

# Latency Minimization for Synchronous Data Flow Graphs\*

A.H. Ghamarian, S. Stuijk, T. Basten, M.C.W. Geilen and B.D. Theelen  
Eindhoven University of Technology, Electronic Systems Group  
a.h.ghamarian@tue.nl

**Abstract.** *Synchronous Data Flow Graphs (SDFGs) are a very useful means for modeling and analyzing streaming applications. Some performance indicators, such as throughput, have been studied before. Although throughput is a very useful performance indicator for concurrent real-time applications, another important metric is latency. Especially for applications such as video conferencing, telephony and games, latency beyond a certain limit cannot be tolerated. This paper proposes an algorithm to determine the minimal achievable latency, providing an execution scheme for executing an SDFG with this latency. In addition, a heuristic is proposed for optimizing latency under a throughput constraint. Experimental results show that latency computations are efficient despite the theoretical complexity of the problem. Substantial latency improvements are obtained, of 24-54% on average for a synthetic benchmark of 900 models, and up to 37% for a benchmark of six real DSP and multimedia models. The heuristic for minimizing latency under a throughput constraint gives optimal latency and throughput results under a constraint of maximal throughput for all DSP and multimedia models, and for over 95% of the synthetic models.*

## 1 Introduction and Related Work

Synchronous Data Flow Graphs (SDFGs, [11, 10]) have been and are being used widely in modeling data flow applications, both sequential DSP applications [2, 14] and concurrent multimedia applications realized on multiprocessor systems-on-chip [12]. The main goal is to provide predictable performance for those applications. Among the performance indicators, throughput is a prominent one; it has been extensively studied in the literature on SDFGs and related models of computation [3, 4, 6, 8, 15, 18]. Other performance indicators are storage requirements and latency. Buffer minimisation for SDFGs has also been studied [5, 15], but latency has so far only been studied for the subclass *homogeneous* SDFGs [14]. Latency is important in interactive applications such as video conferencing, telephony and games, where latency beyond a certain bound becomes annoying to the users. It is in principle possible to compute latency metrics for an SDFG via a conversion to a homogeneous SDFG, which is always possible [14]. However, this conversion might lead to an exponential increase in the number of nodes in the graph, which makes it prohibitively expensive in any SDFG-based design flow aiming at optimizing and predicting performance metrics such as throughput, buffer sizes, and latency [6, 15]. In this paper, we present latency minimization techniques that work directly on SDFGs. One technique can be used to

compute the minimal achievable latency for an SDFG, and it provides an execution scheme that gives the minimal latency. Another, heuristic technique optimizes latency under a throughput constraint. Although it does not always result in the minimal achievable latency, it does give optimal results for all our experiments on six real DSP and multimedia models, and for over 95% of our synthetic models.

The nodes of an SDFG, called *actors*, communicate with *tokens* sent over the edges, called *channels*. Actors typically model application tasks and edges model data or control dependencies. Each time an SDFG actor fires (executes), it consumes a fixed amount of *tokens*, units of control or data, (in a fifo manner) from its input edges and produces a fixed amount of tokens on its output edges. These amounts are called the rates. The production rate of tokens on a channel may differ from the consumption rate on the channel. Therefore, SDFGs are well suited for modeling multi-rate systems. A homogeneous SDFG is an SDFG with all rates equal to one, i.e., it models a single-rate system. If SDFGs are used to analyse the timing behavior of an application, actors are typically annotated with an execution time.

Usually, the dependencies in an SDFG allow some freedom in the execution order of actors. This order determines performance properties like throughput, storage requirements, and latency. An important strength of SDFGs is that they are statically analyzable. This means that the execution order of actors, both for single- and multi-processor implementations, can be fixed at design time via a scheduling scheme, targeting for example minimal buffer sizes or optimal throughput. The class of scheduling schemes providing a static actor execution order is called the class of static order schedulers. An overview of SDFG scheduling techniques can be found in [14].

In this paper, we present a technique to compute the minimal achievable latency between the executions of any two actors in an SDFG. We also present an execution scheme that defines a class of static order schedules that provide minimal latency. Since this scheme may negatively affect throughput, we propose a heuristic to minimize latency under a throughput constraint. We evaluate our schemes in a single-processor context and in a multi-processor context with sufficiently many resources to maximally exploit parallelism, for various buffering schemes. In the multi-processor context, we compare our execution schemes with static order schedules in which all actors fire as soon as they are enabled, called self-timed execution, which is known to provide maximal throughput [14]. In many cases, substantial gains in latency are possible. It further turns out that for all real models and for most synthetic cases minimal latency

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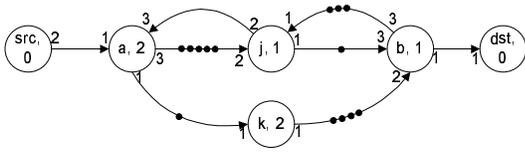


Figure 1. An example SDFG.

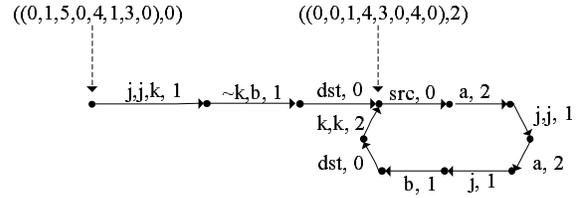


Figure 2. A timed execution of the example SDFG.

and maximal throughput can be achieved simultaneously. We also prove that simultaneously optimal throughput and latency is not achievable in all cases.

The next section provides basic definitions for SDFGs. Section 3 defines a notion of latency for SDFGs, generalizing a definition of latency for homogeneous SDFGs [14]. Section 4 introduces an execution scheme that minimizes latency, while Section 5 presents latency-optimal static order scheduling policies for single-processor systems and for a multi-processor context with sufficient resources to exploit maximal parallelism. Section 6 gives our technique for minimizing latency under a throughput constraint. It also disproves the existence of a general scheduling scheme for achieving simultaneous optimal throughput and latency in all cases by providing a counter example. In Section 7, we experimentally evaluate our techniques. Section 8 concludes. Proofs are omitted and can be found in [7].

## 2 Synchronous Data Flow Graphs

Let  $\mathbb{N}_0 = \{0, 1, \dots\}$  (and  $\mathbb{N} = \mathbb{N}_0 \setminus \{0\}$ ) denote the natural numbers. An SDFG  $G$  is a pair  $(A, C)$  where  $A$  is a set of actors, and  $C \subseteq A^2 \times \mathbb{N}^2$  is a set of channels. Each channel  $(s, d, p, c) \in C$  denotes that actor  $s$  communicates with actor  $d$  where  $p$  and  $c$  are production and consumption rates, respectively. Channels connecting actor  $s$  to some other actor  $d$  are called output channels of  $s$  and input channels of  $d$ . An SDFG where all rates are one is called a homogeneous SDFG (HSDFG).

A (channel) state  $S$  of  $G$  is a mapping  $S : C \mapsto \mathbb{N}_0$  that associates to each channel the number of available tokens on that channel. Each SDFG has an initial state denoted by  $S_0$ , providing the number of initially available tokens on each channel.

To enable performance analysis, an SDFG is annotated with timing information. A *timed SDFG* is a triple  $(A, C, E)$ , with  $(A, C)$  an SDFG and  $E : A \mapsto \mathbb{N}_0$  an execution time mapping that associates to each actor  $a \in A$ , the amount of time  $E(a)$  that it needs for firing, the *execution time of  $a$* .

Figure 1 shows an example of an SDFG. It has six actors  $src, dst, a, b, j, k$ , denoted by circles containing the actor name and its execution time. Arcs represent channels, and are annotated with production and consumption rates. Tokens are depicted as black dots.

The execution of an SDFG is defined based on actor firings, which may change the state of the SDFG, and hence determine the (reachable) state space of the SDFG. An actor  $a \in A$  of SDFG  $(A, C)$  is enabled in state  $S_i$  if  $S_i$  contains at least  $c$  tokens for each input channel  $(s, a, p, c)$  of  $a$ . An

actor can fire when it is enabled. Firing  $a$  changes state  $S_i$  into  $S_{i+1}$ , consuming  $c$  tokens from each input channel  $(s, a, p, c)$  and producing  $p$  tokens on each output channel  $(a, d, p, c)$ .

A timed state of a timed SDFG  $G = (A, C, E)$  is a pair  $(S, \tau)$ , with  $S$  a channel state and  $\tau$  the accumulated time. The initial timed state of  $G$  is  $(S_0, 0)$ . A *timed execution* of  $G$  is a (finite or infinite) sequence of timed states  $(S_0, \tau_0), (S_1, \tau_1), \dots$  where  $\tau_{i+1} \geq \tau_i$ . Each two consecutive timed states correspond to the firing of an actor  $a$  that started its firing in  $\tau_{i+1} - E(a)$  and finishes its firing in  $\tau_{i+1}$ . An execution in which all actors fire as soon as they are enabled is called a *self-timed execution*. Actor firings take time in a timed SDFG which means that they are not atomic. We assume, conservatively, that changes in channel states due to actor firings happen at the end of those firings.

Figure 2 shows a timed execution of the example of Figure 1. A state consists of a pair describing the token distribution over the channels and the accumulated time. The token distribution is given as a tuple, stating the number of tokens per channel, when going through the graph of Figure 1 from left to right and from bottom to top. For clarity, only two states are made explicit. The steps in the execution are labeled with the actor names of active firings, plus the progress of time during that step. For example, in the first step of the execution, actor  $j$  fires two times and actor  $k$  once; time progresses with one time unit. Since  $j$  has an execution time of 1, its firings end upon completion of this step. The firing of  $k$  continues to be active in the next step, which is denoted using a tilde in front of the actor name. It can be observed that the behavior of the example SDFG consists of a transient phase and a periodic phase. It turns out below that the shown execution minimizes the latency between firings of actors  $src$  and  $dst$  (and hence between firings of  $a$  and  $b$ ).

Only SDFGs satisfying the structural property of *consistency* are of interest. Inconsistent graphs either deadlock or need unbounded channel capacities. Consistency can be verified efficiently [11, 1]. A (timed) SDFG  $G = (A, C, E)$  is consistent if and only if it has a non-trivial repetition vector. A repetition vector for  $G$  is a function  $\gamma : A \mapsto \mathbb{N}_0$  such that for every channel  $(s, d, p, c) \in C$ , the so-called balance equation  $p\gamma(s) = c\gamma(d)$  holds. A repetition vector is non-trivial iff it has no zero entries. The smallest non-trivial repetition vector of a consistent SDFG is referred to as *the* repetition vector. An *iteration* is a set of actor firings with as many firings as the repetition vector entry for each actor. The repetition vector  $\gamma$  of the running

example equals  $(src, a, j, k, b, dst) = (1, 2, 3, 2, 1, 1)$ , and the period of the execution shown in Figure 2 consists of precisely one iteration.

### 3 Latency

This section formally defines a notion of latency for timed SDFGs. Generally speaking, latency is the time delay between the moment that a stimulus occurs and the moment that its effect begins or ends. In timed SDFGs, stimuli are actor firings and their effects are the consumptions of produced tokens by some other actors. To define latency, first, we need to define the following.

**Definition 1 [Corresponding Firing]** Let  $a_1, a_2, \dots, a_k \in A$  be actors of a timed SDFG  $(A, C, E)$  on a path  $a_1, a_2, \dots, a_k$  connecting  $a_1$  to  $a_k$ . We say that the  $j_1$ -th firing of  $a_1$  corresponds to the  $j_k$ -th firing of  $a_k$  iff  $j_2$  is the first firing of  $a_2$  which consumes at least one token produced by the  $j_1$ -th firing of  $a_1$ ,  $j_3$  is the first firing of  $a_3$  which uses at least one token produced by the  $j_2$ -th firing of  $a_2$ , and so on. We denote the firing of  $a_k$  corresponding to the  $j_1$ -th firing of  $a_1$  by  $cf(a_1, j_1, a_k)$ .

Note that in general the time that tokens need to travel from some source actor to some destination actor may differ in different firings of the source actor. In an HSDFG, where all production and consumption rates are one, there is a one-to-one correspondence between actor firings of some source and some destination. Because of differing firing rates, this correspondence does not exist, in general, between actors in an SDFG. In order to arrive at a proper definition of latency for SDFGs, we add an explicit source actor to the source of our latency measurement and a destination actor to the intended destination, each of which fires by construction exactly once in every iteration of the graph. If an SDFG already has meaningful input and output actors with repetition vector entries of one, these actors can function as source and destination and no actors need to be added.

**Definition 2 [Latency Graph]** Let  $a, b \in A$  be two actors of a timed SDFG  $(A, C, E)$  with repetition vector  $\gamma$ , and let  $src, dst \notin A$  be two new actors. We define the latency graph for actors  $a$  and  $b$  as  $G_{L(a,b)} = (A_L, C_L, E_L)$ , where  $A_L = A \cup \{src, dst\}$ ,  $C_L = C \cup \{(src, a, \gamma(a), 1), (b, dst, 1, \gamma(b))\}$ , and  $E_L = E \cup \{(src, 0), (dst, 0)\}$ .

The latency between two actors is defined through the latency of different firings of actors  $src$  and  $dst$  in the latency graph. Note that  $src$  and  $dst$  have execution time 0, so that their addition does not influence the timing behavior of the graph. Observe that the example of Figure 1 shows in fact the latency graph for actors  $a$  and  $b$  of the SDFG obtained when omitting the  $src$  and  $dst$  actors. The following proposition shows that there is a one-to-one correspondence between  $src$  and  $dst$  firings (where  $dst$  may have some initial firings without corresponding  $src$  firing)

**Proposition 3** Let  $G_{L(a,b)}$  be some latency graph. The  $k$ -th firing of source actor  $src$  for arbitrary  $k$ , corresponds to

the  $(k + \delta)$ -th firing of  $dst$ , where  $\delta$  is a constant number, i.e., there is a  $\delta \in \mathbb{N}_0$ , such that for all  $k \in \mathbb{N}$ ,  $cf(src, k, dst) = k + \delta$ .

In practice, we are mostly interested in the latency of actors which are considered the input and the output of the system, and these actors often have a repetition vector entry of one already. Furthermore, usually, only executions of complete iterations of graphs are meaningful. Therefore, in case actors have repetition vector entries different from one, we do not look at all firings of those actors. Instead, via the addition of the  $src$  and  $dst$  actors, the latency is defined on the groups of firings of each actor that contain as many firings of the actors as their repetition vector entries.

In the following,  $F_{a,k}^\sigma$  represents the finishing time of the  $k$ -th firing of an actor  $a \in A$  in execution  $\sigma$ . Furthermore, due to resource constraints, such as for example a limited number of processing units, some executions might not be feasible. The set of feasible executions is denoted  $FE$ .

**Definition 4 [Latency]** Let  $a, b \in A$  be two actors of a timed SDFG  $(A, C, E)$  with latency graph  $G_{L(a,b)}$ . The  $k$ -th latency of  $a$  and  $b$  for an execution  $\sigma$  is defined as the time delay between the  $k$ -th firing of  $src$  and its corresponding firing of  $dst$  in  $\sigma$ , and it is denoted by  $L_k^\sigma(a, b)$ :

$$L_k^\sigma(a, b) = F_{dst, cf(src, k, dst)}^\sigma - F_{src, k}^\sigma.$$

The latency of actors  $a$  and  $b$  in execution  $\sigma$ ,  $L^\sigma(a, b)$ , is defined as the maximum  $k$ -th latency of  $a$  and  $b$  for all firings of  $a$ :

$$L^\sigma(a, b) = \max_{k \in \mathbb{N}} L_k^\sigma(a, b).$$

The minimal latency of actors  $a$  and  $b$ ,  $L^{min}(a, b)$ , is defined as the minimum over all feasible executions in  $FE$ :

$$L^{min}(a, b) = \min_{\sigma \in FE} L^\sigma(a, b).$$

Note that this definition implies that latency is measured from the start time of a firing of actor  $a$  (or group of firings), intuitively corresponding to the consumption of some input, to the finishing time of the corresponding (group of) firing(s) of actor  $b$ , intuitively corresponding to the production of output directly related to the consumed input. The current definition is consistent with and generalizes the definition of latency given for HSDFGs in [14]. The latency between actors  $a$  and  $b$  for the execution of Figure 2 of the running example equals 7, being the total delay between the firings of actors  $src$  and  $dst$  in any period.

### 4 Minimum Latency Executions

In [6], a technique to compute throughput of SDFGs based on a state-space traversal is presented, showing that this can be done very efficiently in practice, despite the potentially exponential size of the state space. Therefore, in this section, we introduce an execution scheme to determine the minimal possible latency. As an immediate by-product, we obtain a class of static order schedules that achieve minimum latency. We restrict ourselves to strongly-connected

graphs. In practice, this is not a restriction, because all SDFGs that can be executed within bounded memory can be turned into strongly connected graphs by modeling channel capacity constraints via backward channels [15].

**Definition 5** [Minimum Latency Execution] Let  $G_{L(a,b)}$  be the latency graph of a strongly connected timed SDFG  $G = (A, C, E)$  with actors  $a$  and  $b$ . A feasible execution consisting of the repetition of the following four phases is called a minimum latency execution.

**Phase 1** Execute actors except  $src$  until  $src$  is the only enabled actor. (Note that  $src$  is always enabled because it does not have any inputs.)

**Phase 2** Fire  $src$  once.

**Phase 3** Execute, without any unnecessary delays, the minimum set of required actor firings for enabling  $dst$  for one firing.

**Phase 4** Fire  $dst$  once.

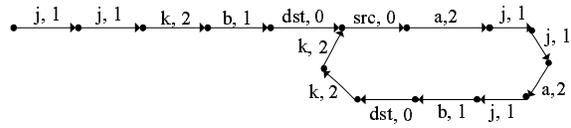
Let  $P_n$  with  $n \in \mathbb{N}$  represent the  $n$ -th execution of the four phases.

Figure 2 shows a minimum latency execution of the running example. In Phases 1 and 3, execution is self-timed (see Sec. 2). Note that the above execution scheme explicitly schedules the  $src$  and  $dst$  actors, which is typically possible for DSP and multimedia applications. Also note that an SDFG may exhibit more executions that realize minimum latency than those defined in Def. 5. However, the defined executions are guaranteed to have minimum latency.

**Proposition 6** Let  $G = (A, C, E)$  be a timed SDFG with  $G_{L(a,b)}$  the latency graph for actors  $a$  and  $b$  in  $A$ . Any minimum latency execution of  $G_{L(a,b)}$  has the following properties.

1.  $P_n$  equals one iteration for all  $n > 1$  and the state reached after Phase 1 is the same for all  $n \geq 1$ .
2. The  $n$ -th firing of  $src$  and its corresponding firing  $cf(src, n, dst)$  of  $dst$  occur in the same  $P_n$ .
3. The set of actor firings between any firing of  $src$  and its corresponding firing of  $dst$  is the smallest possible set among all executions.

Proposition 6 shows that the set of firings in between the designated  $src$  and  $dst$  actors is minimal in any minimum latency execution (prop. 3), that a minimum latency execution is periodic (prop. 1), and that the pairs of corresponding  $src$  and  $dst$  firings that determine the latency always occur in one period (prop. 2). The precise duration of the firings between  $src$  and  $dst$  firings depends on the particular execution. The set of allowed executions may be constrained by the available platform; a single-processor platform, for example, does not allow concurrent execution. If Phase 3 firings are executed within platform constraints without unnecessary delays, the following result follows immediately from Proposition 6.



**Figure 3.** A single-processor minimum latency static order schedule of the example SDFG.

**Theorem 7** [Minimum Latency] Let  $\sigma$  be any minimum latency execution of a latency graph  $G_{L(a,b)}$  taken from the set of feasible executions  $FE$ . Then, we have

$$L^\sigma(a, b) = L^{\min}(a, b).$$

Observe that Proposition 6 proves that a minimum latency execution has a periodic phase consisting of one iteration of the SDFG. An interesting consequence of this is that code size is limited. In general, executions, such as for example self-timed executions that optimize throughput, might have a periodic phase consisting of multiple iterations [6], which implies a larger code size.

Another interesting observation is that the periodic execution of an SDFG allows for a straightforward computation of the *throughput* of an actor, i.e., the average number of firings of the actor per time unit. In the example execution of Figure 2, the throughput of (output) actor  $b$  is  $1/9$ .

## 5 Static Order Scheduling Policies

### 5.1 Single-Processor Scheduling

In the previous section, we have seen that any minimum latency execution leads to a minimum latency between the designated pair of actors. To create a static order schedule for a single processor, it only remains to order the various executions in Phases 1 and 3 of the scheme. If only considering latency, this order can be arbitrary, as long as it satisfies the data and control dependencies specified by the channels in the SDFG. One could decide to try to optimize other constraints such as code size, using for example single appearance scheduling techniques [2, 17]. With respect to throughput, it can be observed that the order in which the individual actors are scheduled in any feasible schedule of a consistent SDFG on a single processor does not impact the average throughput of the application as long as there are no idle periods. Therefore, any minimum latency execution combines minimum latency with the maximal throughput that can be obtained on a single processor.

Figure 3 shows a single-processor static order schedule for our running example that adheres to the minimum latency execution scheme. The latency between actors  $a$  and  $b$  is 8 and the throughput of  $b$  is  $1/12$ . Both latency and throughput are optimal for a single processor. It is interesting to observe that the minimal achievable latency given some arbitrary amount of processing resources is always between the minimal value as defined in Definition 4 under the assumption that all executions are feasible, which gives the limit imposed by the data and control dependencies in the SDFG, and the minimum latency for a single processor.

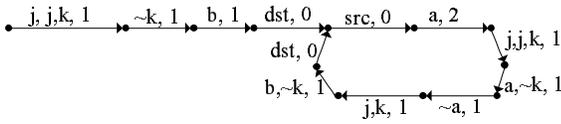


Figure 4. A minimum latency static order schedule using the optimized execution scheme for unlimited resources.

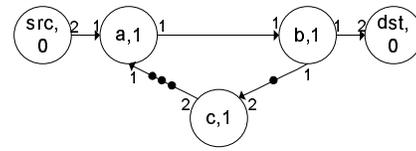


Figure 5. A counter example  $G$  for simultaneously optimizing throughput and latency.

## 5.2 Scheduling with Maximal Parallelism

An interesting case in a multi-processor context, is the case that sufficiently many resources are available to maximally exploit parallelism, or in other words, a context with unlimited processing resources so that any enabled actor can always make progress and all executions are feasible. As mentioned, this allows to determine the minimum achievable latency constrained only by the dependencies in the SDFG. The result can be used as a feasibility check for the application latency in a (multi-processor) design trajectory.

Observe that the crucial point in the 4-phase minimum latency execution scheme is that the actor firings of Phase 1 cannot interfere with the firings in Phase 3. In a single-processor context, this simply means that these two phases have to be executed completely separately. However, in a context with sufficient resources, the two phases can be allowed to execute concurrently, in a self-timed manner (as defined in Sec. 2), because firings of Phase 1 that are executed concurrently with firings of Phase 3 do not interfere with those Phase 3 firings. Furthermore, self-timed execution minimizes the execution time of the critical path of the actor firings in Phase 3. Since also the firing of  $dst$  (Phase 4) can be integrated into this self-timed execution scheme, these observations lead to the following execution scheme.

**Definition 8** [Minimum Latency Execution Scheme with Unlimited Resources]

**Phase 1** Execute actors of the latency graph except  $src$  in a self-timed manner until  $src$  is the only enabled actor.

**Phase 2** Fire  $src$  once, and repeat.

This scheme suggests a concrete multi-processor static order schedule that simply schedules the actor firings in the two phases of this minimum latency execution scheme iteratively in a self-timed manner. Note that the first execution of Phase 1 might be different from the other executions of Phase 1, so that the resulting static order schedule still has a transient part and a periodically repeated part. Figure 4 shows a latency-optimal static order schedule adhering to this scheme. It uses the same conventions as those used for Figure 2. The latency between actors  $a$  and  $b$  is 7, which is of course the same latency as in the execution of Figure 2 which was already optimal given the dependencies inherent in the SDFG. The advantage of the new execution scheme shows in the improved throughput. The throughput of actor  $b$  in the execution of Figure 4 is  $1/7$ , whereas it is  $1/9$  in the execution of Figure 2.

	1	2	3	4	5	6	7	8
src			dst	c	src	dst	src	dst
a	a	b		a	b		a	b
b	a	b		a	b		a	b

Figure 6. Minimal latency execution of  $G$ .

	1	2	3	4	5	6	7	8
(1)	src		src	dst	src	dst		dst
a	a	b	c	a	b	c	a	b
b	a	b		a	b		a	b
(2)	src	dst	c	src	dst	src	dst	
a	a	b		a	b		a	b
b	a	b		a	b		a	b

Figure 7. Maximal throughput execution of  $G$ .

## 6 Throughput Constraints

A multimedia application is often subject to multiple performance constraints such as latency, throughput and memory usage. So far, we have seen several scheduling policies for obtaining minimum latency. The single-processor policy achieves also the maximum throughput since it fully utilizes the only processing unit. As mentioned, the schedule could be further optimized for code size.

The maximum parallelism policy, as the other policies, disallows overlap between multiple iterations of the SDFG. This has a positive effect on code size, but it potentially influences throughput negatively. By allowing simultaneous firings of the source actor in Phase 1 of Definition 8, multiple iterations of the SDFG execution can be scheduled in parallel, which may lead to a higher throughput [9, Sec. 3.4]. However, this might have a negative effect on latency. In fact, we have the following proposition.

**Proposition 9** [Latency and Throughput Optimization] Given an arbitrary SDFG  $G$ ; assume all executions are feasible.  $G$  does not necessarily have an execution that simultaneously minimizes latency and maximizes throughput.

Figure 5 shows an example SDFG  $G$  for which it is not possible to simultaneously optimize latency and throughput. The minimal latency that can be obtained for  $G$  is 2. The minimal latency execution obtained via Definition 8 is shown in Figure 6 in a tabular form.

Figure 7 shows the self-timed execution of this example (split in two parts, as explained later), with the exception that actor  $src$  fires only when actor  $a$  needs tokens for firing. Self-timed execution is known to give maximal achievable throughput [14]. The firings of  $src$  in Figure 7 are scheduled in such a way that they do not constrain throughput, so the execution in Figure 7 achieves maximal throughput. For example, the throughput of actor  $b$  is  $4/3$  firings per time unit. We see that the latency of the execution is 5 (due to the  $src$  and  $dst$  firings in part (1) of the execution).

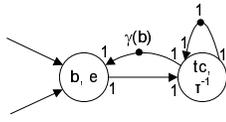


Figure 8. Modeling a throughput constraint.

The self-timed execution of  $G$  can be divided into two parts. Suppose we color the first token on channel  $c-a$  and the token on  $b-c$  blue and the second and third token on channel  $c-a$  red. This coloring implies that firings of actor  $c$  always consume and produce tokens of one color. In Figure 7, (1) and (2) correspond to actor firings involving blue and red tokens on channels  $a-b$ ,  $b-c$ , and  $c-a$  respectively.

The minimal latency execution of Figure 6 follows the schedule of part (2) in Figure 7, i.e., all tokens are processed according to the red scheme. Throughput of  $b$  in the execution of Figure 6 is  $2/3$  firings per time unit. This is the maximum that can be achieved without executing multiple iterations of  $G$  concurrently. (Note that an iteration of  $G$  consists of one firing of  $src$ ,  $dst$ , and  $c$ , and two firings of  $a$  and  $b$ .) However, executing multiple iterations concurrently implies that tokens are necessarily processed according to the blue scheme, part (1), of Figure 7 (or an even slower scheme). This implies that increasing throughput necessarily leads to a higher latency, proving Proposition 9.

A consequence of Proposition 9 is that it is interesting to explore throughput and latency trade-off under the maximal parallelism assumption. In the remainder of this section, we propose a heuristic execution scheme that attempts to minimize latency under a given throughput constraint. That is, the algorithm tries to schedule the SDFG in such a way that the throughput constraint is met, while latency is minimized. If the SDFG is inherently too slow to meet the throughput constraint, the algorithm returns a schedule with maximum throughput and a minimized latency.

An important observation is that a throughput constraint can be modeled in an SDFG, see Figure 8. Assume we want to impose a throughput constraint of  $\tau$  firings per time unit on a designated actor  $b$ , with repetition vector entry  $\gamma(b)$ . This can be achieved by adding a fresh actor  $tc$  to the SDFG with a self-loop containing one token to avoid simultaneous firings of  $tc$  and with an execution time of  $\tau^{-1}$ . By adding two channels between  $b$  and  $tc$  as shown in the figure,  $tc$  on the long run prohibits  $b$  to fire more often than  $\tau$  times per time unit. Note that the rest of the SDFG might slow down  $b$  even more than  $tc$ , so the realized throughput for  $b$  could be lower than  $\tau$ . The number of tokens in the  $tc-b$  channel determines how much short-term deviation in  $b$ -s throughput is allowed. This jitter may influence the minimal achievable latency from any source actor to  $b$ . We have chosen to initialize the channel with  $\gamma(b)$  tokens, which provides maximal firing freedom for  $b$  within the scope of a single iteration. One initial token would disallow any jitter, but might effect latency negatively.

A throughput constraint can be added to the sink actor of a pair of actors for which latency needs to be minimized. This results in the throughput-constrained latency graph.

**Definition 10** [Throughput-constrained Latency Graph] Let  $G_{L(a,b)} = (A_L, C_L, E_L)$  be the latency graph of some SDFG  $G = (A, C, E)$  with actors  $a, b \in A$  and with repetition vector  $\gamma$ . Let  $\tau$  be a throughput constraint on actor  $b$ , and let  $tc \notin A$  be a new actor. We define the  $\tau$ -constrained latency graph for actors  $a$  and  $b$  as  $G_{L(a,b,\tau)} = (A_\tau, C_\tau, E_\tau)$ , where  $A_\tau = A_L \cup \{tc\}$ ,  $C_\tau = C_L \cup \{c_0 = (b, tc, 1, 1), c_1 = (tc, b, 1, 1), c_2 = (tc, tc, 1, 1)\}$ , and  $E_\tau = E_L \cup \{(tc, \tau^{-1})\}$ . The initial state  $S_0$  for the new channels  $c_0, c_1, c_2$  is defined as follows:  $S_0(c_0) = 0$ ,  $S_0(c_1) = \gamma(b)$ , and  $S_0(c_2) = 1$ .

Given a throughput-constrained latency graph, the goal of minimizing latency under the throughput constraint reduces to minimizing latency while maintaining maximal throughput of the throughput-constrained SDFG. As mentioned, maximal throughput can be achieved via self-timed execution. The algorithm presented below essentially performs a self-timed execution, except that the firings of the designated actor  $src$  are delayed. The idea is that latency is minimized by scheduling the firing of  $src$  precisely the minimum achievable latency number of time units before the  $dst$  firing times in self-timed execution. The algorithm does not change the average number of firings over time of any actor in the graph, although it may delay some firings over time. In other words, the maximal throughput of entirely self-timed execution is maintained, but dependencies in the graph may cause the  $dst$  actor to fire at a different moment in time in the schedule produced by the algorithm when compared to the self-timed execution. Consequently, the latency need not be equal to the minimal achievable latency.

**Algorithm** *optimizeThroughputLatency*( $G_{L(a,b,\tau)}$ )

**Input:** A  $\tau$ -constrained latency graph  $G_{L(a,b,\tau)}$  of a strongly connected SDFG  $G$ .

**Output:** “A schedule with maximal throughput (under constraint  $\tau$ ) and (close to) minimal latency”

1. Calculate  $L^{min}(a, b)$  from the execution defined in Def. 8.
2. Execute  $G_{L(a,b,\tau)}$  in the self-timed manner, and store the time of all the firings of actor  $dst$ .
3. Execute  $G_{L(a,b,\tau)}$  as follows
  - Fire all actors but  $src$  if they are enabled.
  - Fire  $src$  (which is always enabled) if the time is  $L^{min}(a, b)$  earlier than the time stored in Line 2 for the corresponding  $dst$  firing.

**return** The schedule obtained from the execution of Line 3.

**Theorem 11** [Maximal Throughput] The schedule returned by algorithm *optimizeThroughputLatency* achieves maximal throughput under the given constraint  $\tau$ .

Figure 9 illustrates algorithm *optimizeThroughputLatency* for the running example. The aim is to achieve maximal throughput. In this case, it is not necessary to explicitly model a throughput constraint in the graph. Figure 9(a)

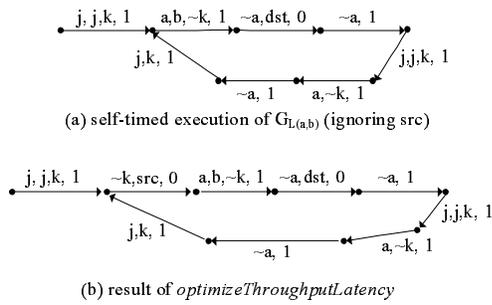


Figure 9. Optimizing latency under maximal throughput.

shows the self-timed execution of the latency graph, ignoring actor *src* (which in principle can fire infinitely often in zero time at time 0 providing unlimited tokens for actor *a*). This execution is known to provide the maximal throughput for *b*, which is  $1/6$ . Actor *dst* fires at times  $2 + 6n$  for every  $n \in \mathbb{N}_0$ . The first of these firings does not need a firing of *src* and can therefore be ignored for latency purposes. Figure 9(b) shows the output of *optimizeThroughputLatency*. Actor *src* is scheduled at times  $1 + 6n$ , i.e., 7 time units (the minimum latency) before every *dst* firing in the self-timed execution except the first one. The result is a schedule that follows self-timed execution, with the *src* actor appropriately inserted. It achieves the minimal achievable latency of 7 and the maximal throughput of  $1/6$ . For each *src* firing, the latency spans the duration till the second subsequent *dst* firing, i.e., the latency exceeds the length of one period.

## 7 Experimental Results

In this section, we evaluate our scheduling schemes. In case of the single processor scheme, static order schedules with an arbitrary order of the concurrently enabled actors are used as a reference point. In the maximal parallelism scenario, the latency and throughput of the schemes of Definition 8 and of algorithm *optimizeThroughputLatency* are compared with those of the self-timed execution. Since truly arbitrary single processor static order schedules can have a very poor latency, for each SDFG in the experiment, the generated static order schedules were constrained allowing only a single iteration of the SDFG in the periodic part of the schedule, and 100 different static orders were tested, choosing the best result.

We created a benchmark containing six real DSP and multimedia models and three sets of 300 synthetic SDFGs, generated using the SDF<sup>3</sup> tool [16]. The first set is composed of arbitrary strongly connected SDFGs. The second set contains graphs in which the dedicated storage capacity for channels is set to the minimum allowing non-zero throughput (computed via techniques from [15]). The third set contains SDFGs in which the buffer sizes for channels are set to the minimum which is enough to obtain the maximal achievable throughput [15]. All experiments were performed on a P4 PC running at 3.4Ghz.

Table 1 shows results for optimal latency single-processor schedules and the randomly generated static or-

Table 1. Results: synthetic, single-processor.

	Min Lat	Arbitr. Order
Strongly Conn. Graphs	4.40	9.65
Min Buff./Throughput	1.36	1.83
Max Throughput	1.31	2.10

Table 2. Results: synthetic, maximal parallelism.

	Latency	Throughput	Execution Time[ms]
Strongly Connected Graphs			
Min latency	1	0.68	5.04
Self-timed	1.43	1	6.29
optThrLat	1.12	1	15.44
Minimum Buffers and Throughput			
Min latency	1	0.90	4.71
Self-timed	1.54	1	6.34
optThrLat	1.10	1	14.14
Maximal Throughput			
Min latency	1	0.78	3.15
Self-timed	1.31	1	3.06
optThrLat	1.01	1	8.73
optThrLat min latency-max throughput: 858/900 (95.3%)			

der schedules. The latency entries are averaged over the entire set of models and normalized wrt minimal achievable latency (without the single-processor constraint). Minimum latency execution improves latency between 26% and 54%. Recall that throughput is the same for both techniques.

Table 2 shows, for the synthetic graphs and the maximal parallelism scheme, the latency, throughput and execution time results of minimum latency execution (Def. 8), self-timed execution and latency optimized maximal throughput execution (Alg. *optimizeThroughputLatency*). All entries show the average numbers taken over all 300 graphs of each set. The entries for latency are normalized wrt the results of the minimum latency schedule and the throughput entries are normalized wrt the throughput achieved in the self-timed execution (i.e., the maximal achievable throughput). The self-timed schedule has a 31-54% higher latency than the minimum latency execution. In other words, minimum latency execution gives a latency reduction of 24-35% compared to self-timed execution. The price to be paid is a decrease in throughput of 10-32%. The latency optimized maximal throughput execution reduces the latency with 22-29% wrt self-timed execution, while guaranteeing maximal throughput. The achieved latency is close to the minimally achievable latency (within 10% on average). In over 95% of the graphs the result combines minimal achievable latency with maximal achievable throughput. The average execution time for a single SDFG for any of the scheduling algorithms is a few milliseconds.

We also experimented with SDFG models of actual DSP and multimedia applications. DSP domain applications are

Table 3. Results: DSP and multimedia benchmark.

	Lat (avg/worst)	Thr (avg/worst)	Exec Time[ms]
Single-processor results			
Min latency	1/1	1/1	3316
Random-order	1/1.03	1/1	3963
Minimum Buffers and Throughput			
Min latency	1/1	0.67/0.34	7339
Self-timed	1.11/1.36	1/1	38262
optThrLat	1/1	1/1	80410
Maximal Throughput			
Min latency	1/1	0.56/0.32	7050
Self-timed	1.11/1.59	1/1	4840
optThrLat	1/1	1/1	10339
optThrLat min latency-max throughput: 6/6 (100%)			

a modem and a sample-rate converter from [2], a channel equalizer, and a satellite receiver [13]. For the multimedia domain, we used an MP3 and an H.263 decoder from [15]. Table 3 shows the results, giving both average and worst-case values for latency and throughput.

The single-processor experiments show only a small latency improvement in one case. Due to the limited parallelism in the graphs, 100 randomly generated static order schedules was in five out of six cases sufficient to achieve optimal results.

Under the maximum parallelism scheme, we considered the application models both with minimal buffers for non-zero throughput and minimal buffers for maximal throughput. The average latency improvement of minimum latency execution wrt self-timed execution is 10%, at a throughput loss of 33-44% on average. The satellite receiver, the modem, and the H.263 decoder do not show any improvement. The channel equalizer (26%, minimal buffers) and the MP3 decoder (37%, maximal throughput) show the largest latency improvements (but also the largest throughput loss). However, applying algorithm *optimizeThroughputLatency* to achieve optimal latency for maximal throughput, achieves maximal throughput and minimal latency simultaneously in all cases. Execution times confirm the feasibility of the proposed techniques.

To test the hypothesis expressed in the introduction that latency optimization via a conversion to homogeneous SD-FGs is often infeasible, we applied our techniques also to the HSDFG equivalents of our DSP and multimedia models. In two cases (satellite receiver, H.263 decoder), self-timed execution of Phase 2 of minimum latency execution (Def. 5), which in essence for HSDFGs is a critical path analysis taking into account parallel and pipelined execution that any potential HSDFG-based latency optimization technique has to perform, takes several hours. This indeed renders HSDFG-based techniques prohibitively expensive in these cases.

## 8 Conclusions and Future Work

We have presented a technique to compute the minimum latency that can be achieved between firings of a designated pair of actors of some SDFG. The technique is based on an execution scheme that guarantees this minimum latency. We presented static-order schedules for single-processor platforms, and for a multi-processor context with sufficient resources to maximally exploit the available parallelism in an SDFG. The latter can be used as a feasibility check for application latency in any multi-processor design trajectory. The experimental evaluation shows that the latency computations and the underlying execution schemes are efficient. Compared to traditional scheduling techniques and execution schemes, substantial reductions in latency can be obtained, sometimes at the price of other performance metrics such as throughput. We showed that it is not always possible to simultaneously optimize latency and throughput. Therefore, we also presented a heuristic for optimizing latency under a throughput constraint. The heuristic gives optimal results for both latency and throughput simultaneously for

all our real DSP and multimedia models, and for over 95% of our synthetic models. Future work includes the development of scheduling schemes for concrete multiprocessor platforms without exploiting maximal parallelism, either because insufficient resources are available or because inter-processor communication is expensive. We also plan to investigate the trade-offs between latency, throughput, code size and storage requirements in more detail.

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