Sensors and Actuators
Introduction to sensors

Sander Stuijk
(s.stuijk@tue.nl)
INDUCTIVE SENSORS

(Chapter 3.4, 7.3)
Inductive sensors
### Sensor classification – type / quantity measured

<table>
<thead>
<tr>
<th>Sensor type</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Position, distance, displacement</td>
</tr>
<tr>
<td>Resistive</td>
<td>Magnetoresistor</td>
</tr>
<tr>
<td></td>
<td>Potentiometer</td>
</tr>
<tr>
<td>Capacitive</td>
<td>Differential capacitor</td>
</tr>
<tr>
<td>Inductive and electro-magnetic</td>
<td>Eddy currents</td>
</tr>
<tr>
<td></td>
<td>Hall effect</td>
</tr>
<tr>
<td></td>
<td>LVDT</td>
</tr>
<tr>
<td></td>
<td>Magnetostriction</td>
</tr>
<tr>
<td>Self-generating</td>
<td>Thermal transport + thermocouple</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>PN junction</td>
<td>Photoelectric sensor</td>
</tr>
</tbody>
</table>

- reactance variation sensors (capacitive and inductive sensors)
- typically require no physical contact
- exert minimal mechanical loading
Magnetic reluctance

- **electrical circuit** may offer resistance to charge flow
  - resistor: $R$
  - resistor dissipates electrical energy
  - current follows path of least resistance
  - total resistance $R_{tot} = R_1 + R_2$

- **magnetic circuit** may offer reluctance to magnetic flux
  - reluctance: $\mathcal{R}$
  - reluctant circuit stores magnetic energy
  - magnetic flux follows path of least reluctance
  - total reluctance computed in similar way as resistance in electrical circuit

$$\mathcal{R}_{tot} = \frac{\mathcal{R}_1}{2} + \frac{\mathcal{R}_2}{2} + \frac{\mathcal{R}_3}{2} + \mathcal{R}_4$$
Magnetic reluctance

- reluctance depends on physical properties of the device

\[ R = \frac{1}{\mu \mu_0 \frac{l}{A}} \]

- \( l \) – length of the device
- \( A \) – cross-sectional area
- \( \mu_0 \) – permeability of free space (4x10^{-7} H/m)
- \( \mu \) – relative permeability of the material
  - “soft” ferromagnetic material (typically 1000 to 10000)
  - permeability of air (approx. 1)

- options to vary reluctance
  - modify length \( l \) (variable gap sensor)
  - modify magnetic permeability \( \mu \) (moving core sensor)
  - modify cross-sectional area \( A \) (not frequently used)
Magnetic reluctance

- reluctance depends on physical properties of the device
  \[ R = \frac{1}{\mu \mu_0} \frac{l}{A} \]

- sensor requires conversion of magnetic signal to electric signal

- Faraday’s law relates magnetic reluctance to electric current
  \[ v = \frac{N^2}{R} \frac{di}{dt} = L \frac{di}{dt} \]
  - change in reluctance changes output voltage
  - self-inductance \( L \) and reluctance are related: \( L = \frac{N^2}{R} \)

- device can also be used as sensor without changing reluctance
  - changing magnetic field causes electrons to move
  - induces additional (eddy) current (eddy current sensor)
Variable gap sensor

- What is the output voltage (in terms of $x$) of a sensor with $N$ windings?

\[
\mathbb{R}_{\text{core}} = \frac{l_{\text{core}}}{\mu_{\text{core}} \mu_0 A}, \quad \mathbb{R}_{\text{object}} = \frac{l_{\text{object}}}{\mu_{\text{object}} \mu_0 A}, \quad \mathbb{R}_{\text{air}} = \frac{x}{\mu_{\text{air}} \mu_0 A}
\]

\[
\mathbb{R}_{\text{total}} = \mathbb{R}_{\text{core}} + \mathbb{R}_{\text{object}} + 2 \cdot \mathbb{R}_{\text{air}}
\]

- Reluctance of core and object are constant

\[
\mathbb{R}_0 = \frac{l_{\text{core}}}{\mu_{\text{core}} \mu_0 A} + \frac{l_{\text{object}}}{\mu_{\text{object}} \mu_0 A}
\]

- Reluctance of the circuit

\[
\mathbb{R}_{\text{total}} = \mathbb{R}_0 + \frac{2x}{\mu_{\text{air}} \mu_0 A} = \mathbb{R}_0 + kx
\]

- Self-inductance of the circuit

\[
L = \frac{N^2}{\mathbb{R}_{\text{total}}} = \frac{N^2}{\mathbb{R}_0 + kx}
\]

- Output voltage of the sensor

\[
v = L \frac{di}{dt} = \frac{N^2}{\mathbb{R}_0 + kx} \frac{di}{dt}
\]
Variable gap sensor

- output voltage of the sensor

\[ v = L \frac{di}{dt} = \frac{N^2}{R_0 + kx \, dt} \frac{di}{dt} \]

- highly non-linear relation between output and displacement \( x \)
- use of sensor limited to proximity sensor
Linear displacement transformer

- two coils in series, moving object
  - increases reluctance in one coil
  - decreases reluctance in other coil
- circuit is differential voltage divider
- impedance of coil is equal to

\[
Z = j\omega L
\]

\[
L = \frac{N^2}{\mathcal{R}} \quad \mathcal{R} = \frac{1}{\mu\mu_0} \frac{l}{A}
\]

\[
\Rightarrow Z = j\omega \frac{N^2}{\mathcal{R}} = j\omega \frac{N^2\mu\mu_0A}{l}
\]

- changing l with a relative amount x

\[
Z = j\omega \frac{N^2\mu\mu_0A}{l(1+x)} = \frac{j\omega L_0}{(1+x)} = \frac{Z_0}{(1+x)}
\]
Linear displacement transformer

- two coils in series, moving object
  - increases reluctance in one coil
  - decreases reluctance in other coil
- circuit is differential voltage divider
- output of the voltage divider
  \[
  v_o = \frac{Z_0 / (1 + x)}{Z_0 / (1 - x) + Z_0 / (1 + x)} v_e = \frac{1 - x}{2} v_e
  \]
  - linear relation between output voltage and displacement
  - offset voltage present
- displacement (x) should be small
- sensor often not practical
Mutual inductance

- **self-inductance**
  - induced voltage due to change in own current
  \[ v = L \frac{di}{dt} \]

- **mutual inductance**
  - induced voltage due to change in current in neighboring circuit
  \[ v_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} \]

- depends on **reluctance** of the space between the coils

- changing reluctance between coils alters mutual inductance
  - device usable as sensor
  - two coil solution still not practical (large offset, small fluctuation)
Linear Variable Differential Transformer (LVDT)

- Two secondary coils in series-opposition
- Linear relation between output voltage and core displacement
- Operation based on mutual inductance
Linear Variable Differential Transformer

- assume sinusoidal excitation of primary circuit
  \[ v_1(t) = V_1 \sin(\omega t) \]
- output voltage of secondary circuit
  \[ v_o(t) = S_\omega \cdot x \cdot V_1 \sin(\omega t + \varphi) \]
  - \( S_\omega \) – sensitivity at frequency \( \omega \)
  - \( x \) – displacement of the core from center
  - \( \varphi \) – phase shift (in voltage) from primary to secondary circuit

- \( S_\omega \) and \( \varphi \) depend on
  - load \( R_L \) of measurement circuit
  - excitation frequency \( \omega \)
- phase shift can be compensated
Signal conditioning for LVDT sensors

- Output signal of LVDT is amplitude modulated AC signal.

\[ v_o (x = x_0) \]
\[ v_o (x = 2x_0) \]
\[ v_o (x = -2x_0) \]

- Amplitude indicates magnitude of displacement.
- Phase indicates direction of displacement.

\[ L_1 \quad v_1 \quad L_2 \quad M_1 \quad v_0 \quad M_2 \quad L'_2 \]

\[ x \]

\[ \text{Output voltage} \]
Linear Variable Differential Transformer

- output voltage (no load connected to secondary winding)
  - no current in secondary circuit ($I_2 = 0$)

$$V_1 = I_1(R_1 + j\omega L_1) \iff I_1 = \frac{V_1}{j\omega L_1 + R_1}$$

$$V_o = I_1(-j\omega M_1 + j\omega M_2) = j\omega(M_2 - M_1)I_1$$

$$\Rightarrow V_o = \frac{j\omega(M_2 - M_1)V_1}{j\omega L_1 + R_1}$$

- primary current $I_1$ independent of core position
- output voltage $V_o$ proportional to core position
Linear Variable Differential Transformer

- output voltage (no load connected to secondary winding)
  \[ V_o = j \omega k_x x I_1 = \frac{j \omega k_x x V_1}{j \omega L_1 + R_1} \]

- sensitivity
  \[ S = \frac{|V_o/V_1|}{x} = \left| \frac{j \omega k_x}{j \omega L_1 + R_1} \right| = \left| \frac{k_x}{L_1 + \frac{R_1}{j \omega}} \right| = \left| \frac{k_x}{L_1 - j \frac{R_1}{\omega}} \right| = \sqrt{\frac{k_x}{L_1^2 + \frac{R_1^2}{\omega^2}}} \]
  sensitivity increases with increasing frequency

- phase shift
  - output voltage 90° out of phase with primary current
  - phase shift between \( V_1 \) and \( V_0 \)
    \[ \varphi = 90° - \arctan \frac{\omega L_1}{R_1} \]
  - consider phase shift when recovering position