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Adaptive Switching Controllers for Tracking with Hybrid Communication Protocols

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Abstract— **The focus of this paper is on the co-design of control and communication protocol for the control of multiple applications with unknown parameters using a distributed embedded system. The co-design consists of an adaptive switching controller and a hybrid communication architecture that switches between a time-triggered and event-triggered protocol. It is shown that the overall co-design leads to an overall switching adaptive system that has bounded solutions and ensures tracking in the presence of a class of disturbances.**

I. Introduction

Embedded control systems are ubiquitous and can be found in several applications including aircraft, automobiles, process control, and buildings. An embedded control system is one in which the computer system is designed to perform dedicated functions with real-time computational constraints [1]. Typical features of such embedded control systems are the control of multiple applications, the use of shared networks used by different components of the systems to communicate with each other for control, a large number of sensors as well as actuators, and their distributed presence in the overall system.

The most common feature of such distributed embedded control systems (DES) is shared resources. Constrained by space, speed, and cost, often information has to be transmitted using a shared communication network. In order to manage the flow of information in the network, protocols that are time-triggered [2] and event-triggered [3, 4] have been suggested over the years. Associated with each of these communication protocols are different set of advantages and disadvantages. The assignment of time-triggered (TT) slots to all control-related signals has the advantage of high quality of control (QoC) due to the possibility of reduced or zero delays, but leads to poor utilization of the communication bandwidth, high cost, overall inflexibility, and infeasibility as the number of control applications increase. On the other hand, event-triggered (ET) schedules often result in poor control performance due to the unpredictable temporal behavior of control messages and the related large delays which occurs due to the lack of availability of the bus. These imply that a hybrid protocol that suitably switches between these two schedules offers the possibility of exploiting their combined advantages of high QoC, efficient resource utilization, and low cost [5]. Such a hybrid protocol is the focus of this paper. To combine the advantage of TT and ET policies, hybrid protocols are increasingly being studied in recent years. Examples of such protocols are FlexRay and TTCAN [6, 7], used extensively in automotive systems.

While several papers have considered control using TT protocols (see for example, [2, 8]) and ET protocols (see for example, [3, 4]), control using hybrid protocols has not been studied in the literature until recently. The co-design problem has begun to be addressed of late as well (see for example, [9–14]). In [11–14], the design of scheduling policies that ensure a good Quality of Control (QoC) is addressed. In [11], the schedulability analysis of real-time tasks with respect to the stability of control functions is discussed. In [12], modeling the real-time scheduling process as a dynamic system, an adaptive self-tuning regulator is proposed to adjust the bandwidth of each single task in order to achieve an efficient CPS utilization. The focus of most of the papers above are either on a simple platform or on a single processor. A good survey paper on co-design can be found in [15]. Our focus in this paper is on the co-design of adaptive switching controllers and hybrid protocols so as to ensure good tracking in the presence of parametric uncertainties in the plant being controlled while utilizing minimal resources in the DES.

The solution to the problem of co-design of an adaptive swtiched controller and switches in a hybrid protocol was partially considered in [16], where the control goal was one of stabilization. In this paper, we consider tracking, which is a non-trivial extension of [16]. The main reason for this lies in the trigger for the switch, which corresponds to a system error becoming small. In order to ensure that this error continues to remain small even in the presence of a non-zero reference signal, we needed to utilize fundamental properties of the adaptive system with persistent excitation, and derive additional properties in the presence of reference signals with an invariant persistent excitation property. These properties in turn are suitably exploited and linked with the switching instants, and constitute the main contribution of this paper.

In Section II the problem is formulated, and preliminaries

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related to adaptive control and persistent excitation are presented. In Section III, the switching adaptive controller is described and the main result of global boundedness is proved. The main result is illustrated by a numerical example in Section IV and concluding remarks are given in Section V.

II. Problem Formulation

A. The plant model

The problem that we address in this paper is the simultaneous control of n plants, C_i, $i = 1, \dots, n$, in the presence of impulse disturbances that occur sporadically, using a hybrid communication protocol. We assume that each of these n applications have the following problem statement.

The plant to be controlled is assumed to have a discrete time model described by

$$\mathcal{C}_i: \; y(k) = -\sum_{l=1}^{m_1} a_l y(k-l) + b_0 u(k-d) + \sum_{l=1}^{m_2} b_l u(k-l-d) + D(k-d) \quad (1)$$

where $u(k)$ and $y(k)$ are the input and output of the i-th control application, respectively, at the time-instant t_k and $d \geqslant 1$ is a time-delay. The disturbance $D(k)$ are assumed to be impulses that can occur occasionally with their inter-arrival time lower-bounded by a finite constant. The parameters of the i-th plant are given by a_l, $l = 1, \dots, m_1$, b_l,$l = 0, \dots, m_2$ and are assumed to be unknown. It is further assumed that the sampling time of the controller is a constant h, so that $t_{k+1} = t_k + h$. The goal is to choose the control input u such that $y(k)$ tracks a desired signal $y_{\text{ref}}(k)$, with all signals remaining bounded.

The model in (1) can be expressed as

$$A(q^{-1})y(k) = q^{-d}(B(q^{-1})u(k) + D(k)); \qquad k \geq 0 \quad (2)$$

where q^{-1} is the backward shift operator and the polynomials A and B are given by

$$A(q^{-1}) = 1 + \sum_{l=1}^{m_1} a_l q^{-l} \qquad B(q^{-1}) = b_0 + \sum_{l=1}^{m_2} b_l q^{-l} \quad (3)$$

The following assumptions are made regarding the plant poles and zeros:

Assumption 1: 1) An upper bound for the orders of the polynomials in (3) is known and 2) all zeros of $B_i(q^{-1})$ lie strictly inside the closed unit disk.

For any delay d, Eq. (1) can be expressed in a *predictor form* as follows [17]:

$$y(k+d) = \alpha(q^{-1})y(k) + \beta(q^{-1})u(k) + D(k) \quad (4)$$

with

$$\begin{aligned} \alpha(q^{-1}) &= \alpha_0 + \alpha_1 q^{-1} + \ldots + \alpha_{m_1-1} q^{-(m_1-1)} \\ \beta(q^{-1}) &= F(q^{-1})B(q^{-1}) \\ &= \beta_0 + \beta_1 q^{-1} + \ldots + \beta_{m_2+d-1} q^{-(m_2+d-1)} \end{aligned} \quad (5)$$

where $F(q^{-1})$ and $\alpha(q^{-1})$ are the unique polynomials that satisfy the equation

$$1 = F(q^{-1})A(q^{-1}) + q^{-d}\alpha(q^{-1}). \quad (6)$$

Equation (4) can be expressed as

$$\begin{aligned} y(k+d) &= \theta_d^{*T}\Phi_d(k) + D(k) &(7)\\ &= \vartheta_d^* \phi_d(k) + \beta_0^d u(k) + D(k) &(8) \end{aligned}$$

where $\phi_d(k)$, ϑ_d^*, $\Phi_d(k)$, and θ_d^* are defined as

$$\phi_d(k) = \begin{bmatrix} y(k) \\ \vdots \\ y(k-m_1+1) \\ u(k-1) \\ \vdots \\ u(k-m_2-d+1) \end{bmatrix} \qquad \vartheta_d^* = \begin{bmatrix} \alpha_0^d \\ \vdots \\ \alpha_{m_1-1}^d \\ \beta_1^d \\ \vdots \\ \beta_{m_2+d-1}^d \end{bmatrix} \quad (9)$$

$$\Phi_d(k) = \begin{bmatrix} \phi_d(k) \\ u(k) \end{bmatrix} \text{ and } \theta_d^* = \begin{bmatrix} \vartheta_d^* \\ \beta_0^d \end{bmatrix} \quad (10)$$

with $\phi_d(k) \in \mathbb{R}^{m_1+m_2+d-1}$, $\vartheta_d^* \in \mathbb{R}^{m_1+m_2+d-1}$, $\Phi_d(k) \in \mathbb{R}^{m_1+m_2+d}$, $\theta_d^* \in \mathbb{R}^{m_1+m_2+d}$, and $\alpha_j^d, j = 0, \dots, m_1-1$ and $\beta_j^d, j = 0, \dots, m_2+d-1$ the coefficients of the polynomials in (5) with respect to the delay d and finite initial conditions

$$\begin{aligned} y(k-i) &= y_0(i) \quad i = 0, \dots, m_1-1, \\ u(k-i) &= u_0(i) \quad i = 1, \dots, m_2+d-1. \end{aligned} \quad (11)$$

From Eqs. (7)-(10), we observe that a feedback controller of the form

$$u(k) = \frac{1}{\beta_0^d}\left(y_{\text{ref}}(k+d) - \vartheta_d^{*T}\phi_d(k)\right) \quad (12)$$

realizes the objective of stability and follows the desired bounded trajectory $y_{\text{ref}}(k)$ in the absence of disturbances. Designing a stabilizing controller $u(k)$ essentially boils down to a problem of implementing (12) with the controller gain ϑ_d^*. Two things should be noted: (i) Controller (12) is not realizable as ϑ_d^* and β_0^d are not known, and (ii) the dimension of $\phi_d(k)$, ϑ_d^* as well as the entries of ϑ_d^* depend on the delay d.

B. Baseline adaptive controller

Since ϑ_d^* and β_0^d are unknown, we replace them with their parameter estimates and derive the following adaptive control input

$$u(k) = \frac{1}{\hat{\theta}_{d,\nu}(k)}\left(y_{\text{ref}}(k+d) - \hat{\vartheta}_d(k)^T\phi_d(k)\right) \quad (13)$$

where $\hat{\theta}_{d,\nu}(k)$ denotes the (m_1+m_2+d)-th element of the parameter estimation $\hat{\theta}_d(k)$ and is the estimate of β_0^d. $\hat{\theta}_d(k)$ is adjusted according to the adaptive update law [17]:

$$\hat{\theta}_d(k) = \hat{\theta}_d(k-1) + \frac{a(k)\Phi_d(k-d)\varepsilon_d(k)}{1 + \Phi_d(k-d)^T\Phi_d(k-d)} \quad (14)$$

$$a(k) = \begin{cases} 1 & \text{if } \nu\text{-th element of right-hand side of (14) evaluated using } a(k) = 1 \text{ is } \neq 0 \\ \gamma_d & \text{otherwise, where } 0 < \gamma_d < 2, \gamma_d \neq 1 \end{cases} \quad (15)$$

$$\varepsilon_d(k) = y(k) - \hat{\theta}_d(k-1)^T\Phi_d(k-d), \quad (16)$$

with $\hat{\theta}_d(k) = \begin{bmatrix} \hat{\vartheta}_d(k)^T & \hat{\theta}_{d,\nu}(k) \end{bmatrix}^T$. Equation (15) is necessary to avoid division by zero in the control law (13). Theorem 1 addresses the stability of the adaptive system given by (4), (13), and (14)-(16). The reader is referred to

Theorem 6.3.1 in [17] or Theorem 5.1 in [18] for the proof of Theorem 1.

Theorem 1: Let $D(k) \equiv 0$. Subject to Assumption 1 and given a fixed delay d, the adaptive controller (13) with the update law (14) guarantees that the plant given by (4) follows the reference y_{ref}, i.e., $\lim_{k\to\infty}(y(k) - y_{\text{ref}}(k)) = 0$, and that the sequences $\{\hat{\theta}_d(k)\}$, $\{y(k)\}$ and $\{u(k)\}$ are bounded $\forall k$.

C. Persistent excitation and sufficient richness

The following definitions related to persistent excitation are needed to introduce our switching controller. We define the terms *persistently exciting* and *sufficiently rich* in the following way:

Definition 1 ([19]): A sequence $x(t) \in \mathbb{R}^n$ is said to be *persistently exciting (PE) (in N steps)*, if there exists $N \in \mathbb{Z}^+, \alpha > 0$ such that

$$\sum_{t=t_0+1}^{t_0+N} x(t)x(t)^T \geqslant \alpha I \tag{17}$$

uniformly in t_0.

Definition 2 ([19]): A sequence $x(t) \in \mathbb{R}^n$ is said to be *sufficiently rich (SR) of order m (in N steps)*, if there exists $N \in \mathbb{Z}^+, \alpha > 0$ such that

$$\sum_{t=t_0+1}^{t_0+N} \xi_m(t)\xi_m(t)^T \geqslant \alpha I \tag{18}$$

with $\xi_m(t) = \begin{bmatrix} x(t+1) & x(t+2) & \cdots & x(t+m)\end{bmatrix}^T$ uniformly for all t_0.

Lemma 1: Consider the discrete time system

$$X(t+1) = AX(t) + BU(t) \tag{19}$$

with $X(t) \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times l}$, and $U(t) \in \mathbb{R}^l$. Assume that (19) is completely reachable and that the input $U(t)$ is SR of order $1 \le p \le n$. Then,

$$\operatorname{rank}\left(\sum_{t=t_0+1}^{t_0+n+N} X(t)X(t)^T\right) = p \tag{20}$$

for all $t_0 \geqslant 0$.

Proof: See [20]. ∎

Remark: Much of the existing results pertaining to persistent excitation pertain to the case when the external input $U(t)$ is SR of order n. Lemma 1 above as well as Corollary 1 stated below address the case when $U(t)$ is SR of order p, where $p \in (1, n)$, which to our knowledge has not been examined in the literature. As our goal is tracking of an arbitrary signal and not identification, we do not need the SR-order to be n, but arbitrary and fixed at some p.

Corollary 1: Consider the discrete time system

$$X(t+1) = AX(t) + BU(t) \tag{21}$$

with $X(t) \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times l}$, and $U(t) \in \mathbb{R}^l$. Assume that (21) is completely reachable and that the input $U(t)$ is SR of a fixed order $1 \le p \le n$. Then, there exists a subspace $\Omega_p \subset \mathbb{R}^n$ such that

$$X(t) \in \Omega_p \; \forall t \ge t_0 \quad \text{with } \dim \Omega_p = p. \tag{22}$$

That is, the columns of $X(t)$ span the subspace Ω_p.

Proof: This follows directly from Lemma 1. ∎

We make the following assumption which refers to an invariant property of persistent excitation.

Assumption 2: $y_{\text{ref}}(k)$ is sufficiently rich of constant order $1 \le p \le M$ for all k.

Theorem 2 connects the sufficient richness of y_{ref} with the tracking error and the parameter convergence in an adaptive system.

Theorem 2: Let $D(k) \equiv 0$. Suppose the adaptive controller (13)-(16) is used to control the plant in (8) and let Assumptions 1 and 2 hold. Then

(i) $\lim_{k\to\infty} e(k) = \lim_{k\to\infty} y(k) - y_{\text{ref}}(k) = 0$, and
(ii) $\Phi_d(k) \in \Omega_p \subset \mathbb{R}^M$ as $k \to \infty$
(iii) $\tilde{\theta}_d(k) = \hat{\theta}_d(k) - \theta_d^*$ converges to $\bar{\Omega}_{M-p}$ where $\bar{\Omega}_{M-p}$ is defined as

$$\bar{\Omega}_{M-p} := \left\{x \mid \Phi_d(k)^T x = 0 \text{ for } k \to \infty\right\} \tag{23}$$

where $\Phi_d(k)$ is given in (10).

Proof: Item (i) follows directly from Theorem 1 as it is independent of any persistent excitation of the reference signal y_{ref}. Item (ii) follows by noting that the adaptive system in (1) and (13)-(16) becomes asymptotically linear, and this linear system in turn has a state that satisfies (22) due to Assumption 2. Item (iii) follows from (i) and the fact that $e(k) = \Phi_d(k-d)^T\tilde{\theta}_d(k)$. ∎

III. THE SWITCHING ADAPTIVE CONTROLLER

A. Hybrid Communication Protocols

The focus of this problem is the simultaneous control of several applications for stabilization. That is, the goal is to choose u, the input of the ith control application such that $y(k)$, its output, converges to $y_{\text{ref}}(k)$ which is zero. In the context of the problem under consideration, all control applications are partitioned into a sensor task T_s, a controller task T_c, and an actuator task T_a. We consider a communication protocol where each communication cycle is divided into time-triggered and event-triggered segments. Using *time-triggered* communication schedules, denoted as M_{TT}, applications are allowed to send messages only at their assigned slots and the tasks are triggered synchronously with the bus, i.e., we assume that the communication delay due to the finite speed of the bus is negligible and hence the delay d in (4) is equal to 1. On the other hand, in an *event-triggered* schedule, denoted as M_{ET}, the tasks are assigned priorities in order to arbitrate for access to the bus. Note that in our setup, multiple control applications share the same bus and hence multiple control messages have to be sent using a common bus and thus the messages might experience a communication delay τ when the higher priority tasks access the event-triggered segment. We choose the event-triggered communication schedules such that the sensor-to-actuator delay τ is within $(d_2 - 1)$ sample intervals, i.e., $0 < \tau \le (d_2 - 1)h$ for the control-related messages and hence the delay d is at most equal to d_2 with $d_2 \geqslant 2$. In summary, the delay $d = 1$ if $\mathcal{M}_{\text{Bus}}(k) = M_{\text{TT}}$ and $d = d_2$ if $\mathcal{M}_{\text{Bus}}(k) = M_{\text{ET}}$ where $\mathcal{M}_{\text{Bus}}(k)$ denotes the protocol used at time k.

The properties of the varying delay of the TT and ET protocol are directly exploited in the control design in the following way. Whenever the error between the plant output and its desired value is above some threshold e_{th}, we send the control messages over the TT protocol, as this guarantees an aggressive control action with minimal communication delay. Otherwise, the control messages are sent using the ET protocol. That is,

$$\mathcal{M}_{\text{Bus}}(k) = \begin{cases} M_{\text{ET}} & \text{if } |y(k) - y_{\text{ref}}(k)| \leqslant e_{\text{th}} \\ M_{\text{TT}} & \text{if } |y(k) - y_{\text{ref}}(k)| > e_{\text{th}}. \end{cases} \tag{24}$$

That is, the protocol switches depending on the state of the control application, as in (24).

B. Controller design

Commensurate with the switching protocol in (24), we propose a switch in the adaptive controller as well, and is defined below:

$$\left.\begin{aligned} u(k) &= \frac{1}{\hat{\theta}_{1,\nu}(k)}\Big(y_{\text{ref}}(k+1) - \hat{\vartheta}_1(k)^T\phi_1(k)\Big) \\ \varepsilon_1(k) &= y(k) - \hat{\theta}_1(k-1)^T\Phi_1(k-1) \\ \hat{\theta}_1(k) &= \hat{\theta}_1(k-1) + \frac{a(k)\Phi_1(k-1)\varepsilon_1(k)}{1+\Phi_1(k-1)^T\Phi_1(k-1)} \\ a(k) &= \begin{cases} 1 & \text{if } \nu\text{-th element of right-hand side of update law evaluated using } a(k)=1 \text{ is} \neq 0 \\ \gamma_1 & \text{otherwise, where } 0<\gamma_1<2, \gamma_1 \neq 1 \end{cases} \end{aligned}\right\} \text{if } M_{\text{TT}} \tag{25}$$

where $\phi_1(k)$ is given in Eq. (9), $\Phi_1(k)$ is given in Eq. (10), $\hat{\theta}_1(k) = \begin{bmatrix}\hat{\vartheta}_1(k)^T & \hat{\theta}_{1,\nu}(k)\end{bmatrix}^T$ is the estimation of the controller gains θ_1^* (Eq. 10), and $\gamma_1 \in (0,2)$.

If $\mathcal{M}_{\text{Bus}}(k) = M_{\text{ET}}$, the adaptive controller is given by

$$\left.\begin{aligned} u(k) &= \frac{1}{\hat{\theta}_{2,\nu}(k)}\Big(y_{\text{ref}}(k+d_2) - \hat{\vartheta}_2(k)^T\phi_2(k)\Big) \\ \varepsilon_2(k) &= y(k) - \hat{\theta}_2(k-1)^T\Phi_2(k-d_2) \\ \hat{\theta}_2(k) &= \hat{\theta}_2(k-1) + \frac{a(k)\Phi_2(k-d_2)\varepsilon_2(k)}{1+\Phi_2(k-d_2)^T\Phi_2(k-d_2)} \\ a(k) &= \begin{cases} 1 & \text{if } \nu\text{-th element of right-hand side of update law evaluated using } a(k)=1 \text{ is} \neq 0 \\ \gamma_2 & \text{otherwise, where } 0<\gamma_2<2, \gamma_2 \neq 1 \end{cases} \end{aligned}\right\} \text{if } M_{\text{ET}} \tag{26}$$

where $\phi_2(k)$ is given in Eq. (9), $\Phi_2(k)$ is given in Eq. (10), $\hat{\theta}_2(k) = \begin{bmatrix}\hat{\vartheta}_2(k)^T & \hat{\theta}_{2,\nu}(k)\end{bmatrix}^T$ is the estimation of the controller gains θ_2^* (Eq. 10), and $\gamma_2 \in (0,2)$.

C. Main Result

The following definitions are useful for the rest of the paper. We denote the instants of time when the switch from TT to ET occurs with k_p, $p = 1,3,5,\ldots$, and the instants of time when the switch from ET to TT occurs with k_p, $p = 2,4,6,\ldots$. That is, the TT protocol is applied for $k \in [k'_{2p}; k_{2p+1}], p \in \mathbb{N}_0$ and the ET protocol is applied for $k \in [k'_{2p+1}; k_{2p}], p \in \mathbb{N}_0$ with $k'_p := k_p + 1$ and switches occurring between $[k_p; k'_p], p \in \mathbb{N}$ (see Figure 1).

Assumption 3: The disturbance $D(k)$ in (4) is an impulse train, with the distance between any two consecutive impulses greater than a constant T_{dw}.

This is the main result of the paper:

Theorem 3: Let the plant and disturbance D in (4) satisfy Assumptions 1, 2, and 3. Consider the switching adaptive controller in (25) and (26) with the hybrid protocol in (24) and the following parameter estimate selections at the switching instants

$$\hat{\theta}_1(k_p) = 0, \qquad p = 0,2,4,\ldots \tag{27}$$

$$\hat{\theta}_2(k_1) = 0 \tag{28}$$

$$\hat{\theta}_2(k_p + l) = \hat{\theta}_2(k_{p-1}), \quad p = 3,5,7,\ldots, \tag{29}$$

$$l = 0,1,\ldots,m_2+d-1 \tag{30}$$

Then there exists a positive constant T^*_{dw} such that for all $T_{dw} \geq T^*_{dw}$, the closed loop system has globally bounded solutions.

Proof of Theorem 3: We define an equivalent reference signal $y'_{\text{ref}}(k)$ that combines the effect of both $y_{\text{ref}}(k)$ and the disturbance D as

$$y'_{\text{ref}}(k) := y_{\text{ref}}(k) + D'(k) \tag{31}$$

where $D'(k)$ is given by $D'(k) := G^{-1}(q^{-1})D(k)$ and $G(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})}$ is the transfer function of the plant (1). Also, we define a reference model signal ϕ_i^* given by

$$\phi_i^*(q^{-1}) = G_i(1+\vartheta_i^{*T}G_i)^{-1}y'_{\text{ref}} \tag{32}$$

where the transfer functions $G_i, i = 1,2$ is given by

$$G_i(q^{-1}) = \begin{bmatrix}W(q^{-1})q^{-1} & \cdots & W(q^{-1})q^{-m_1+1} \\ & & q^{-1} & \cdots & q^{-m_2-d_i+1}\end{bmatrix}^T \tag{33}$$

and the optimal feedback gain $\vartheta_i^*, i = 1,2$ is given by (9). We note that when there is no disturbance, the output ϕ_i^* corresponds to the desired regressor vector, and its first element of the vector corresponds to y_{ref}.

When the algorithm is in mode M_{TT}, the underlying error equation is given by

$$e_1(k+1) = \Big(\vartheta_1^* - \hat{\vartheta}_1(k)\Big)^T\phi_1(k) = \tilde{\vartheta}_1(k)^T\phi_1(k). \tag{34}$$

with $\tilde{\vartheta}_1(k) = \vartheta_1^* - \hat{\vartheta}_1(k)$. When the system is in mode M_{ET} and using $\tilde{\theta}_2(k) = \theta_2^* - \hat{\theta}_2(k)$, the error equation is given by

$$e_2(k+2) = \Big(\vartheta_2^* - \hat{\vartheta}_2(k)\Big)^T\phi_2(k) = \tilde{\vartheta}_2(k)^T\phi_2(k). \tag{35}$$

Define θ_a^* as

$$\theta_a^* = \begin{cases} \begin{bmatrix}\theta_1^{*T} & 0\end{bmatrix}^T & \text{if } \mathcal{M}_{\text{Bus}}(k) = M_{\text{TT}} \\ \theta_2^* & \text{if } \mathcal{M}_{\text{Bus}}(k) = M_{\text{ET}} \end{cases} \tag{36}$$

Choose Lyapunov function $V(k) = \tilde{\theta}_a(k)^T\tilde{\theta}_a(k)$ where $\tilde{\theta}_a(k) = \theta_a^* - \hat{\theta}_a(k)$. Let $\Delta V(k) = V(k) - V(k-1)$.

The proof consists of four stages and we note that the proofs of Stages 1, 3, and 4 are identical to that in [16] and are therefore omitted here. Since Stage 2 differs significantly from its counterpart in [16] due to $y_{\text{ref}} \neq 0$, we provide its proof in detail below.

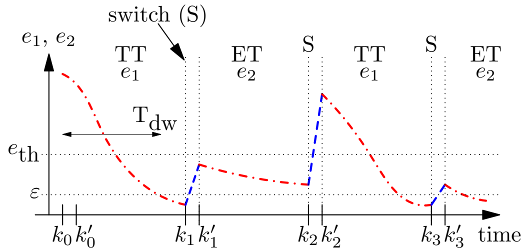

Fig. 1. Schematic evolution of the error with a given sequence of switching times. Impulses in $D(k)$ are assumed to occur at $k_p, p = 0, 2, 4, \ldots$.

Step 2-1: $\lim_{k\to\infty} |y(k) - y_{ref}(k)| = 0 \Rightarrow \lim_{k\to\infty} \|\phi_i(k) - \phi_i^*(k)\| = 0$

Proof of Step 2-1: In this step we show that if the tracking error $|y(k) - y_{\text{ref}}(k)|$ is small the state signal error $\|\phi_i(k) - \phi_i^*(k)\|$ is also small.

The signal $\phi_i(k) - \phi_i^*(k)$ is the output produced by the following transfer function H with $|y(k) - y_{\text{ref}}(k)|$ as the input:

$$H = \begin{bmatrix} 1 \\ \vdots \\ q^{-m_1+1} \\ W^{-1}(q^{-1})q^{-1} \\ \vdots \\ W^{-1}(q^{-1})q^{-m_2-d_i+1} \end{bmatrix} \tag{37}$$

where $W^{-1}(q^{-1})$ is the inverse of the plant transfer function $W(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})}$ with the input signal $y(k) - y_{\text{ref}}(k)$ and $\phi_i^*(k)$ given in (32). From Assumption 1, it follows that $W^{-1}(q^{-1})$ is a stable transfer function. Hence, as $|y(k) - y_{\text{ref}}(k)|$ tends to zero, $\phi_i(k) - \phi_i^*(k)$ also tends to zero.

Step 2-2: *If* $|y(k) - y_{ref}(k)| \to 0$*, then* $\phi_i(k_j) \in \Omega_i$

Proof of Step 2-2: We first show that $\phi_d^*(k) \in \Omega_i$ for $i = 1, 2$ and $j = 1, 2, 3, \ldots$. We note that the reference model given in (32) is a linear system and hence there exists a state space representation

$$\phi_i^*(k+1) = R\phi_i^*(k) + Sy'_{\text{ref}} \tag{38}$$

with (R, S) being completely reachable. Then it follows directly from Corollary 1 that $\phi_i^*(k_j) \in \Omega_i$ for $i = 1, 2$ and $j = 1, 2, 3, \ldots$. Together with Step 2-1 it follows that if $|y(k) - y_{\text{ref}}(k)| \to 0$, then $\phi_i(k_j) \in \Omega_i$.

Step 2-3: $|e_2(k'_p)| = |\tilde{\vartheta}_2(k_{p-1})^T \phi_2(k'_p - d_2) + \tilde{\theta}_{2,\nu}(k_{p-1}) y_{ref}(k'_p)| \leqslant e_{th}$ *for* $p = 3, 5, 7, \ldots$

Proof of Step 2-3: First, we show that the error of the signal generated by the reference model signal $\phi_2^*(k'_p - d_2)$ together with the last parameter estimation value $\vartheta_2(k_{p-1})$ at the end of the previous ET phase is small and therefore the output error $e_2(k'_p), p = 3, 5, 7, \ldots$ is below the threshold e_{th}.

From Step 2-2 we know that $\phi_2^*(k'_p - d_2)$ is in the same subspace Ω_2 as $\phi_2^*(k_{p-1})$. From Step 2-1 we know that $\phi_2^*(k_{p-1})$ is close to $\phi_2(k_{p-1})$ which in turn generates together with $\tilde{\vartheta}_2(k_{p-1})$ and $\tilde{\theta}_{2,\nu}(k_{p-1})$ an error which is $\leqslant \varepsilon$ according to Theorem 1. Hence,

$$|\tilde{\vartheta}_2(k_{p-1})^T \phi_2^*(k'_p - d_2)| + \tilde{\theta}_{2,\nu}(k_{p-1}) y_{\text{ref}}(k'_p) \leqslant \varepsilon.$$

From Step 2-1 we know that $\phi_2(k'_p - d_2)$ is close to $\phi_2^*(k'_p - d_2)$. Hence, according to Step 2-4 we have $|e_2(k'_p)| \leqslant e_{\text{th}}, p = 3, 5, 7, \ldots$.

Step 2-4: $|e_2(k'_p + l)| \leqslant e_{th}, p = 3, 5, 7, \ldots$ *and* $l = 1, 2, \ldots, m_2 + d_2 - 1$

Proof of Step 2-4: From Step 2-3 we know that $|e_2(k'_p)| \leqslant e_{\text{th}}$. According to the parameter choice in (29), the controller uses a constant initial value for the first $m_2 + d_2 - 1$ steps. Thus, the error $|e_2(k'_p + l)| = |\tilde{\vartheta}_2(k_{p-1})^T \phi_2(k'_p + l - d_2) + \tilde{\theta}_{2,\nu}(k_{p-1}) y_{\text{ref}}(k'_p + l)| \leqslant e_{\text{th}}$ because Steps 2-1 to 2-5 can be applied.

D. Comments on the Main Result

Theorem 3 implies that the plant in (4) can be guaranteed to have bounded solutions with the proposed adaptive switching controller in (25) and (26) and the hybrid protocol in (24), in the presence of disturbances. The latter is assumed to consist of impulse-trains, with their inter-arrival lower bounded. We note that if no disturbances occur, then the choice of the algorithm in (24) implies that these switches cease to exist, and the event-triggered protocol continues to be applied. And switching continues to occur with the onset of disturbances, with Theorem 3 guaranteeing that all signals remain bounded with the tracking errors e converging to e_{th} before the next disturbance occurs.

The nature of the proof is similar to that of all switching systems, in some respects. A common Lyapunov function $V(k)$ was used to show the boundedness of parameter estimates, which are a part of the state of the overall system (in Stage 3). The additional states were shown to be bounded using the boundedness of the tracking errors e_1 and e_2 (in Stage 1) and the control input using the method of induction (in Stage 4). Since the switching instants themselves were functions of the states of the closed-loop system, we needed to show that indeed these switching sequences exist, which was demonstrated in Stage 2. To this end, the sufficient richness properties of the reference signal were utilized to show that the signal vectors of a reference model and the system converge to the same subspace. Next, it was shown that the error generated by the reference model is small and thus concluded that the tracking error at the switch from TT to ET stays below the threshold e_{th}. It is the latter that distinguishes the adaptive controller proposed in this paper, as well as the methodology used for the proof, from existing adaptive switching controllers and their proofs in the literature.

IV. Numerical Example

We illustrate the co-design presented with a numerical example. We consider the discrete-time plant

$$y(k+1) = -0.73y(k) - 0.64y(k-1) + 1.05u(k) + D(k). \tag{39}$$

According to Eqs. (4)-(6), the ET model is given by

$$\begin{aligned} y(k+2) = &-0.107y(k) + 0.467y(k-1) \\ &+ 1.05u(k) - 0.767u(k-1) + D(k). \end{aligned} \tag{40}$$

The initial conditions are given by

$$y(k-i) = 1 \text{ and } u(k-i) = 0 \quad i = 0, 1 \tag{41}$$

Impulse disturbances with a magnitude $D(k)$ generated using a uniformly distributed random variable between $[-2, 2]$, were introduced at time instants $k = 150, 213, 325$. We use the adaptive switching scheme given in Eqs. (25) and (26) together with the hybrid protocol in (24) with a threshold $e_{\text{th}} = 0.01$ to track the reference signal $y_{\text{ref}}(k) = -0.5$ with the plant in (39). Figure 2 shows the evolution of the output $y(k)$. It can be seen that the goal of tracking is achieved and that whenever a disturbance occurs the system switches to the TT protocol and once the error is below the threshold it switches back to the ET protocol. It can also be seen that when the system switches to the ET protocol for the first time at $k = 30$, the system does not switch to the TT protocol although the error $e(k) \approx 1.4$ which is larger than the threshold e_{th}. As discussed in Section III, during this interval, the adaptive algorithm enables the system to learn until the tracking error becomes small. This is illustrated during subsequent switches from TT to ET at $t = 180, 243, 355$ where it can be seen that $|e(t)| \leq e_{\text{th}}$. It is also clear that the tracking error goes to zero when disturbances cease. A comparison of this adaptive switching controller with non-adaptive switching controllers as well as adaptive non-switching controllers showed that the proposed controller resulted in minimal usage of the TT protocol compared to all other controllers.

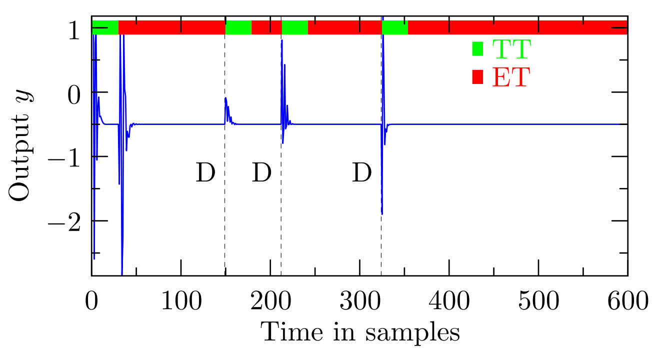

Fig. 2. Evolution of the output y. Disturbances are marked with D.

V. Summary

In this work we considered the control of multiple control applications using a hybrid communication protocol for sending control-related messages. These protocols switch between time-triggered and event-triggered methods, with the switches dependent on the closed-loop performance, leading to a co-design of the controller and the communication architecture. In particular, this co-design consisted of switching between a TT and ET protocol depending on the amplitude of the tracking error, and correspondingly between two different adaptive controllers that are predicated on the resident delay associated with each of these protocols. These delays were assumed to be fixed and equal to 1 for the TT protocol and greater than 2 for the ET protocol. It was shown that for any reference input whose order of sufficient richness stays constant, the signal vector and the parameter error vector converge to subspaces which are orthogonal to each other. The overall adaptive switching system was shown to track such reference signals, with all solutions remaining globally bounded, in the presence of an impulse-train of disturbances with the inter-arrival time between any two impulses greater than a finite constant. A numerical example was presented that illustrates the main result.

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