Sensors and Actuators
Introduction to sensors

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CAPACITIVE SENSORS

(Chapter 3.2, 7.2, 9.2, 10.6, 13.1, 13.2)
### Sensor classification – type / quantity measured

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- reactance variation sensors (capacitive and inductive sensors)
- typically requires no physical contact
- exerts minimal mechanical loading
Capacitive sensors
Capacitive sensor

- charge on two plates separated by a dielectric
  \[ Q = C \cdot V \]

- current \( I \) is the change in charge \( Q \) per time unit
  \[ I = \frac{dQ}{dt} = C \frac{dV}{dt} \]

- device possibly usable as sensor (why?)
  - device produces electric signal
  - \( C \) depends on physical device properties

- alternating current (AC) signal required to produce output signal
  - resistive sensors operate using direct current signal
  - capacitor requires potentially more complex interface circuit
Capacitive sensor

- capacitance defined by \( Q = C \cdot V \) and \( I = C \frac{dV}{dt} \)
- capacitance depends on physical properties
  \[ C = \varepsilon_r \varepsilon_0 \frac{A}{z} \]
  - \( \varepsilon_0 \) – dielectric constant for vacuum (8.85 pF/m)
  - \( \varepsilon_r \) – relative dielectric constant
- device usable as sensor (why?)
  - changing \( A, z, \varepsilon_r \) changes capacitance
  - change in capacitance can be sensed
- capacitance of a cylindrical capacitor
  \[ C = \varepsilon_r \varepsilon_0 \frac{2\pi \cdot l}{\ln(b/a)} \]
Capacitive level sensor

example – capacitive level sensor

- sensor based on two concentric cylinders \( (d_1 = 8\text{mm}, d_2 = 40\text{mm}) \)
- cylindrical storage tank \( (L = 50\text{cm}, H = 1.2\text{m}) \)
- stored liquid has \( \varepsilon_r = 2.1 \)

what is the sensitivity of the sensor \( (\text{pF}/\text{L}) \) when used in the storage tank?

hint: capacitance of two cylindrical concentric electrodes is equal to

\[
C = \varepsilon_r \varepsilon_0 \frac{2\pi \cdot h}{\ln(d_2 / d_1)}
\]
Capacitive level sensor

example – capacitive level sensor

- capacitance of two cylindrical concentric electrodes
  \[ C = \varepsilon_r \varepsilon_0 \frac{2\pi \cdot h}{\ln(d_2 / d_1)} \]

- relative permittivity varies with height
  \[ C = \varepsilon_0 \frac{2\pi \cdot (\varepsilon_1 h_1 + \varepsilon_2 h_2)}{\ln(d_2 / d_1)} \]

- use: \( h_2 = h \) (height of liquid), \( h_1 = H - h \)

- capacitance is equal to
  \[ C = \frac{2\pi}{\ln(d_2 / d_1)} (\varepsilon_0 (H - h) + \varepsilon_0 \varepsilon_r h) = \frac{2\pi}{\ln(d_2 / d_1)} (\varepsilon_0 H + \varepsilon_0 (\varepsilon_r - 1)h) \]

- min, max capacitance
  \[ C_{\text{min}} = \frac{2\pi \varepsilon_0 H}{\ln(d_2 / d_1)} = \frac{2\pi (8.85 \text{ pF/m})(1.2\text{ m})}{\ln(40\text{ mm}/8\text{ mm})} = 41.46 \text{ pF} \]
  \[ C_{\text{max}} = \frac{2\pi \varepsilon_0 \varepsilon_r H}{\ln(d_2 / d_1)} = 41.46 \text{ pF} \cdot 2.1 = 87.07 \text{ pF} \]
Capacitive level sensor

example – capacitive level sensor

- volume of the storage tank

\[ V = \frac{\pi \cdot L^2}{4}, \quad H = \frac{\pi \cdot (0.5m)^2}{4} \cdot (1.2m) = 235.6L \]

- sensitivity

\[ S = \frac{C_{\text{max}} - C_{\text{min}}}{V} = \frac{87.07\, pF - 41.46\, pF}{235.6L} = 0.19\, pF/L \]
Dielectric material

example – capacitive level sensor

![Capacitive Level Sensor Diagram]
Capacitive sensor

- capacitance depends on physical properties
  \[ C = \varepsilon_r \varepsilon_0 \frac{A}{z} \]
  - \( \varepsilon_0 \) – dielectric constant for vacuum (8.85 pF/m)
  - \( \varepsilon_r \) – relative dielectric constant

- device usable as sensor
  - changing \( A, z, \varepsilon_r \) changes capacitance
  - change in capacitance can be sensed

- capacitive level sensor changes \( \varepsilon_r \)
  - sensor assumes that \( \varepsilon_r \) is a (material) constant

\[ C_x = \frac{2\pi}{\ln(d_2 / d_1)} \left( \varepsilon_0 H + \varepsilon_0 (\varepsilon_r - 1)h \right) = C_0 (1 + x) \]
Dielectric material

- Dielectric materials are electrical insulators (share pair of electrons) with a high polarizability.
- Dielectric materials have a permanent dipole moment.
- External field aligns molecules (dielectric polarization).
  - Dipoles form electric field opposite to external field.
  - Capacitance depends on properties of dielectric material.

- Relative dielectric constant depends on:
  - Material (air, water, ...)
  - Temperature.
Dielectric material

- example – temperature sensor based on barium titanate (BaTiO$_3$)
- relative permittivity of ferromagnetic materials
  \[ \varepsilon_r = \frac{k}{T - T_c} \]
  - $T_c$ – Curie temperature
  - $k$ – material depend constant

- $\varepsilon_r$ (BaTiO$_3$) = 1250 at 20°C
- $\varepsilon_r$ (BaTiO$_3$) = 10000 at 120°C

- (+) sensitive sensor (TCR = -7%/°C @ 20°C)
- (+) simple to integrate in silicon process
Dielectric material

example – relative humidity (RH) sensor (HC1000)

- relative permittivity
  - $\varepsilon_r$ (air) = 1
  - $\varepsilon_r$ (water) = 88 at 0°C, $\varepsilon_r$ (water) = 55.33 at 100°C
- use dielectric that absorbs and exudes water without hysteresis
- capacitance of the sensor
  $$C = C_{76}[1 + \alpha_{76}(RH - 76)]$$
  - $C_{76} = 500\text{pF}$
  - $\alpha_{76} = (2900) \times 10^{-6}/(\%\text{RH})$

(-) sensitivity 1.40pF/\%RH
(+) linearity error < 1.5\%RH
(+ ) temperature dependency $\Delta RH = -0.003 \cdot RH \cdot (T - 20^\circ C)$
Capacitive sensor

- Capacitance depends on physical properties:
  \[ C = \varepsilon_r \varepsilon_0 \frac{A}{z} \]
  - \( \varepsilon_0 \) – dielectric constant for vacuum (8.85 pF/m)
  - \( \varepsilon_r \) – relative dielectric constant
- Device usable as sensor:
  - Changing \( A, z, \varepsilon_r \) changes capacitance
  - Change in capacitance can be sensed
- Capacitance defined by:
  \[ Q = CV \quad \text{and} \quad I = C \frac{dV}{dt} \]
Capacitor – impedance and admittance

- Impedance (reluctance to charge flow)
  \[ Z = R + jX \]
  - \( R \) – resistance, \( X \) – reactance
- Impedance of a capacitor
  \[ Z = \frac{1}{j\omega C} = \frac{1}{j2\pi fC} \]
- Admittance (conductance)
  \[ Y = \frac{1}{Z} = \frac{1}{R + jX} \]
- Admittance of a capacitor
  \[ Y = j\omega C \]
- Interface circuit measures impedance (through voltage) or admittance (through current)
  \[ V = ZI = \frac{1}{j\omega C} I \]
  \[ I = \frac{V}{Z} = YV = j\omega CV \]
Capacitive sensor – sensitivity

- measure impedance (voltage drop) or admittance (measure current)
  \[ V = \frac{1}{j \omega C} I \quad I = j \omega CV \quad C = \varepsilon_r \varepsilon_0 \frac{A}{z} \]

- sensor is linear when changing \( \varepsilon_r \) or \( A \) while measuring admittance
  - \( C \) linear with respect to \( \varepsilon_r \) and \( A \)
  - sensitivity does not depend on changing parameter
    \[ \frac{dC}{d\varepsilon_r} = \varepsilon_0 \frac{A}{z} \quad \frac{dC}{dA} = \varepsilon_r \varepsilon_0 \frac{1}{z} \]

- sensor is non-linear when changing \( z \)
  \[ \frac{dC}{dz} = \frac{d}{dz} \varepsilon_r \varepsilon_0 \frac{A}{z} = -\varepsilon_r \varepsilon_0 \frac{A}{z^2} = -\frac{C_0}{z} \]
Capacitive sensor

- sensor is non-linear when changing $z$
  \[
  \frac{dC}{dz} = d \varepsilon_r \varepsilon_0 \frac{A}{z} = -\varepsilon_r \varepsilon_0 \frac{A}{z^2} = -\frac{C_0}{z}
  \]

- solution: allow only small displacement of plates
  - total distance between plates ($d + z$)
  - small displacement ($z$) allowed

- capacitance equal to
  \[
  C = \varepsilon_r \varepsilon_0 \frac{A}{d + z} \\
  x = \frac{z}{d}
  \]
  \[
  \Rightarrow C = \varepsilon_r \varepsilon_0 \frac{A}{d(1 + x)}
  \]

- sensitivity
  \[
  \frac{dC}{dz} = -\frac{\varepsilon_r \varepsilon_0 A}{d^2 (1 + x)^2} = -\frac{C_0}{d(1 + x)^2} \approx -\frac{C_0}{d}\left(1 - 2x + 3x^2 - 4x^3 + \ldots\right)
  \]

- non-linear sensitivity (sensitivity depends on $z$ through $x$)
- sensitivity increases when $d$ and $z$ are small (choose small $d$)
- distance $d$ limited by dielectric breakdown (30kV/cm for air)
Capacitive sensor

- sensor is non-linear when changing $z$
  \[
  \frac{dC}{dz} = \frac{d}{dz} \varepsilon_r \varepsilon_0 \frac{A}{z} = -\varepsilon_r \varepsilon_0 \frac{A}{z^2} = -\frac{C_0}{z}
  \]

- alternative: non-linearity improved by adding dielectric
  - two capacitors $C_z$ and $C_0$ in series
    \[
    C = \frac{C_0 C_z}{C_0 + C_z} = \varepsilon_r \varepsilon_0 \frac{A}{(d + \varepsilon_r z)}
    \]
  - sensitivity of series capacitor
    \[
    \frac{dC}{dz} = -\frac{\varepsilon_r \varepsilon_0 A \varepsilon_r}{(d + \varepsilon_r z)^2} = -\frac{\varepsilon_r^2 \varepsilon_0 A}{d^2} \frac{1}{(1 + \frac{\varepsilon_r z}{d})^2}
    \]
    \[
    \approx -\frac{C_0}{d} \varepsilon_r \left[1 - 2\varepsilon_r \frac{z}{d} + 3 \left(\varepsilon_r \frac{z}{d}\right)^2 - \ldots\right]
    \]
  - first term independent of $z$
  - sensor is more linear then sensor without dielectric material
Differential capacitor

- differential sensor with two capacitors
  \[ C_1 = \frac{\varepsilon_r \varepsilon_0 A}{d + z} \quad C_2 = \frac{\varepsilon_r \varepsilon_0 A}{d - z} \]

- voltage drop across capacitors
  \[ V_1 = \frac{1}{j \omega C_1} V_r = \frac{C_2}{C_1 + C_2} V_r \quad V_2 = \frac{C_1}{C_1 + C_2} V_r \]

- substitute capacitor values
  \[ V_1 = \frac{1/(d - z)}{1/(d + z) + 1/(d - z)} V_r = \frac{d + z}{2d} V_r \quad V_2 = \frac{1/(d + z)}{1/(d + z) + 1/(d - z)} V_r = \frac{d - z}{2d} V_r \]

- use differential amplifier to subtract voltages
  \[ V_1 - V_2 = \left( \frac{d + z}{2d} - \frac{d - z}{2d} \right) V_r = \frac{z}{d} V_r \]

- linear relation between displacement (z) and output voltage
Differential capacitor versus single capacitor
Differential capacitor

- differential capacitor with changing area
  \[ C_1 = \varepsilon \frac{w(z_0 + z)}{d} = \varepsilon \frac{w \cdot z_0}{d} \frac{z_0 + z}{z_0} = C_0 \frac{z_0 + z}{z_0} \]
  \[ C_2 = \varepsilon \frac{w(z_0 - z)}{d} = \varepsilon \frac{w \cdot z_0}{d} \frac{z_0 - z}{z_0} = C_0 \frac{z_0 - z}{z_0} \]

- voltage difference
  \[ V_1 = \frac{C_2}{C_1 + C_2} V_r = \frac{(z_0 - z)/z_0}{(z_0 + z)/z_0 + (z_0 - z)/z_0} V_r = \frac{z_0 - z}{2z_0} V_r \]
  \[ V_2 = \frac{C_1}{C_1 + C_2} V_r = \frac{(z_0 + z)/z_0}{(z_0 + z)/z_0 + (z_0 - z)/z_0} V_r = \frac{z_0 + z}{2z_0} V_r \]
  \[ V_1 - V_2 = \left( \frac{z_0 - z}{2z_0} - \frac{z_0 + z}{2z_0} \right) V_r = -\frac{z}{z_0} V_r \]

- linear relation between displacement \((z)\) and output voltage
Differential capacitor

example – capacitive rotation sensor

- two equal sized parallel circular plates separated by an insulator
- one pair of plates acts as rotor, other pair as stator
- sensor is placed in bridge circuit

show that the output voltage $v_o$ is proportional to the angle of rotation $\Theta$

![Diagram of differential capacitor with rotor and stator plates, insulator gap, and bridge circuit](image)
Differential capacitor

example – capacitive rotation sensor

- ¼ overlap when $\Theta = 0$ rad

$$A = \frac{\pi R^2}{4}$$

- capacitance is then equal to

$$C = \frac{\varepsilon_r \varepsilon_0 \pi R^2}{4d}$$

- maximal overlap when $\Theta = \pi/2$ rad
- capacitance $C_1$ and $C_3$ are maximal
- $C_1$ and $C_3$ have ½ overlap, hence

$$C_1 = C_3 = \frac{\varepsilon_r \varepsilon_0 \pi R^2}{4d} \left( 1 + \frac{2\Theta}{\pi} \right)$$

- capacitance $C_2$ and $C_4$ have no overlap at $\Theta = \pi/2$ rad, hence

$$C_2 = C_4 = \frac{\varepsilon_r \varepsilon_0 \pi R^2}{4d} \left( 1 - \frac{2\Theta}{\pi} \right)$$
Differential capacitor

example – capacitive rotation sensor

- capacitance proportional to angle

\[
C_1 = C_3 = \frac{\varepsilon_r \varepsilon_0 \pi R^2}{4d} \left(1 + \frac{2\Theta}{\pi}\right)
\]

\[
C_2 = C_4 = \frac{\varepsilon_r \varepsilon_0 \pi R^2}{4d} \left(1 - \frac{2\Theta}{\pi}\right)
\]

- capacitors in bridge circuit

\[
V_0 = \left(\frac{C_1}{C_1 + C_2} - \frac{C_4}{C_3 + C_4}\right) V_r = \left(\frac{C_1 - C_4}{C_1 + C_2}\right) V_r
\]

\[
= \frac{\left(1 + \frac{2\Theta}{\pi}\right) - \left(1 - \frac{2\Theta}{\pi}\right)}{\left(1 + \frac{2\Theta}{\pi}\right) + \left(1 - \frac{2\Theta}{\pi}\right)} V_r = \frac{2\Theta}{\pi} V_r
\]

- output proportional to angle
Fringe effect – error source

- capacitance of a flat plate capacitor is equal to
  \[ C = \varepsilon_r \varepsilon_0 \frac{A}{d} \]
- only when \( d \ll A \)
- error source (fringe effect)
  - electric field does not end at edge
  - field bends outside the plates
  - real capacitance larger than formula suggests
- reduce fringe effect with outer guard ring at same voltage
Stray capacitance – error source

- only one of two capacitor plates can be grounded
- other plate can form capacitor with any nearby conductor
- **stray capacitance** exists between each pair of conductors
- stray capacitance reduces sensitivity of the sensor
- stray capacitance can be reduced using **shielding**
- shielding creates another capacitor in parallel with the sensor
Stray capacitance – error source

- capacitor has high output impedance
  \[ Z = \frac{1}{j\omega C} \]
- requires
  - processing circuit with high input impedance
  - sensor used at high frequency
- frequency is limited by stray capacitances
- solution: place processing circuit close to sensor
Signal processing

- variable reactance sensors
  - single varying capacitance \((C_0 \pm \Delta C)\)
  - differential capacitance \((C_0 + \Delta C, C_0 - \Delta C)\)

- voltage / current relation for capacitor
  \[
  I = C \frac{dV}{dt} \Leftrightarrow I = j\omega CV
  \]

- AC voltage or current source needed
- typically \(C_0 \sim 100\) pF
- excitation frequency typically between 10 kHz to 100 MHz to get reasonable impedance
Signal processing

- Output voltage of sensor
  \[ v_o = v_1 - v_2 = \frac{z}{d} v_r \]
  \[ v_r = V_r \sin(\omega t) \]  \[ \implies v_o = \frac{z}{d} V_r \sin(\omega t) \]

- Information present in
  - Amplitude (magnitude)
  - Phase shift (direction)
Signal processing

- parallel plate capacitor
  \[ C_x = \varepsilon_r \varepsilon_0 \frac{A}{d(1+x)} = \frac{C_0}{1+x} \]
  - non-linear relation between capacitance and distance

- circuit for linear impedance changes
  - R provides bias current path
    - allows \( C_x \) to discharge
    - impedance \( R \gg \text{impedance } C_x \) at excitation frequency
  - output voltage
    \[ v_o = -\frac{Z_x}{Z} v_e = -\frac{C}{C_0} (1+x) v_e \]
  - linear relation between output voltage and displacement
  - offset voltage present in output
Signal processing

- capacitive level sensor
  \[ C_x = \frac{2\pi}{\ln(d_2 / d_1)} \left( \varepsilon_0 H + \varepsilon_0 (\varepsilon_r - 1)h \right) = C_0 (1 + x) \]

- circuit for linear admittance changes
  - R provides bias current path
  - output voltage
    \[ v_o = -\frac{C_x}{C} v_e = -\frac{C_0}{C} (1 + x)v_e \]
  - linear relation between output voltage and input signal x
  - offset voltage present in output
  - circuit known as charge amplifier
    - response larger compared to measuring voltage drop over \( C_x \)
Signal processing

- stray capacitance influences output signal
- output voltage (ignoring stray capacitance)
  \[ v_o = -\frac{C_x}{C} v_e \]
- stray capacitance \( C_{s1} \) and \( C_{s2} \) do not affect output voltage
  - \( C_{s1} \) in parallel with \( v_e \)
  - both ends of \( C_{s2} \) at same voltage
- output voltage (considering stray capacitance)
  \[ v_o = -\frac{(C_x + C_{s3})}{C} v_e \]
- shielding reduces \( C_{s3} \)