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Adaptive Switching Controllers for Systems with Hybrid Communication Protocols

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Abstract— **Cyber-Physical Systems (CPS) are ubiquitous, motivated by the need to integrate control, computing, and communication, increasing capabilities and declining costs of the underlying technologies, and increased requirements on safety, reliability, and performance. The focus of this paper is the co-design of hybrid communication protocol and adaptive switching control that enables stabilization of multiple control applications using minimal communication resources. The hybrid protocol chosen is one that switches between time-triggered and event-triggered methods in order to realize their combined advantages. A control algorithm is co-designed along with this hybrid protocol so as to exploit the properties of the individual protocols as well as the switches between them. The controller is chosen to be adaptive in order to cope with any uncertainties that may be present. The resulting adaptive, switching controller is shown to be stable in the presence of a class of disturbances.**

I. INTRODUCTION

Of late, there has been significant interest in cyber-physical systems that have a high degree of adaptability, autonomy, efficiency, functionality, reliability, safety, and usability. One area that has witnessed significant activity is the co-design of control and communication architectures in a CPS with a goal of realizing efficient, reliable, high performance systems [1–4]. Our focus in this paper is on the design of a switching adaptive controller that is directly aligned with the design of a hybrid communication protocol such as the TTCAN [5] and FlexRay [6]. The latter is considered to be the future de-facto standard for automotive communication systems.

Time-triggered and event-triggered protocols are two common methods for scheduling messages in a distributed embedded system [7]. Communication activities in event-triggered protocols are triggered by the occurrence of specific events, whereas time-triggered protocols schedule communication activities at predetermined time windows, which are commonly referred to as *slots*. Associated with each of these communication protocols are different set of advantages and disadvantages. The assignment of time-triggered slots to all control-related signals has the advantage of high quality of control (QoC) due to the possibility of reduced or zero delays, but leads to poor utilization of the communication bandwidth, high cost, overall inflexibility, and infeasibility as the number of control applications increase. On the other hand, event-triggered schedules often result in poor control performance due to the unpredictable temporal behavior of control messages and the related large delays which occurs due to the lack of availability of the bus. These imply that a hybrid protocol that suitably switches between these two schedules offers the possibility of exploiting their combined advantages of high QoC, efficient resource utilization, and low cost. Such a hybrid protocol is the focus of this paper.

While several papers have considered control using TT protocols (see for example, [8, 9]) and ET protocols (see for example, [10, 11]), control using hybrid protocols has not been studied in the literature. In [2, 12–14], the co-design of control in CPS has been addressed, with particular focus on the design of scheduling policies for effective Quality of Control (QoC). Control theoretic principles based on linear systems, feedback control, and optimization are used to determine parameters such as sampling period and resource allocation so as to maintain both an efficient control performance and CPS utilization. In [12], the schedulability analysis of real-time tasks with respect to the stability of control functions is discussed. In [13], modeling the real-time scheduling process as a dynamic system, an adaptive self-tuning regulator is proposed to adjust the bandwidth of each single task in order to achieve an efficient CPS utilization. The focus of most of the papers above are either on a simple platform or on a single processor. A good survey paper on co-design can be found in [15]. Unlike the above papers, we focus on a hybrid protocol in this paper.

Our concern is the design of a controller in the presence of a hybrid protocol that switches between a TT and a ET scheme. The controller is to be designed for multiple control applications, each of which is subjected to a parametric uncertainty. An adaptive methodology is introduced to accommodate these uncertainties. Together with the presence of a hybrid protocol, the resulting controller is an adaptive, switching kind. The main result of this paper is the demonstration that this switching adaptive controller is stable.

The main challenges in demonstrating the stability of the underlying switching adaptive controller is the fact that the underlying plant-model has varying delays which, in turn, are due to the hybrid use of the TT and ET protocols. This in turn causes the use of existing results in switched and tuned

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adaptive systems (for example, [16–18]) inadequate. The proof that we provide directly makes use of the underlying properties of the closed-loop system with the TT and ET protocols and the construction of a single Lyapunov function that decreases during each of these protocols together with a bounded jump during the switches.

In Section II the problem is formulated. In Section III, the switching adaptive controller is designed and the main result is proved. Concluding remarks are presented in section V. Finally, the main result is illustrated with a numerical example in Section IV.

II. Problem Formulation

A. The plant model

The problem that we address in this paper is the simultaneous stabilization of n plants, C_i, $i = 1, \ldots, n$, in the presence of impulse disturbances that occur sporadically, using a hybrid communication protocol. We assume that each of these n applications have the following problem statement.

The application to be controlled, denoted as a *plant*, is assumed to have a discrete time model described by

$$\begin{aligned}\mathcal{C}_i:\ y(k) = &-\sum_{l=1}^{m_1} a_l y(k-l) \\ &+ u(k-d) + \sum_{l=1}^{m_2} b_l u(k-l-d) + D(k-d) \quad (1)\end{aligned}$$

where $u(k)$ and $y(k)$ are the input and output of the i-th control application, respectively, at the time-instant t_k and $d \geqslant 1$ is a time-delay. The disturbance $D(k)$ are assumed to be impulses that can occur occasionally with their inter-arrival time lower-bounded by a finite constant. The parameters of the i-th plant are given by a_l, $l = 1, \ldots, m_1$, $b_l, l = 1, \ldots, m_2$ and are assumed to be unknown but identical for all i. It is further assumed that the sampling time of the controller is a constant h, so that $t_{k+1} = t_k + h$. The goal is to choose the control input u such that $y(k)$ tends to zero, with all signals remaining bounded.

The model in (1) can be expressed as

$$A(q^{-1})y(k) = q^{-d}(B(q^{-1})u(k) + D(k)); \qquad k \geq 0 \quad (2)$$

where q^{-1} is the backward shift operator and the polynomials A and B are given by

$$A(q^{-1}) = 1 + \sum_{l=1}^{m_1} a_l q^{-l} \qquad B(q^{-1}) = 1 + \sum_{l=1}^{m_2} b_l q^{-l} \quad (3)$$

The following assumptions are made regarding the plant poles and zeros:

Assumption 1: 1) An upper bound for the orders of the polynomials in (3) is known and 2) all zeros of $B_i(q^{-1})$ lie strictly outside the closed unit disk.

As different values for the time-delay d occur in our setup, it is advantageous to express (1) in a *predictor form* [19]:

$$y(k+d) = \alpha(q^{-1})y(k) + \beta(q^{-1})u(k) + D(k) \quad (4)$$

with

$$\begin{aligned}\alpha(q^{-1}) &= \alpha_0 + \alpha_1 q^{-1} + \ldots + \alpha_{m_1-1} q^{-(m_1-1)} \\ \beta(q^{-1}) &= F(q^{-1})B(q^{-1}) \\ &= 1 + \beta_1 q^{-1} + \ldots + \beta_{m_2+d-1} q^{-(m_2+d-1)}\end{aligned} \quad (5)$$

where $F(q^{-1})$ and $\alpha(q^{-1})$ are the unique polynomials that satisfy the equation

$$1 = F(q^{-1})A(q^{-1}) + q^{-d}\alpha(q^{-1}). \quad (6)$$

Equation (4) can be expressed as

$$\begin{aligned}y(k+d) &= \theta_d^{*T}\Phi_d(k) + D(k) \quad (7)\\ &= \vartheta_d^*\phi_d(k) + u(k) + D(k) \quad (8)\end{aligned}$$

where $\phi_d(k)$, ϑ_d^*, $\Phi_d(k)$, and θ_d^* are defined as

$$\phi_d(k) = \begin{bmatrix} y(k) \\ \vdots \\ y(k-m_1+1) \\ u(k-1) \\ \vdots \\ u(k-m_2-d+1)\end{bmatrix} \qquad \vartheta_d^* = \begin{bmatrix} \alpha_0^d \\ \vdots \\ \alpha_{m_1-1}^d \\ \beta_1^d \\ \vdots \\ \beta_{m_2+d-1}^d \end{bmatrix} \quad (9)$$

$$\Phi_d(k) = \begin{bmatrix}\phi_d(k) \\ u(k)\end{bmatrix} \text{ and } \theta_d^* = \begin{bmatrix}\vartheta_d^* \\ 1\end{bmatrix} \quad (10)$$

with $\phi_d(k) \in \mathbb{R}^{m_1+m_2+d-1}$, $\vartheta_d^* \in \mathbb{R}^{m_1+m_2+d-1}$, $\Phi_d(k) \in \mathbb{R}^{m_1+m_2+d}$, $\theta_d^* \in \mathbb{R}^{m_1+m_2+d}$, and $\alpha_j^d, j = 0, \ldots, m_1-1$ and $\beta_j^d, j = 1, \ldots, m_2+d-1$ the coefficients of the polynomials in (5) with respect to the delay d and finite initial conditions

$$\begin{aligned} y(k-i) &= y_0(i) \quad i = 0, \ldots, m_1-1, \\ u(k-i) &= u_0(i) \quad i = 1, \ldots, m_2+d-1. \end{aligned} \quad (11)$$

From Eqs. (7)-(10), we observe that a feedback controller of the form

$$u(k) = y_{\text{ref}}(k+d) - \vartheta_d^{*T}\phi_d(k) \quad (12)$$

realizes the objective of stability and follows the desired bounded trajectory $y_{\text{ref}}(k)$ in the absence of disturbances. Since our goal is stabilization, we set $y_{\text{ref}}(k)$ to zero. Designing a stabilizing controller $u(k)$ essentially boils down to a problem of implementing (12) with the controller gain ϑ_d^*. Two things should be noted: (i) Controller (12) is not realizable as ϑ_d^* is not known, and (ii) the dimension of $\phi_d(k)$, ϑ_d^* as well as the entries of ϑ_d^* depend on the delay d.

B. Design of the basic adaptive controller

Consider the plant in (1) which can be considered, equivalently, in a predictor form as in Eq. (4). Then, by using the certainty equivalence principle, we can derive the following adaptive control input:

$$u(k) = y_{\text{ref}}(k+d) - \bar{\vartheta}_d(k)^T\phi_d(k) \quad (13)$$

where $\bar{\theta}_d(k)$ is the estimate of θ_d^* with an adaptive update law:

$$\bar{\theta}_d(k) = \bar{\theta}_d(k-1) + \frac{\gamma_d \Phi_d(k-d)\varepsilon_d(k)}{1+\Phi_d(k-d)^T\Phi_d(k-d)}, \quad (14)$$

$\Phi_d(k)$ given in (10) and $\varepsilon_d(k) = y(k) - \bar{\theta}_d(k-1)^T\Phi_d(k-d)$.

Theorem 1: Let $D(k) \equiv 0$. Subject to Assumption 1 and given a fixed delay d, the adaptive controller (13) with the update law (14) guarantees that the plant given by (4) follows the reference y_{ref}, i.e., $\lim_{k\to\infty}(y(k) - y_{\text{ref}}(k)) = 0$, and that the sequences $\{\bar{\theta}_d(k)\}$, $\{y(k)\}$ and $\{u(k)\}$ are bounded for all k.

Theorem 1 shows that the adaptive system given by (4), (13), and (14) guarantees global boundedness of the overall adaptive system and convergence of the the tracking error to zero. The reader is referred to Theorem 5.1 in [19] for the proof of Theorem 1.

III. The switching adaptive controller

A. Hybrid Communication Protocols

The focus of this problem is the simultaneous control of several applications for stabilization. That is, the goal is to choose u, the input of the ith control application such that $y(k)$, its output, converges to $y_{\text{ref}}(k)$ which is zero. In the context of the problem under consideration, all control applications are partitioned into a sensor task T_s, a controller task T_c, and an actuator task T_a. We consider a communication protocol where each communication cycle is divided into time-triggered and event-triggered segments. Using *time-triggered* communication schedules, denoted as M_{TT}, applications are allowed to send messages only at their assigned slots and the tasks are triggered synchronously with the bus, i.e., we assume that there is only a negligible communication delay τ due to the finite speed of the bus and hence the delay d in (4) is equal to 1. On the other hand, in an *event-triggered* schedule, denoted as M_{ET}, the tasks are assigned priorities in order to arbitrate for access to the bus. Note that in our setup, multiple control applications share the same bus and hence multiple control messages have to be sent using a common bus and thus the messages might experience a communication delay τ when the higher priority tasks access the event-triggered segment. We choose the event-triggered communication schedules such that the sensor-to-actuator delay τ is within one sample interval, i.e., $0 < \tau \le h$ for the control-related messages and hence the delay d is at most equal to 2. In summary, the delay $d = 1$ if $\mathcal{M}_{\text{Bus}}(k) = M_{\text{TT}}$ and $d = 2$ if $\mathcal{M}_{\text{Bus}}(k) = M_{\text{ET}}$ where $\mathcal{M}_{\text{Bus}}(k)$ denotes the protocol used at time k.

The main idea of this paper is to send control-related messages over the time-triggered protocol whenever the error between the state of the system and the reference signal is above some threshold e_{th} as this guarantees an aggressive control action with minimal communication delay. That is,

$$\mathcal{M}_{\text{Bus}}(k) = \begin{cases} M_{\text{ET}} & \text{if } |y(k) - y_{\text{ref}}| \leqslant e_{\text{th}} \\ M_{\text{TT}} & \text{if } |y(k) - y_{\text{ref}}| > e_{\text{th}}. \end{cases} \quad (15)$$

B. Controller design

The adaptive controller has a switching structure, and is given by

$$\left.\begin{aligned} u(k) &= -\bar{\vartheta}_1(k)^T\phi_1(k) \\ \varepsilon_1(k) &= y(k) - \bar{\theta}_1(k-1)^T\Phi_1(k-1) \\ \bar{\theta}_1(k) &= \bar{\theta}_1(k-1) + \frac{\gamma_1\Phi_1(k-1)\varepsilon_1(k)}{1+\Phi_1(k-1)^T\Phi_1(k-1)} \end{aligned}\right\} \begin{array}{l}\text{if}\\ M_{\text{TT}}\end{array} \quad (16)$$

where $\Phi_1(k)$ is given in Eq. (10), $\bar{\theta}_1(k)$ is the estimation of the controller gains θ_1^* (Eq. 10), and $\gamma_1 \in (0, 2)$.

If $\mathcal{M}_{\text{Bus}}(k) = M_{\text{ET}}$, the adaptive controller is given by

$$\left.\begin{aligned} u(k) &= -\bar{\vartheta}_2(k)^T\phi_2(k) \\ \varepsilon_2(k) &= y(k) - \bar{\theta}_2(k-1)^T\Phi_2(k-2) \\ \bar{\theta}_2(k) &= \bar{\theta}_2(k-1) + \frac{\gamma_2\Phi_2(k-2)\varepsilon_2(k)}{1+\Phi_2(k-2)^T\Phi_2(k-2)} \end{aligned}\right\} \begin{array}{l}\text{if}\\ M_{\text{ET}}\end{array} \quad (17)$$

where $\Phi_2(k)$ is given in Eq. (10), $\bar{\theta}_2(k)$ is the estimation of the controller gains θ_2^* (Eq. 10), and $\gamma_2 \in (0, 2)$.

C. Main Result

The following definitions are useful for the rest of the paper. We denote the instants of time when the switch from TT to ET occurs with k_p, $p = 1, 3, 5, \ldots$, and the instants of time when the switch from ET to TT occurs with k_p, $p = 2, 4, 6, \ldots$. Also, we denote $k'_p := k_p + 1$ for all $p \in \mathbb{N}$. That is, let the sequence of finite switching times $\{k_l\}_{l\in\mathbb{N}}$ be such that for $k \in [k'_{2p}; k_{2p+1}], p \in \mathbb{N}_0$ the TT protocol is applied and for $k \in [k'_{2p+1}; k_{2p}], p \in \mathbb{N}_0$ the ET protocol is applied with switches occurring between $[k_p; k'_p], p \in \mathbb{N}$ (see Figure 1). Theorem 2 contains the main result of the paper.

Theorem 2: Let Assumption 1 be satisfied. Consider the closed-loop adaptive system given by the plant in (4), where $D(k)$ are impulse disturbances with their inter-arrival time greater than T_{dw}, together with the switching adaptive controller in (16) and (17) with the hybrid protocol in (15) and the following parameter estimate selections at the switching instants

$$\bar{\theta}_1(k_p) = 0, \qquad p = 0, 2, 4, \ldots \quad (18)$$

$$\bar{\theta}_2(k_p - 1) = 0, \qquad p = 1, 3, 5, \ldots \quad (19)$$

$$\bar{\theta}_2(k_p) = 0, \qquad p = 1, 3, 5, \ldots, \quad (20)$$

Then there exists a positive constant T_{dw}^* such that for all $T_{dw} \ge T_{dw}^*$, the closed loop system has globally bounded solutions. That is, the sequences $\{y(k)\}$, $\{u(k)\}$, and the parameter estimation error sequences $\{\tilde{\theta}_1(k)\}$ and $\{\tilde{\theta}_2(k)\}$ are bounded for all k. If there are no disturbances, the ET protocol will stay en vogue and the tracking error $y(k) - y_{\text{ref}}(k)$ converges to zero.

Proof of Theorem 2: In what follows, the effect of the impulse disturbances is represented as a jump in the plant output at the instances where the disturbances occur. When the algorithm is in mode M_{TT}, the underlying error equation is given by

$$e_1(k+1) = \left(\vartheta_1^* - \bar{\vartheta}_1(k)\right)^T\phi_1(k) = \tilde{\vartheta}_1(k)^T\phi_1(k). \quad (21)$$

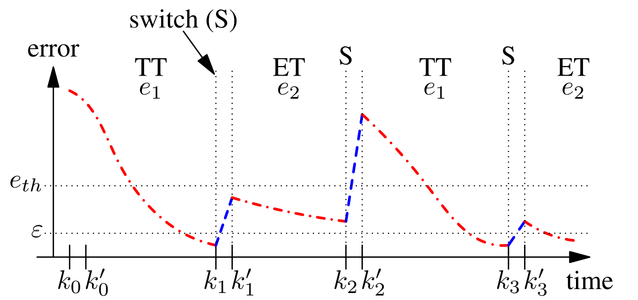

Fig. 1. Schematic evolution of the error with a given sequence of switching times. Impulses in $D(k)$ are assumed to occur at $k_p, p = 0, 2, 4, \ldots$.

with $\tilde{\vartheta}_1(k) = \vartheta_1^* - \bar{\vartheta}_1(k)$. When the system is in mode M_{ET}, the error equation is given by

$$e_2(k+2) = \left(\vartheta_2^* - \bar{\vartheta}_2(k)\right)^T \phi_2(k) = \tilde{\vartheta}_2(k)^T \phi_2(k). \quad (22)$$

with $\tilde{\vartheta}_2(k) = \vartheta_2^* - \bar{\vartheta}_2(k)$.

Define θ_a^* as

$$\theta_a^* = \begin{cases} \begin{bmatrix} \theta_1^{*T} & 0 \end{bmatrix}^T & \text{if } \mathcal{M}_{\text{Bus}}(k) = M_{\text{TT}} \\ \theta_2^* & \text{if } \mathcal{M}_{\text{Bus}}(k) = M_{\text{ET}} \end{cases} \quad (23)$$

Choose Lyapunov function $V(k) = \tilde{\theta}_a(k)^T \tilde{\theta}_a(k)$ where $\tilde{\theta}_a(k) = \theta_a^* - \bar{\theta}_a(k)$. Let $\Delta V(k) = V(k) - V(k-1)$.

For ease of exposition, we divide the proof in several steps.

Step 1-1: *There exists a* $\Delta \in \mathbb{N}$ *such that* $\forall \varepsilon \in]0; e_{th}]$: $|e_1(k_1)| < \varepsilon \leqslant e_{th}$ *where* $k_1 = k_0 + \Delta + m_1$.

Proof of Step 1-1: Let $k = k_0$ when an impulse disturbance occurs, and the system is in mode M_{TT}. It follows that $|e_1(k_0)| = y(k_0) + D_{\max}$ where $D_{\max}$ is an upper bound on $D(k)$. Since $D(k) \equiv 0$ for $k \leq k_0 + T_{dw}^*$, Theorem 1 implies that there exists a $\Delta \in \mathbb{N}$ such that for $k \geqslant k_0 + \Delta$, $|e_1(k)| < \varepsilon \leqslant e_{\text{th}}$. Choose $k_1 = k_0 + T_{dw}^*$, with

$$T_{dw}^* = \Delta + m_1. \quad (24)$$

Then, it follows that no disturbance can occur between k_0 and k_1, and

$$|e_1(k_1 - i)| < \varepsilon \leq e_{\text{th}} \forall 0 \leq i \leq m_1. \quad (25)$$

We note that the same arguments as above can be applied for the interval $[k_{2p}, k_{2p+1}], p = 1, 2, \ldots$. For $k \in [k_1+1; k_2]$ the ET protocol with controller (17) is applied.

Step 1-2: *During M_{TT} (M_{ET}), the error $e_1(k)$ ($e_2(k)$) is bounded.*

Proof of Step 1-2: Theorem 1 ensures that during M_{TT} (M_{ET}), the sequence $\{y(k)\}$ is bounded. Hence, the error $e_1(k)$ ($e_2(k)$) is also bounded during M_{TT} (M_{ET}). $e_2(k)$ ($e_1(k)$), by definition, is bounded during M_{TT} (M_{ET}).

Step 1-3: *There exists a constant $M_1 < \infty$ with $|e_1(k_p')| \leqslant M_1$, for $p = 2, 4, 6, \ldots$.*

Proof of Step 1-3: Let $M_1 = e_{\text{th}} + D_{\max}$. Then, since $|e_1(k_{2p})| \leqslant e_{\text{th}}$ by definition, it follows that

$$|e_1(k_{2p}')| \leqslant M_1 \quad \forall p \in \mathbb{N} \quad (26)$$

Step 2-1: $\forall e_{th} > 0\ \exists \varepsilon_p \in]0; e_{th}]$ *such that* $(|e_1(k_p)| \leqslant \varepsilon_p \Rightarrow |e_2(k_p')| \leqslant e_{th})$ *for* $p = 1, 3, 5, \ldots$

Proof of Step 2-1: By Step 1-1, we know that for all $\varepsilon \leqslant e_{\text{th}}$ there exists a k_1 such that $|e_1(k_1)| < \varepsilon \leqslant e_{\text{th}}$. Now, it needs to be ensured that the jump in the output error is no larger than e_{th} and hence the ET protocol stays en vogue. This is established by showing that (i) there exists a $\varepsilon < e_{\text{th}}$ such that when $|e_1(k_1)| \leqslant \varepsilon$ then $|e_2(k_1')| \leqslant e_{\text{th}}$ and (ii) $|e_2(k_p')| \leqslant e_{\text{th}}$ for $p = 3, 5, \ldots$.

Consider the switch from TT to ET at time k_p', $p = 1, 3, 5, \ldots$. Then,

$$\begin{aligned} |e_2(k_p')| &= \left|\tilde{\vartheta}_2(k_p - 1)^T \phi_2(k_p - 1)\right| \\ &= \left|\left(\vartheta_2^* - \bar{\vartheta}_2(k_p - 1)\right)^T \phi_2(k_p - 1)\right| \end{aligned} \quad (27)$$

Choosing $\bar{\vartheta}_2(k_p - 1)$ according to (19), we get

$$|e_2(k_p')| \leqslant \sum_{i=0}^{m_1-1} |\alpha_i^2||y(k_p - 1 - i)| + \sum_{i=1}^{m_2+1} |\beta_i^2||u(k_p - 1 - i)| \quad (28)$$

Replacing $|u(k_p - 1 - i)|$ by (16), using (25), and noting that

$$\begin{aligned} &|\bar{\vartheta}_1(k_p - 1 - i)^T \phi_1(k_p - 1 - i)| \\ &\qquad \leqslant |y(k_p - i)| + |\varepsilon_1(k_p - i)| \leqslant 2\varepsilon, \end{aligned} \quad (29)$$

$i = 1, \ldots, m_2 + 1$, we get

$$|e_2(k_p')| \leqslant \varepsilon \left(\sum_{i=0}^{m_1-1} |\alpha_i^2| + 2 \sum_{i=1}^{m_2+1} |\beta_i^2|\right) \leqslant M_0 \varepsilon \quad (30)$$

as the second term in the last equation in (30) is constant and thus smaller than some constant M_0.

For $\varepsilon = \dfrac{e_{\text{th}}}{1 + M_0}$, it follows that

$$|e_2(k_p')| \leqslant \frac{M_0}{1 + M_0} e_{\text{th}} \leqslant e_{\text{th}}. \quad (31)$$

Step 2-2: $|e_2(k_p' + 1)| \leqslant e_{th}$ *for* $p = 1, 3, 5, \ldots$

Proof of Step 2-2: Using (20), the error $e_2(k_p' + 1) = e_2(k_p + 2)$ is bounded by

$$\begin{aligned} |e_2(k_p + 2)| \leqslant &\sum_{i=0}^{m_1-1} |\alpha_i^2||y(k_p - i)| \\ &+ \sum_{i=1}^{m_2+1} |\beta_i^2||\bar{\vartheta}_1(k_p - i)^T \phi_1(k_p - i)| \end{aligned} \quad (32)$$

Noting that $|y(k_p - i)|, i = 0, \ldots, m_1 - 1$ is smaller than ε and that

$$\begin{aligned} &|\bar{\vartheta}_1(k_p - i)^T \phi_1(k_p - i)| \\ &\qquad \leqslant |y(k_p + 1 - i)| + |\varepsilon_1(k_p + 1 - i)| \leqslant 2\varepsilon, \end{aligned} \quad (33)$$

$i = 1, \ldots, m_2 + 1$, we get

$$|e_2(k_p')| = \varepsilon \left(\sum_{i=0}^{m_1-1} |\alpha_i^2| + 2 \sum_{i=1}^{m_2+1} |\beta_i^2|\right) \quad (34)$$

With the same ε and M_0 as in Step 2.1, it follows that

$$|e_2(k_p + 2)| \leqslant \frac{M_0}{1 + M_0} e_{\text{th}} \leqslant e_{\text{th}}. \quad (35)$$

at $k=k_p$, $|e_2(k)| \le e_{\text{th}}$ for at least $k \in [k'_p, k'_p+1]$.

Step 3-1: $\Delta V(k'_p) \le M_2 < \infty$, $p=1,3,\dots$ *and* $\Delta V(k'_p) \le M_3 < \infty$, $p=2,4,\dots$.

Proof of Step 3-1: Let $p=1,3,5,\dots$. Using (20), it can be shown that

$$\Delta V(k'_p) \leqslant \theta_2^{*T}\theta_2^* - 2\gamma_2 \frac{\theta_2^{*T}\Phi_2(k_p-1)\theta_2^{*T}\Phi_2(k_p-1)}{1+\Phi_2(k_p-1)^T\Phi_2(k_p-1)} + \frac{\gamma_2^2\left(\theta_2^{*T}\Phi_2(k_p-1)\right)^2\Phi_2(k_p-1)^T\Phi_2(k_p-1)}{\left(1+\Phi_2(k_p-1)^T\Phi_2(k_p-1)\right)^2} \tag{36}$$

It can be shown that the sum of the last two terms in (36) is negative. Hence,

$$\Delta V(k'_1) \leqslant \theta_2^{*T}\theta_2^* \leqslant M_2 \le \infty \tag{37}$$

where M_2 is a finite constant. Hence, the jump of V at k_p, from TT to ET, is bounded.

Let $p=2,4,6,\dots$. At k'_p, when the controller switches from ET to TT we get by using (18)

$$\Delta V(k'_p) \leqslant \theta_1^{*T}\theta_1^* - 2\gamma_1 \frac{\theta_1^{*T}\Phi_1(k_p)\theta_1^{*T}\Phi_1(k_p)}{1+\Phi_1(k_p)^T\Phi_1(k_p)} + \frac{\gamma_1^2\left(\theta_1^{*T}\Phi_1(k_p)\right)^2\Phi_1(k_p)^T\Phi_1(k_p)}{\left(1+\Phi_1(k_p)^T\Phi_1(k_p)\right)^2} \tag{38}$$

Any disturbance occurring at k'_p is bounded by $D_{\max}$ and hence, when the TT algorithm is employed $|\theta_1^{*T}\Phi_1(k_2)| \leqslant |y(k_2+1)| + D_{\max}$. Then,

$$\Delta V(k'_p) \leqslant \theta_1^{*T}\theta_1^* + \frac{\gamma_1^2\left(e_{\text{th}}+D_{\max}\right)^2}{1+\Phi_1(k_p)^T\Phi_1(k_p)} \leqslant M_3 \le \infty \tag{39}$$

where we use the fact that $|y(k_2+1)| \leqslant e_{\text{th}}$ which follows since $y(k_2+1)$ is computed before the disturbance occurs and M_3 is a finite constant. The above implies that the jumps at k_p and $k_{p+1}, \forall p \in \mathbb{N}$, corresponding to TT to ET and ET to TT, respectively, are bounded.

Step 3-2: $\Delta V(k) \leqslant 0$ *during* M_{TT} *and during* M_{ET}

Proof of Step 3-2: For $k \in [k'_p; k_{p+1}], p=0,2,4,\dots$ and $d=1$ and for $k \in [k'_p; k_{p+1}], p=1,3,5,\dots$ and $d=2$, $\Delta V(k)$ is given by

$$\Delta V(k) = \frac{\gamma_d \varepsilon_d(k)^2}{1+\Phi_d(k-d)^T\Phi_d(k-d)} \times \left(\frac{\gamma_d\Phi_d(k-d)^T\Phi_d(k-d)}{1+\Phi_d(k-d)^T\Phi_d(k-d)} - 2\right) \leqslant 0 \tag{40}$$

with $0 < \gamma_d < 2$. This shows that the parameter estimation error decreases during M_{TT} and M_{ET}.

Step 4-1: $|u(k)| \leqslant M_4 \quad \forall k$

Proof of Step 4-1: We will show that if in the interval $[k'_{p-1}; k_p]$ the boundedness condition $|u(k)| \leqslant \gamma_{p-1}Q_{p-1}$ is satisfied, then $|u(k)| \leqslant \gamma_p Q_p$ for $p=1,2,3,\dots$ where all γ_n, Q_n are bounded $\forall n \in \mathbb{N}$.

Let $p=1$. As all initial conditions (11) are bounded, i.e., $|u(k'_0)| \leqslant Q_0$, it follows from Theorem 1 that there exists a constant γ_0 with $0 < \gamma_0 < \infty$ such that $|u(k)| \leqslant \gamma_0 Q_0$ for $k \in [k'_0; k_1]$. Consider the switch from TT to ET at k'_1. Then, there exits a finite constant Q_1 such that $|u(k'_1)| < Q_1$ as the components of $|u(k'_1)|$ are all finite values. Hence, the interval $[k'_1; k_2]$ begins with bounded values and thus it follows from Theorem 1 that there exists a γ_1 such that $|u(k)| \leqslant \gamma_1 Q_1$ for $k \in [k'_1; k_2]$.

By induction and using similar arguments as above, we can conclude that there exist finite constants γ_p and $Q_p, p=0,1,2,3,\dots$ such that

$$|u(k)| \leqslant \gamma_p Q_p \tag{41}$$

for all k. Since all γ_p and Q_p are finite there exists a constant $M_4 = \max_{p=0,1,2,3,\dots}\{\gamma_p Q_p\}$ such that $u(k) \leqslant M_4$ for all k.

Step 4-2: *All signals are bounded.*

Proof of Step 4-2: In Steps 1-2 to 2-2, it is shown that the error $e(k)$ is bounded for all k and hence the output $y(k)$ is also bounded for all k. In Step 4-1 it is shown that $u(k)$ is bounded for all k. Thus, $\Phi_1(k)$ and $\Phi_2(k)$ are bounded for all k. In Steps 3-1 and 3-2 it is shown that the parameter estimation error $\tilde{\theta}(k)$ is bounded for all k. Hence, also the parameter estimation $\bar{\theta}(k)$ is bounded for all k. Overall, all signals are bounded and the theorem is proved if a T^*_{dw} exists as in (24). ■

D. Comments on the Main Result

Theorem 2 implies that the plant in (4) can be guaranteed to have bounded solutions with the proposed adaptive switching controller in (16) and (17) and the hybrid protocol in (15), in the presence of disturbances. The latter is assumed to consist of impulse-trains, with their inter-arrival lower bounded. We note that if no disturbances occur, then the choice of the algorithm in (15) implies that these switches cease to exist, and the event-triggered protocol continues to be applied. And switching continues to occur with the onset of disturbances, with Theorem 2 guaranteeing that all signals remain bounded with the tracking errors e_1 and e_2 converging to e_{th} before the next disturbance occurs.

The nature of the proof is similar to that of all switching systems. Multiple Lyapunov functions represented by one switching function $V(k)$ were chosen, and were shown to have a finite jump during switches, and to decrease in between switches, in Stage 3. Unlike linear switching controllers, in the adaptive controller, these Lyapunov functions only ensure the boundedness of parameter estimates, which are a part of the state of the overall system. The additional states were shown to be bounded using the boundedness of the tracking errors e_1 and e_2 (in Stage 1) and the parameter estimates (in Stage 3), and the control input using the method of induction (in Stage 4). Since the switching instants themselves were functions of the states of the closed-loop system, we needed to show that indeed these switching sequences exist, which was demonstrated in Stage 2. It is the latter that distinguishes the adaptive controller proposed in this paper as well as the methodology used for the proof from existing adaptive switching controllers and their proofs in the literature.

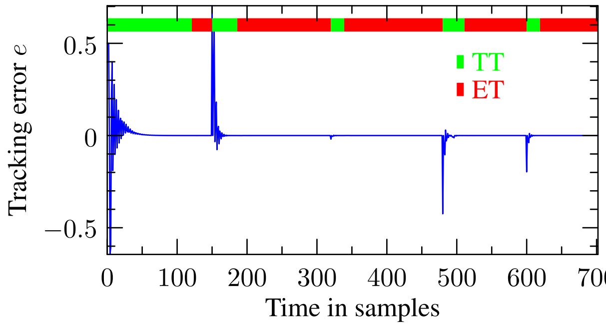

Fig. 2. Evolution of the error e

It should be noted that the presented approach can be easily extended to cases where the difference between the delays in the TT and ET phase is greater than 1. For ease of exposition, this was not done here.

IV. Numerical Example

We illustrate the co-design presented with a numerical example. We consider the discrete-time plant

$$y(k+1) = y(k) - 0.5y(k-1) + u(k) - 2u(k-1) + D(k). \quad (42)$$

According to Eqs. (4)-(6), the ET model is given by

$$y(k+2) = y(k) + 0.5y(k-1) + u(k) + 3u(k-1) + 2u(k-2). \quad (43)$$

The initial conditions are given by

$$y(k-i) = 0.5 \text{ and } u(k-i) = 0 \quad i = 0, 1 \quad (44)$$

Impulse disturbances with a magnitude $D(k)$ generated using a uniformly distributed random variable between $[-1, 1]$, were introduced at time instants $k = 150, 320, 480, 600$. We use the adaptive switching scheme given in Eqs. (16) and (17) together with the hybrid protocol in (15) to stabilize the plant in (42). Figure 2 shows the evolution of the tracking error $e(k)$, which coincides with $e_1(k)$ during TT and $e_2(k)$ during ET. It can be seen that the goal of stabilization is achieved and that whenever a disturbance occurs the system switches to the TT protocol and once the error is below the threshold it switches back to the ET protocol. It is clear that the tracking error goes to zero when disturbances cease. A comparison of this adaptive switching controller with non-adaptive switching controllers as well as adaptive non-switching controllers showed that the proposed controller resulted in minimal usage of the TT protocol compared to all other controllers.

V. Summary

In this work we considered the control of multiple control applications using a hybrid communication protocol for sending control-related messages. These protocols switch between time-triggered and event-triggered methods, with the switches dependent on the closed-loop performance, leading to a co-design of the adaptive controller and the communication architecture. In particular, this co-design consisted of switching between a TT and ET protocol depending on the amplitude of the tracking error, and correspondingly between two different adaptive controllers that are predicated on the resident delay associated with each of these protocols. The resulting adaptive switching controller was shown to result in bounded solutions for the task of stabilization, in the presence of a disturbance of impulse-trains, with the inter-arrival time between any two impulses greater than a finite constant. The class of plants to be controlled was assumed to be of the form of (1), with a unity high-frequency gain. Extensions to tracking and plants with unknown high-frequency gain are topics of on-going research.

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