

- response becomes linear when $R_r >> R$ (i.e. $k >> 1+x$)
Increasing $k$ is good for linearity, but what about sensitivity?
Voltage divider – sensitivity

- measure fractional change in resistance $x$
  - sensor: $R = R_0(1+x)$
  - load resistance: $R_r = R_0k$

- sensitivity

$$S = \frac{dv_o}{dx} = \frac{d}{dx} \left( \frac{1+x}{k+1+x} V_r \right) = \frac{(k+1+x)-(1+x)}{(k+1+x)^2} V_r = \frac{k}{(k+1+x)^2} V_r$$

- maximal sensitivity

$$\frac{dS}{dk} = 0 \implies \frac{d}{dk} \left( \frac{k}{(k+1+x)^2} V_r \right) = 0$$

$$(k+1+x)^2 - 2k(k+1+x) = \frac{x+1-k}{(k+1+x)^4} = 0 \implies k = x+1$$

- maximal sensitivity reached when $R = R_r$
- same situation as when self-heating error is maximal
- maximal transfer of power (at $R = R_r$) leads to
  - maximal sensitivity and maximal self-heating
Voltage divider – sensitivity and linearity

- for many sensors $x < 1$
  - sensitivity largest for $k = 1$
  - sensitivity may be considered constant if maximal value of $x << 1$
Voltage divider – output voltage

- maximal sensitivity when $k = 1$
- output voltage

\[
V_o = \frac{1+x}{k+1+x} V_r = \frac{1+x}{2+x} V_r
\]

- offset voltage present in output
Voltage divider

- disadvantages of voltage dividers
  - may have offset voltage at desired reference point (e.g. temperature)

- Wheatstone bridge
  - offset voltage can be removed at desired reference temperature
  - sign of output indicates direction of change with respect to reference point (e.g. temperature)

\[ R_1 \cdot V_r \cdot R_2 \cdot R_3 = R_0(1+x) \cdot R_4 \cdot v_o \]
Wheatstone bridge – balance measurement

- **Balance or null** measurement method
  - $R_4$ adjusted till $V_o = 0$
  - Balance condition $v_o = 0 \Rightarrow \frac{R_3}{R_2} = \frac{R_4}{R_1} \iff R_3 = R_4 \frac{R_2}{R_1}$

- Advantages of null measurement method
  - Insensitive to $V_r$ fluctuations
  - Sensor may have non-linear resistance-input relation

- Fast auto balancing method for $R_4$ is required
Wheatstone bridge – balance measurement

- DAC outputs two current sources
  - $I_o$ – current corresponding to digital input
  - $I_o'$ – current corresponding to complement of digital input
- bridge imbalance exceeding comparator threshold changes counter
- change in counter value $\rightarrow$ change in current $\rightarrow$ change in voltages
  - on one arm the voltage increases
  - on other arm the voltage decreases
- process of changing counter continues till balance is restored
Wheatstone bridge – deflection measurement

- deflection measurement method
  - measure voltage difference on bridge arms
  - measure current through a detector between both arms
- deflection measurement is much faster than null measurement

- bridge output voltage

\[ v_o = \left( \frac{R_3}{R_3 + R_2} - \frac{R_4}{R_4 + R_1} \right) V_r \]

- assume bridge balanced \((v_o = 0)\) when \(x = 0\)
- bridge is then balanced when

\[ k = \frac{R_1}{R_4} = \frac{R_2}{R_0} \]

- what is \(v_o\) in terms of \(k\), \(x\) and \(V_r\)?
Wheatstone bridge – deflection measurement

- bridge output voltage in terms of $k$, $x$, and $V_r$

\[
\begin{align*}
  v_o &= \left( \frac{R_3}{R_3 + R_2} - \frac{R_4}{R_4 + R_1} \right) V_r \\

  R_3 &= R_0(1 + x) = R_0 + R_0 x
\end{align*}
\]

\[
\Rightarrow v_o = \left( \frac{R_0 + R_0 x}{R_0 + R_0 x + R_2} - \frac{R_4}{R_4 + R_1} \right) V_r
\]

\[
\Leftrightarrow v_o = \left( \frac{R_0}{R_0 + R_0 x} + \frac{R_0}{R_0 x + R_2} \right) - \frac{R_4}{R_4 + R_1} \right) V_r
\]

\[
\Leftrightarrow v_o = \left( \frac{1}{1 + x + k} - \frac{1}{1 + k} \right) V_r
\]

\[
\Leftrightarrow v_o = \frac{(1 + x)(1 + k) - (1 + x + k)}{(k + 1)(k + 1 + x)} V_r
\]

\[
\Leftrightarrow v_o = \frac{k x}{(k + 1)(k + 1 + x)} V_r
\]
Wheatstone bridge – deflection measurement

- Bridge output voltage
  
  \[ v_o = \frac{kx}{(k+1)(k+1+x)} V_r \]

- Output proportional to x when x << k+1

- Change in \( R_3(x) \) must be small compared to \( R_1/R_4 \) (k)

- Sensitivity of the bridge
  
  \[ S = \frac{dv_o}{dx} = \frac{d}{dx} \left( \frac{kx}{(k+1)(k+1+x)} V_r \right) \]
  
  \[ = \left( \frac{k}{(k+1)(k+1+x)} - \frac{kx}{(k+1)(k+1+x)^2} \right) V_r \]
  
  \[ = \frac{k(k+1)}{(k+1)(k+1+x)^2} V_r = \frac{k}{(k+1+x)^2} V_r \]
Wheatstone bridge – deflection measurement

- maximal sensitivity
  \[
  \left. \frac{dS}{dk} \right|_{x=0} = 0 \Rightarrow \frac{d}{dk} \left( \frac{k}{(k+1)^2} V_r \right) = 0
  \]
  \[
  \Rightarrow \frac{1-k}{(k+1)^3} V_r = 0 \Rightarrow k = 1
  \]

- max sensitivity when
  - \( R_1 = R_4 \)
  - \( R_2 = R_3 \)

- bridge output voltage (k=1)
  \[
  v_o = \frac{x}{4+2x} V_r \approx \frac{x}{4} V_r
  \]

- \( x \) in denominator can be ignored when \( x \ll 1 \)
- otherwise more advanced signal processing required
Wheatstone bridge versus voltage divider

- sensitivity
  \[ S = \frac{k}{(k+1+x)^2} V_r \]
- max sensitivity when
  \( k=1: R_1=R_4=R_2=R_0 \)
- output voltage (\( k=1 \))
  \[ v_o = \frac{x}{4+2x} V_r \approx \frac{x}{4} V_r \]
- bridge removes DC offset

\[ S = \frac{k}{(k+1+x)^2} V_r \]
\( k=1: R_1=R_0 \)
\[ v_o = \frac{1+x}{2+x} V_r \approx \frac{1}{2} V_r + \frac{x}{2} V_r \]
Wheatstone bridge versus voltage divider

- sensitivity is equal, but DC offset makes response look “flat”
- output of Wheatstone bridge can easily be boosted with amplifier
Wheatstone bridge versus voltage divider

- Output voltage \((k=1)\)
  \[
  v_o = \frac{x}{4+2x} V_r \approx \frac{x}{4} V_r
  \]

- Response of bridge output to change in \(x\) only half of response when using divider

- Can we change the bridge to get the same response?
  - Use operational amplifier (also amplifies non-linearity error)
  - Use an additional sensor

\[
\begin{align*}
R_3 &= R_0(1+x) \\
R_2 &= R_0(1+x)
\end{align*}
\]
Wheatstone bridge

- **increase sensitivity** by adding sensor on other side of opposing arm
- **bridge output voltage**

\[
V_o = \left( \frac{R_3}{R_3 + R_2} - \frac{R_4}{R_4 + R_1} \right) V_r
\]

\[
R_1 = R_3 = R_0(1 + x) = R_0 + R_0 x
\]

\[
\Rightarrow V_o = \left( \frac{R_0 + R_0 x}{R_0 + R_0 x + R_2} - \frac{R_4}{R_4 + R_0 + R_0 x} \right) V_r
\]

\[
\Leftrightarrow V_o = \left( \frac{1 + x}{1 + x + R_2/R_0} - \frac{1}{1 + R_0/R_4 x + R_0/R_4} \right) V_r
\]

\[
\Rightarrow V_o = \left( \frac{1 + x}{1 + x + k} - \frac{1}{1 + x + k} \right) V_r
\]

\[
\Rightarrow V_o = \left( \frac{x}{2 + x} V_r \right)
\]

\[
k = 1
\]

\[
\Rightarrow V_o \approx \frac{x}{2} V_r
\]
Wheatstone bridge

- **measure difference** by adding sensor on same side of opposing arm
- bridge output voltage

\[
R_3 = R_0(1 + x_1), \quad R_4 = R_0(1 + x_2)
\]

\[
v_o = \left( \frac{R_3}{R_3 + R_2} - \frac{R_4}{R_4 + R_1} \right) V_r
\]

\[
\Rightarrow v_o = \left( \frac{1 + x_1}{1 + x_1 + k} - \frac{1 + x_2}{1 + x_2 + k} \right) V_r
\]

\[
\Rightarrow v_o = \left( 1 + x_2 + k + x_1 + x_1 x_2 + k x_1 - 1 - x_1 - k x_1 - x_2 + x_2 x_1 + x_2 k \right) V_r
\]

\[
\Rightarrow v_o = \left( 1 + x_2 + k + x_1 + x_1 x_2 + k x_1 - 1 - x_1 - k x_1 - x_2 + x_2 x_1 + x_2 k \right) V_r
\]

\[
\Rightarrow v_o = \frac{k(x_1 - x_2)}{(1 + x_1 + k)(1 + x_2 + k)} V_r
\]

when \( x_1, x_2 << k + 1 \)

\[
\Rightarrow v_o = \frac{k(x_1 - x_2)}{(1 + k)^2} V_r
\]
Wheatstone bridge

- different sensor placements in Wheatstone bridge allow
  - increasing sensitivity
  - create differential sensor
  - create averaging sensor
  - compensate error sources (strain, temperature, ...)

\[ V_r \]

\[ V_r = \frac{kR_0}{3} \]

\[ R_1 = \frac{R_0}{3} \]

\[ R_2 = R_0(1-x) \]

\[ R_3 = R_0(1+x) \]

\[ R_4 = R_0(1+x_1)/3 \]

\[ R_5 = R_0(1+x_2)/3 \]

\[ R_6 = R_0(1+x_3)/3 \]