Sensors and Actuators
Introduction to sensors

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BRIDGES AND RESISTIVE STRAIN SENSORS
(Chapter 5.9-5.12, 9.1)
Applications of resistive strain sensors
### Strain, force, pressure sensors

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Resistive effect in strain gauge plays an important role in many sensors for different quantities.
Strain gauge

- resistance of a wire
  \[ R = \rho \frac{l}{a} = \frac{m}{n\varepsilon^2\tau} \frac{l}{a} \]

- changing temperature affects resistance (thermoresistive effect)
- changing dimensions affects resistance (piezoresistive effect)

- strain gauges use piezoresistive effect to sense mechanical stress
- sensor based on strain gauges convert mechanical energy to electrical energy

- thermoresistive effect is an error source
force leads to deformation of object

measuring deformation provides opportunity to sense mechanical force, which in turn is related to
torque
pressure
acceleration
mass
...
Stress and strain

- force leads to deformation of object

- deformation depends on force per area which is called stress ($\sigma$)
  \[ \sigma = \frac{F}{a} \]

- deformation also depends on
  - material properties
  - length or volume of object

- deformation per unit length (or volume) is called strain ($\varepsilon$)
  \[ \varepsilon = \frac{dl}{l} \]
Stress and strain

- object deform under force and restores to original state when force is removed (elasticity)
- materials resist deformation (rigidity)

- change in length due to force $F$ given by Hooke’s law

$$\sigma = \frac{F}{a} = E\varepsilon = E\frac{dl}{l}$$

- $E$ – Young’s modulus, which depends on
  - material
  - temperature
- $\varepsilon$ – strain (unit deformation, dimensionless)

strain and stress are proportional in elastic zone

stress, $\sigma = F/a$
rupture
elastic limit
elastic zone
strain, $\varepsilon$
Strain gauge

- resistance of a wire
  \[ R = \rho \frac{l}{a} \]

- stretching wire longitudinally changes resistance
  \[ \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} + \frac{d(1/a)}{a} = \frac{d\rho}{\rho} + \frac{dl}{l} - \frac{da}{a} \]

- change in length due to force F given by Hooke’s law
  \[ \sigma = \frac{F}{a} = E\varepsilon = E\frac{dl}{l} \]
  - E – Young’s modulus, which depends on
    - material
    - temperature
  - \( \varepsilon \) – strain (unit deformation, dimensionless)
  - strain and stress are proportional in elastic zone

\[ F \]

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**Strain gauge**

- **longitudinal stress changes**
  - length of wire \( l \)
  - thickness of wire \( t \)

- **Poisson ratio** gives relation between change in length and thickness

\[ \nu = -\frac{dt/t}{dl/l} \]

- Poisson ratio of perfectly compressible material: 0.0 (e.g. cork)
  - deformation in one direction does not change other direction
- Poisson ratio of incompressible material: 0.5 (e.g. rubber)
  - volume of this material is constant when stress is applied
Strain gauge

- longitudinal stress changes
  - length of wire (l)
  - thickness of wire (t)

- Poisson ratio gives relation between change in length and thickness

\[ \nu = -\frac{dt/t}{dl/l} \]

- Poisson ratio of metals: \(0 < \nu < 0.5\)
  - volume of metal changes when deformed
  - consider a circular wire with diameter t (and thus radius: t/2)
  - change in volume per unit volume is then equal to

\[
\begin{align*}
V &= \pi \cdot \left(\frac{t}{2}\right)^2 \cdot l \\
\Rightarrow \frac{dV}{V} &= \frac{dl}{l} + \frac{2dt}{t} \\
\Rightarrow \frac{dV}{V} &= \frac{dl}{l} - 2\nu \frac{dl}{l} = \frac{dl}{l} \left(1 - 2\nu\right)
\end{align*}
\]
Strain gauge

- Longitudinal stress changes
  - Length of wire (l)
  - Thickness of wire (t)

- Poisson ratio gives relation between change in length and thickness
  \[ \nu = -\frac{dt/t}{dl/l} \]

- Poisson ratio of metals: \(0 < \nu < 0.5\)
  - Volume of metal changes when deformed
  - Cross-sectional area changes when metals are deformed

Using same approach as used for volume we can show

\[
\begin{align*}
 a &= \pi \cdot \left(\frac{t}{2}\right)^2 \Rightarrow \frac{da}{a} = \frac{2dt}{t} \\
 \nu &= -\frac{dt/t}{dl/l} \Rightarrow \frac{dt}{t} = -\nu \frac{dl}{l}
\end{align*}
\]

\[ \Rightarrow \frac{da}{a} = -2\nu \frac{dl}{l} \]
Strain gauge

- Longitudinal stress changes
  - Length of wire (l)
  - Thickness of wire (t)

- Poisson ratio gives relation between change in length and thickness
  \[ \nu = -\frac{dt/t}{dl/l} \]

- Poisson ratio of metals: \( 0 < \nu < 0.5 \)
  - Volume of metal changes when deformed
  - Because of volume change
    - Amplitude of vibrations in metal lattice changes
  - Results in change of specific resistivity (for metals)
    \[ \frac{d\rho}{\rho} = C \frac{dV}{V} \]
  - \( C \) – Bridgman’s constant
Strain gauge

- longitudinal stress changes
  - length of wire \((l)\)
  - thickness of wire \((t)\)

- stretching wire longitudinally changes resistance

\[
\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} + \frac{d(1/a)}{a} = \frac{d\rho}{\rho} + \frac{dl}{l} - \frac{da}{a}
\]

- using results found so far we find

\[
\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} - \frac{da}{a} \quad \Rightarrow \quad \frac{dR}{R} = C \frac{dV}{V} + \frac{dl}{l} - \frac{da}{a}
\]

\[
\frac{d\rho}{\rho} = C \frac{dV}{V}
\]

\[
\frac{dV}{V} = \frac{dl}{l} (1-2\nu), \quad \frac{da}{a} = -2\nu \frac{dl}{l}
\]

\[
\Rightarrow \frac{dR}{R} = \frac{dl}{l} \left[C(1-2\nu) + 1 + 2\nu \right]
\]

\[
\Leftrightarrow \frac{dR}{R} = G \frac{dl}{l}
\]

- \(G\) – gauge factor
Strain gauge

- change in resistance related to change in length
  \[
  \frac{dR}{R} = G \frac{dl}{l}
  \]
- remember Hooke’s law (relates stress $\sigma$ to strain $\varepsilon$)
  \[
  \sigma = \frac{F}{a} = E\varepsilon = E \frac{dl}{l}
  \]
- change in resistance related to force (per unit area) and strain
  \[
  \frac{dR}{R} = G\varepsilon = G \frac{F}{aE}
  \]
- strain gauge can be used to sensor force and its derived quantities
- gauge factor is constant for metals, hence
  \[
  R = R_0 + dR = R_0 \left(1 + \frac{dR}{R_0}\right) = R_0 (1 + G\varepsilon) = R_0 (1 + x)
  \]
- typically $x < 0.002$
example – strain gauge attached to aluminum strut

- strain gauge with \( R = 350 \ \Omega \) and \( G = 2.1 \)
- aluminum strut has \( E = 73 \ \text{GPa} \)
- outer diameter of the strut: \( D = 50 \ \text{mm} \)
- inner diameter of the strut: \( d = 47.5 \ \text{mm} \)

What is the change in resistance when the strut supports a load of 1000 kg?

- area supporting the force
  \[
a = \frac{\pi(D^2 - d^2)}{4} = \frac{\pi((50\ \text{mm})^2 - (47.5\ \text{mm})^2)}{4} = 191\ \text{mm}^2
\]
- change in resistance
  \[
  \Delta R = RG\varepsilon = RG \frac{F}{aE} = (350\Omega)(2.1)\frac{9800\text{N}}{(191\cdot10^{-6}\text{m}^2)\cdot(73\cdot10^9\text{Pa})} = 0.5\Omega
  \]
- change in resistance is less than 0.15% of the initial resistance
Metal strain gauge

- Change in resistance due to stress depends on:
  \[
  \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} - \frac{da}{a} = \frac{d\rho}{\rho} + (1 + 2\nu) \frac{dl}{l}
  \]
  - Change in conductivity
  - Deformation of the conductor

- Metals
  - Change in conductivity related to deformation:
    \[
    \frac{d\rho}{\rho} = C \frac{dV}{V}
    \]
  - Change in conductivity small compared to dimensional change:
    \[
    \frac{d\rho}{\rho} \ll (1 + 2\nu) \frac{dl}{l}
    \]
  - Metals have linear relation between \(\Delta R\) and \(\Delta l\)
Silicon strain gauge

- change in resistance due to stress depends on
  \[
  \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} - \frac{da}{a} = \frac{d\rho}{\rho} + (1 + 2\nu) \frac{dl}{l}
  \]
  - change in conductivity
  - deformation of the conductor

- silicon
  - change in conductivity dominates dimensional change
    \[
    \frac{d\rho}{\rho} \gg (1 + 2\nu) \frac{dl}{l}
    \]
  - stress influences
    - lattice spacing between atoms
    - changes band gap energy
    - influences number of carriers and average mobility of carriers
  - silicon has a larger non-linearity compared to metal
Typical characteristics

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<th>Metal</th>
<th>Semiconductor</th>
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<tr>
<td>Gauge factor</td>
<td>2.0</td>
<td>-100 to 150</td>
</tr>
<tr>
<td>Base resistance (Ω)</td>
<td>100 - 1000</td>
<td>&gt;200</td>
</tr>
<tr>
<td>Non-linearity</td>
<td>0.1%</td>
<td>1%</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>5mΩ/Ω</td>
<td>5mΩ/Ω</td>
</tr>
<tr>
<td>Deformation</td>
<td>2-3μm/m</td>
<td>2-3μm/m</td>
</tr>
<tr>
<td>Breaking strain</td>
<td>25000</td>
<td>5000</td>
</tr>
<tr>
<td>Fatigue life</td>
<td>10 million reversals</td>
<td>10 million reversals</td>
</tr>
<tr>
<td>TCR (10⁻⁶/°C)</td>
<td>10 - 2000</td>
<td>90000</td>
</tr>
</tbody>
</table>

- semiconductor strain gauges have
  - high gauge factor
  - strong temperature dependency
  - magnitude and sign of gauge factor depends on doping
Construction

- bonded strain gauges
  - wires cemented onto a backing or
  - thin film resistor deposited on a substrate
  - resistor forms a long, meandering wire

- strain gauge connected to test object
Interface circuit

- strain gauge in resistive divider
- stress applied to gauge (resistive change ‘x’)
- what is the output voltage \( v_o \) of the circuit?

\[
v_o = \left( \frac{R_2}{R_1 + R_2} \right) V_r = \left( \frac{R_0(1+x)}{kR_0 + R_0(1+x)} \right) V_r = \left( \frac{1+x}{k+1+x} \right) V_r
\]

- for strain gauges holds that \( x \ll k \) (typically \( x < 0.002 \))

\[
v_o \approx \left( \frac{1+x}{k+1} \right) V_r = \frac{1}{k+1} V_r + \frac{x}{k+1} V_r
\]

- maximal sensitivity when \( k = 1 \)

\[
v_o \approx \frac{1}{2} V_r + \frac{x}{2} V_r
\]
Interface circuit

- strain gauge in resistive divider
- stress applied to gauge (resistive change ‘x’)
- two problems when measuring output voltage
  - non-linearity in response
  - offset voltage present

\[ V_r = kR_0 \\
R_2 = R_0(1+x) \]

\frac{|v_o/V_r|}{v_o when ignoring non-linearity}

\frac{v_o measured (includes non-linearity)}{offset voltage}
Interface circuit

- remove offset voltage by placing strain gauge in bridge
- stress applied to gauge (resistive change ‘x’)
- **what is the output voltage** $v_o$? (assume $k = \frac{R_1}{R_4} = \frac{R_2}{R_0} = 1$)

\[
v_o = \left( \frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3} \right) V_r = \left( \frac{R_0}{2R_0} - \frac{R_0(1+x)}{R_0(2+x)} \right) V_r = \left( \frac{1}{2} - \frac{1+x}{2+x} \right) V_r = \left( \frac{-x}{4+2x} \right) V_r \approx -\frac{x}{4} V_r
\]

- intermezzo: two sources of non-linearity
  - strain gauge itself does not adhere to $R = R_0(1+x)$
  - interface circuit causes non-linear resistance – voltage relation

\[
R_1 \quad \text{\begin{tikzpicture}
\draw (0,0) -- (1,0) -- (1,1) -- (0,1) -- cycle;
\end{tikzpicture}} \quad R_2
\]

\[
R_4 \quad \text{\begin{tikzpicture}
\draw (0,0) -- (1,0) -- (1,1) -- (0,1) -- cycle;
\end{tikzpicture}} \quad R_3 = R_0(1+x)
\]

\[
V_r \quad \text{\begin{tikzpicture}
\draw (0,0) -- (1,0) -- (1,1) -- (0,1) -- cycle;
\end{tikzpicture}} \quad V_0
\]

non-linearity
Interface circuit

- remove offset voltage by placing strain gauge in bridge
- stress applied to gauge (resistive change ‘x’)
- **what is the output voltage \( v_o \)? (assume \( k = \frac{R_1}{R_4} = \frac{R_2}{R_0} = 1 \))

\[
v_o = \left( \frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3} \right) V_r = \left( \frac{R_0}{2R_0} - \frac{R_0(1+x)}{R_0(2+x)} \right) V_r = \left( \frac{1}{2} - \frac{1+x}{2+2} \right) V_r = \left( \frac{-x}{4+2x} \right) V_r \approx -\frac{x}{4} V_r
\]

- compare output voltage bridge and divider

\[
v_{o,\text{divider}} \approx \frac{1}{2} V_r + \frac{x}{2} V_r
\]

- bridge removes offset
- bridge reduces sensitivity
Interface circuit

- **increase sensitivity** by adding extra strain gauge to bridge
- stress applied to gauge (resistive change ‘x’)
- **what is the output voltage** $v_o$? (assume $k = \frac{R_0}{R_4} = \frac{R_2}{R_0} = 1$)

$$v_o = \left( \frac{\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3}}{R_0(2 + x)} \right) V_r = \left( \frac{\frac{R_0}{R_0(2 + x)} - \frac{R_0(1 + x)}{R_0(2 + x)}}{R_0(2 + x)} \right) V_r = \left( \frac{-x}{2 + x} \right) V_r \approx -\frac{x}{2} V_r$$

- compare output voltage to single sensor solution

$$v_{o,\text{single}} = \frac{-x}{4 + 2x} V_r \approx -\frac{x}{4} V_r$$

- extra sensor **increases sensitivity**
Interface circuit

- remove non-linearity by adding applying opposing signal to gauge
- stress applied to gauge (resistive change ‘x’)
- what is the output voltage $v_o$? (assume $k = R_1/R_4 = R_2/R_0 = 1$)

$$v_o = \left( \frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3} \right)V_r = \left( \frac{R_0}{2R_0} - \frac{R_0(1+x)}{R_0(1-x) + R_0(1+x)} \right)V_r = \left( \frac{R_0}{2R_0} - \frac{R_0(1+x)}{2R_0} \right)V_r = -\frac{x}{2}V_r$$

- sensitivity equal to previous arrangement
- non-linearity removed

![Diagram of interface circuit]
Interface circuit

- add more strain gauges to increase sensitivity
- stress applied to gauge (resistive change ‘x’)
- what is the output voltage $v_o$?

$$v_o = \left( \frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3} \right) V_r = \left( \frac{R_0(1-x)}{R_0(1+x) + R_0(1-x)} - \frac{R_0(1+x)}{R_0(1-x) + R_0(1+x)} \right) V_r$$

$$= \left( \frac{1-x}{2} - \frac{1+x}{2} \right) V_r = \frac{-2x}{2} V_r = -xV$$