Sensors and Actuators
Sensor Physics

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PN-JUNCTION SENSORS

(Chapter 16.5)
## Temperature sensors

<table>
<thead>
<tr>
<th>placement</th>
<th>excitation</th>
<th>physical effect</th>
<th>material</th>
<th>thermal sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>contact</td>
<td>passive</td>
<td>thermal expansion</td>
<td>metal</td>
<td>bimetal</td>
</tr>
<tr>
<td>contact</td>
<td>active</td>
<td>resistive effect</td>
<td>metal</td>
<td>RTD</td>
</tr>
<tr>
<td>contact</td>
<td>active</td>
<td>resistive effect</td>
<td>semiconductor</td>
<td>silicon resistive</td>
</tr>
<tr>
<td>contact</td>
<td>active</td>
<td>resistive effect</td>
<td>polymer or ceramic</td>
<td>thermistor</td>
</tr>
<tr>
<td>contact</td>
<td>passive</td>
<td>thermoelectric effect</td>
<td>conductor</td>
<td>thermocouple</td>
</tr>
<tr>
<td>contact</td>
<td>active</td>
<td>PN junction</td>
<td>semiconductor</td>
<td>silicon bandgap</td>
</tr>
<tr>
<td>non-contact</td>
<td>passive</td>
<td>pyroelectric effect</td>
<td>pyroelectric</td>
<td>pyroelectric</td>
</tr>
<tr>
<td>non-contact</td>
<td>active</td>
<td>ultrasound</td>
<td>-</td>
<td>acoustic</td>
</tr>
</tbody>
</table>
Semiconductor P-N junction sensor

- semiconductor material doped with P and N material
  - P doping
  - N doping

- electrons deplete from N to P material
  - P
  - N

- internal field develops across PN junction which stops recombination

- place large enough (and opposite) external field across PN junction
  - recombination process can continue
  - current flows through PN junction
Semiconductor P-N junction sensor

- semiconductor p-n junction in diodes have temperature dependency
- p-n junction connected to current source

- voltage depends on
  - temperature
  - current

Forward biased (-2mV/°C)
Semiconductor P-N junction sensor

- semiconductor p-n junction in diodes have temperature dependency
- p-n junction connected to current source
- voltage across the P-N junction

\[ V = \frac{kT}{q} \ln\left(\frac{I}{I_0}\right) \]

- \( k \) – Boltzmann’s constant (8.617 \cdot 10^{-5} \text{ eV/K})
- \( q \) – electron charge (1 eV)
- \( I_0 \) – saturation current

\[ I_0 = BT^3 e^{-E_g/kT} \]

- \( E_g \) – band gap energy
- \( B \) – material dependent constant

![Graph showing voltage vs. temperature for semiconductor P-N junction sensor]
Semiconductor P-N junction sensor

- Voltage across the P-N junction
  \[ V = \frac{kT}{q} \ln \left( \frac{I}{I_0} \right) \]

- Saturation current \( I_0 \) has strong temperature dependency
  \[ I_0 = BT^3 e^{-E_g/kT} \]
  \[ V = \frac{kT}{q} \ln \left( \frac{I}{I_0} \right) \]
  \[ \Rightarrow V = \frac{kT}{q} \ln \left( \frac{I}{BT^3} \right) + \frac{kT}{q} \frac{E_g}{kT} \]
  \[ \Leftrightarrow V = \frac{E_g}{q} - \frac{kT}{q} \left[ \ln (BT^3) - \ln (I) \right] \]

- Temperature sensitivity
  \[ \frac{dV}{dT} = -\frac{3k}{q} - \frac{k}{q} \left[ \ln (BT^3) - \ln (I) \right] \]

- Accurate sensor requires constant current
- P-N junction has non-linear temperature dependency
  - Non-linearity is limited in operating range (-50 to +150°C)
Semiconductor P-N junction sensor

- disadvantages of using diodes for temperature measurement
  - stable, temperature independent current source needed
  - building diodes in semiconductor with little variation is difficult
- both issues impact sensitivity and repeatability of device

- solution for second problem: bipolar transistor
  - collector current
    \[ I = I_0 \left( \frac{qV_{be}}{e^{kT}} - 1 \right) - I_{cs} \left( \frac{qV_{cb}}{e^{kT}} - 1 \right) \]
  - connect base to collector \((V_{cb} = 0)\)
    \[ I = I_0 \left( \frac{qV_{be}}{e^{kT}} - 1 \right) \Rightarrow V_{be} = \frac{kT}{q} \ln \left( \frac{I + I_0}{I_0} \right) \]
  - when forward biased \((I_0 \sim 10^{-12} \text{A hence } I \gg I_0)\)
    \[ V_{be} \approx \frac{kT}{q} \ln \left( \frac{I}{I_0} \right) \]
Semiconductor P-N junction sensor

- disadvantages of using diodes for temperature measurement
  - stable, temperature independent current source needed
  - building diodes in semiconductor with little variation is difficult
  - both issues impact sensitivity and repeatability of device

- solution for second problem: bipolar transistor
  - device behaves as a diode
  - using same approach as with diode we find
    \[
    v_{be} = \frac{E_g}{q} - \frac{kT}{q} \left[ \ln(BT^3) - \ln(I) \right]
    \]

  - B depends on
    - geometry
    - doping concentration
  - eliminate B through one calibration point
Semiconductor P-N junction sensor

- calibration: reference output \( V_{be0} \) with collector current \( I_{C0} \) at \( T_0 \)

\[
v_{be} = v_{be} - \frac{T}{T_0} V_{be0} + \frac{T}{T_0} V_{be0} = v_{be} - \frac{T}{T_0} \left[ \frac{E_g}{q} + \frac{kT_0}{q} \ln \left( \frac{I_{C0}}{BT_0^3} \right) \right] + \frac{T}{T_0} V_{be0}
\]

\[
= \frac{E_g}{q} + \frac{kT}{q} \left[ \ln \left( \frac{I}{BT^3} \right) \right] - \frac{T}{T_0} \frac{E_g}{q} - \frac{T}{T_0} \frac{kT_0}{q} \left[ \ln \left( \frac{I_{C0}}{BT_0^3} \right) \right] + \frac{T}{T_0} V_{be0}
\]

\[
= \frac{kT}{q} \left[ \ln \left( \frac{I}{I_{C0} BT^3} \right) \right] - \frac{kT}{q} \left[ \ln \left( \frac{I_{C0}}{I_{C0} BT_0^3} \right) \right] + \frac{T}{T_0} \left( V_{be0} - \frac{E_g}{q} \right) + \frac{E_g}{q}
\]

\[
= \frac{kT}{q} \left[ \ln \left( \frac{I}{I_{C0} \left( \frac{T}{T_0} \right)^3} \right) \right] + \frac{T}{T_0} \left( V_{be0} - \frac{E_g}{q} \right) + \frac{E_g}{q}
\]

- output has non-linear relation with temperature
- output depends on current
Semiconductor P-N junction sensor

- sensitivity of the transistor (assume constant collector current)

\[
\frac{dv_{be}}{dT} \bigg|_{I=I_{c0}} = V_{be0} - \frac{E_g}{q} \frac{k}{T_0} - \frac{k}{q} \left( \frac{T}{T_0} \right)^3 + \ln \left( \frac{T}{T_0} \right)^3
\]

- typical values for silicon
  - sensitivity: -2.2mV/°C
  - non-linearity: 0.34mV/°C
Semiconductor P-N junction sensor

example – diode connected transistor

- $v_{be} = 0.595\text{V}$ at $25^\circ\text{C}$ and sensitivity of $-2.265\text{mV/}^\circ\text{C}$
- collector current $I_0 = 100\mu\text{A}$
- design the circuit to obtain an output from $0\text{V}$ to $10\text{V}$ over the temperature range from $0^\circ\text{C}$ to $100^\circ\text{C}$
Semiconductor P-N junction sensor

example – diode connected transistor

- $v_{be} = 0.595V$ at $25^\circ C$ and sensitivity of $-2.265\text{mV/}^\circ C$
- collector current $I_0 = 100\mu\text{A}$
- design the circuit to obtain an output from 0V to 10V over the temperature range from 0$^\circ C$ to 100$^\circ C$

output voltage

$$v_o = I_0 \cdot R_0 \left(1 + \frac{R_2}{R_1}\right) - v_{be} \frac{R_2}{R_1}$$

- base-emitter voltage

$$v_{be}(T) = 0.595V - \left(2.265\text{mV/}^\circ C\right) (T - 25^\circ C)$$

- two constraints

$$0V = I_0 \cdot R_0 \left(1 + \frac{R_2}{R_1}\right) - \left[0.595V - \left(2.265\text{mV/}^\circ C\right) (0^\circ C - 25^\circ C)\right] \frac{R_2}{R_1}$$

$$10V = I_0 \cdot R_0 \left(1 + \frac{R_2}{R_1}\right) - \left[0.595V - \left(2.265\text{mV/}^\circ C\right) (100^\circ C - 25^\circ C)\right] \frac{R_2}{R_1}$$
Semiconductor P-N junction sensor

example – diode connected transistor

- $v_{be} = 0.595\text{V}$ at $25^\circ\text{C}$ and sensitivity of $-2.265\text{mV/}^\circ\text{C}$
- collector current $I_0 = 100\mu\text{A}$

- **design the circuit to obtain an output from 0V to 10V over the temperature range from 0$^\circ\text{C}$ to 100$^\circ\text{C}$**

- substitute all values

\[
0V = (100\mu\text{A}) \cdot R_0 \left(1 + \frac{R_2}{R_1}\right) - (0.6516V) \frac{R_2}{R_1}
\]

\[
10V = (100\mu\text{A}) \cdot R_0 \left(1 + \frac{R_2}{R_1}\right) - (0.4251V) \frac{R_2}{R_1}
\]

- solving these constraints gives
  - $R_0 = 6372\Omega$, $R_1/R_2 = 44.15$
  - choose $R_1 = 1\text{k}\Omega$, then $R_2 = 44.1\text{k}\Omega$
Semiconductor P-N junction sensor

- Two issues need to be considered when using a P-N junction sensor:
  - Base-emitter voltage has non-linear temperature dependency.
  - Collector current must be constant with time and temperature.

- Solution: use two identical transistors kept at the same temperature.
  - Emitter currents have constant ratio (\(I_{c1}/I_{c2}\) constant).
  - Difference in base-emitter voltages:
    \[
    v_d = v_{be1} - v_{be2} = \frac{kT}{q} \ln \frac{I_{c1}}{I_{01}} - \frac{kT}{q} \ln \frac{I_{c2}}{I_{02}}
    \]
  - Saturation currents are equal:
    \[
    I_{02} \approx I_{01} \Rightarrow v_d = \frac{kT}{q} \ln \frac{I_{c1}}{I_{c2}}
    \]

- Output (\(v_d\)) proportional to temperature when ratio \(I_{c1}/I_{c2}\) is constant.
  - Non-linear temperature dependency removed.
Semiconductor P-N-junction sensor

- Collector currents can be combined into a single current source
  - Combine two transistors with different emitter areas
  - Very simple, but accurate temperature sensor
- $Q_3$ and $Q_4$ are equal and form current mirror

$$I_{c1} = I_{c2} = \frac{I_T}{2}$$

- $Q_2$ consists of 8 transistors in parallel
  - Current density in $Q_1$ is 8 times larger than $Q_2$
Semiconductor P-N-junction sensor

- Collector currents can be combined into a single current source
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\[ I_{c1} = I_{c2} = \frac{I_T}{2} \]

- $Q_2$ consists of 8 transistors in parallel
  - Current density in $Q_1$ is 8 times larger than $Q_2$
  - Voltage difference between $v_{be}$ $Q_2$ and $v_{be}$ $Q_1$
  - Similar to solution on previous slide

\[ V_T = v_{be1} - v_{be2} = \frac{kT}{q} \ln \frac{I_1}{I_{01}} - \frac{kT}{q} \ln \frac{I_2}{I_{02}} \]

- Hence

\[ V_T = \frac{kT}{q} \ln \frac{I_1}{I_2} = \frac{kT}{q} \ln \frac{8I}{1I} = \frac{kT}{q} \ln(8) = T \cdot 179 \mu V/K \]
Semiconductor P-N-junction sensor

- collector currents can be combined into a single current source
  - combine two transistors with different emitter areas
  - very simple, but accurate temperature sensor

- voltage difference between $v_{be} \ Q_2$ and $v_{be} \ Q_1$
  \[
  V_T = \frac{kT}{q} \ln \frac{I_1}{I_2} = \frac{kT}{q} \ln \frac{8I}{1I} = \frac{kT}{q} \ln(8) = T \cdot 179 \ \mu V/K
  \]

- input current
  \[
  I_T = 2I_{c2} = \frac{2V_T}{R}
  \]

- $R$ can be fixed during manufacturing (e.g. $358\Omega$)
- gives following current-temperature relation
  \[
  \frac{I_T}{T} = 1 \mu A/K
  \]
Semiconductor P-N junction sensor

- temperature sensor is placed in series with large resistor (e.g. 10kΩ)
  - current through sensor is temperature dependent
  - resistor translates current to voltage
  - voltage across resistor is measure for temperature

- current-to-voltage cure for different temperatures

![Graph showing current-to-voltage relationship for different temperatures and a circuit diagram with a voltage sensor and resistor.]
Semiconductor P-N junction sensors

example - LM35Z from national semiconductors

- sensor output
  \[ V_{out} = V_0 + aT \]

- sensitivity
  \[ a = 10 \text{mV/}^\circ C \]

- ideal sensor would have no offset voltage
  \[ V_0 = 0 \]

- part-to-part variations cause
  \[ V_0 = \pm 10 \text{mV} \]
  \[ a = [9.9 \text{mV/}^\circ C, 10.1 \text{mV/}^\circ C] \]

- offset error
  \[ \varepsilon = 1^\circ C \]

- non-linearity error
  \[ \varepsilon = \pm 0.1^\circ C \]