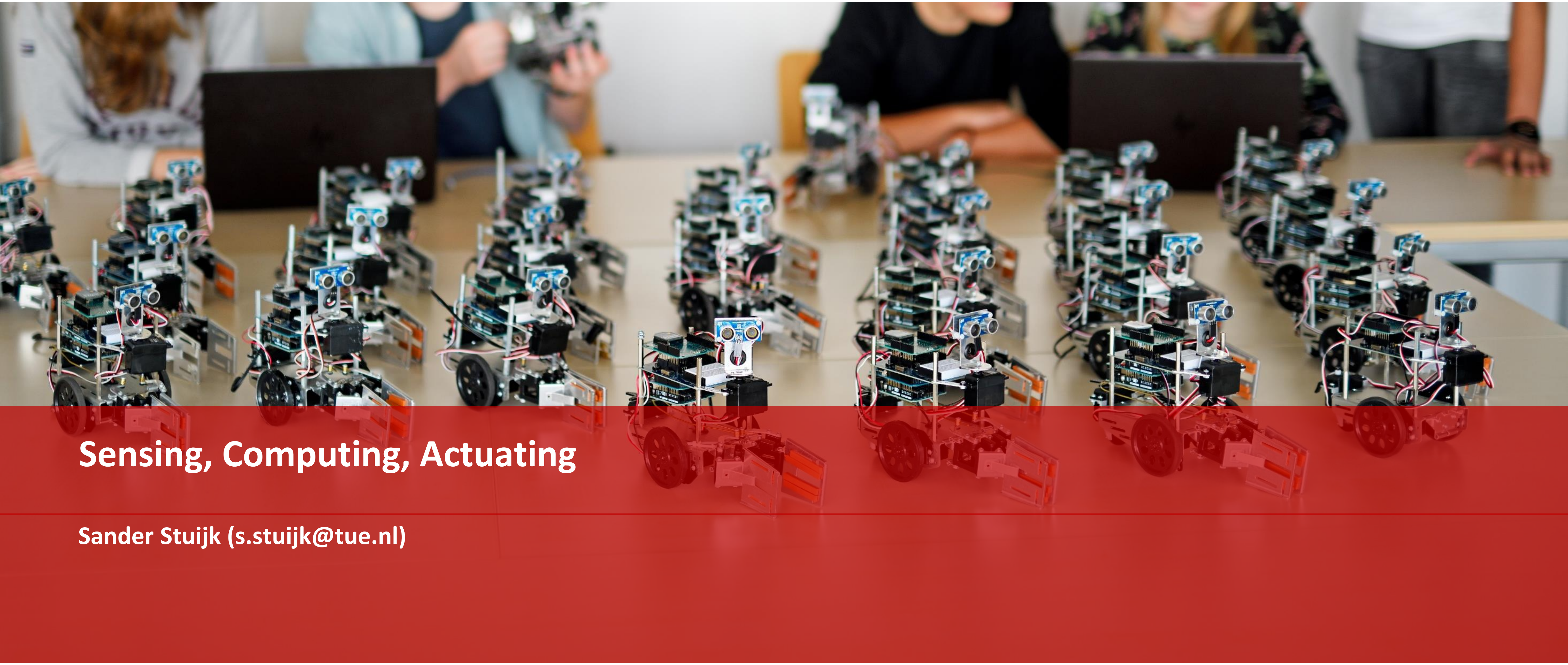

Sensing, Computing, Actuating

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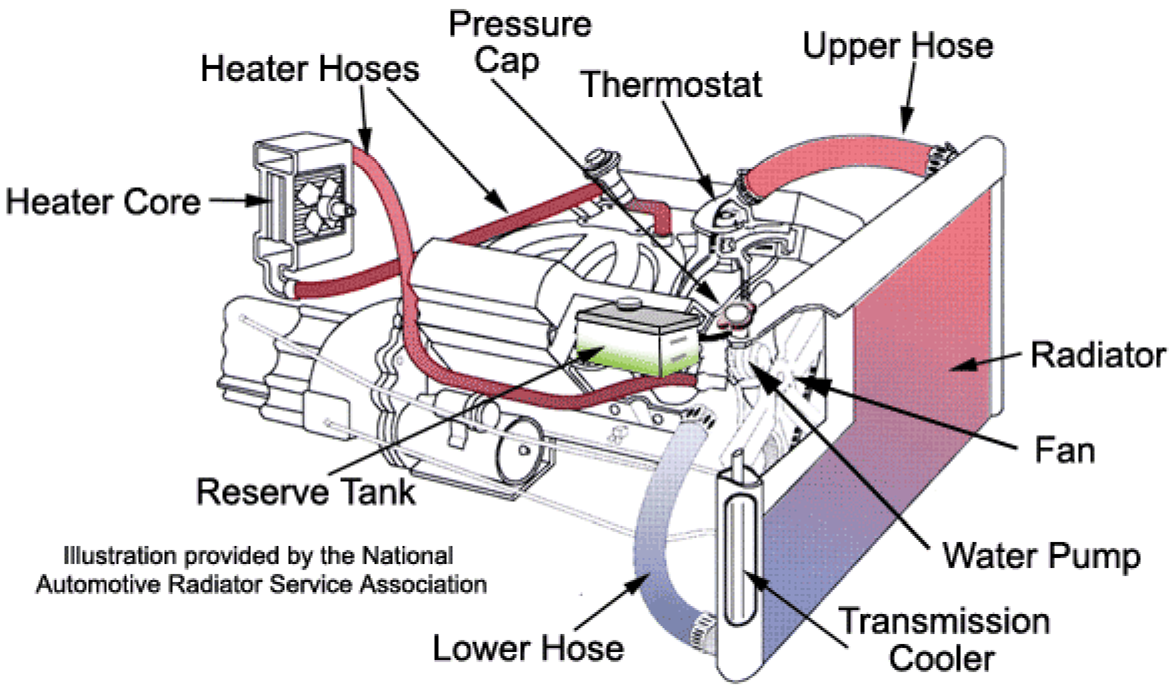
SENSING TEMPERATURE, SELF-HEATING

(Chapter 2.1, 2.2, 5.11)

Engine coolant temperature sensor

- https://www.youtube.com/watch?v=Q56k37FsRcA



Applications

discharge air temperature
(NTC thermistor)

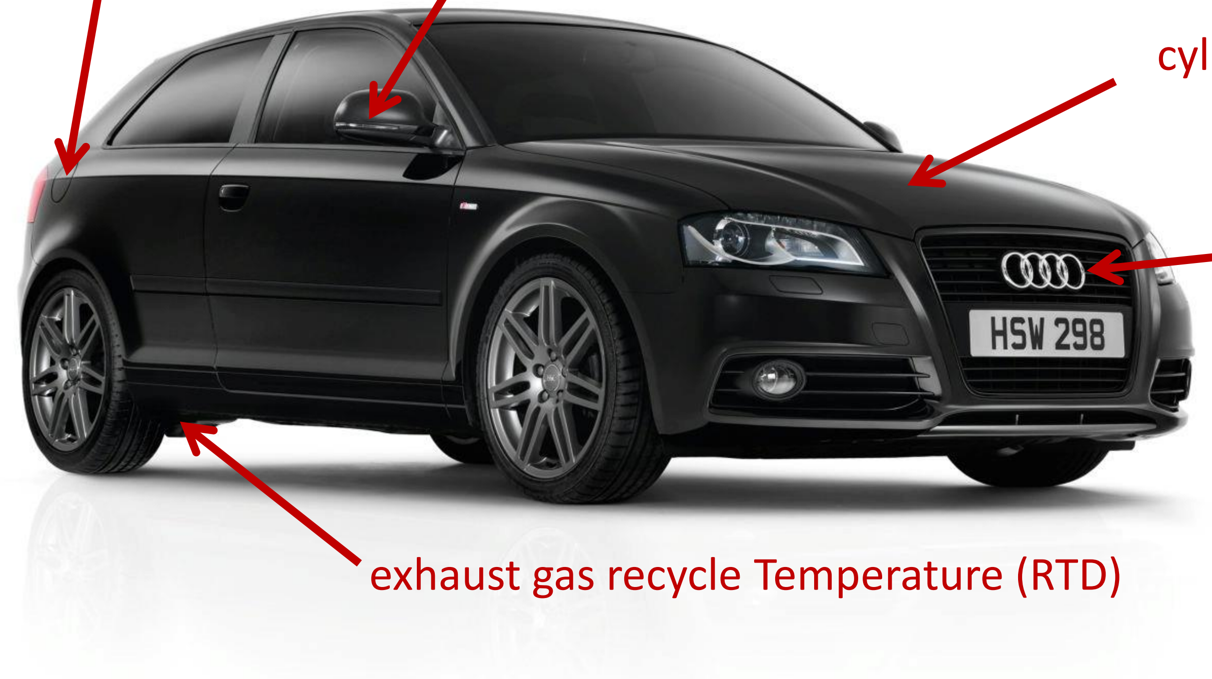
active in-car temperature
(NTC thermistor)

fuel temperature
(NTC thermistor)

outside air temperature
(NTC thermistor)

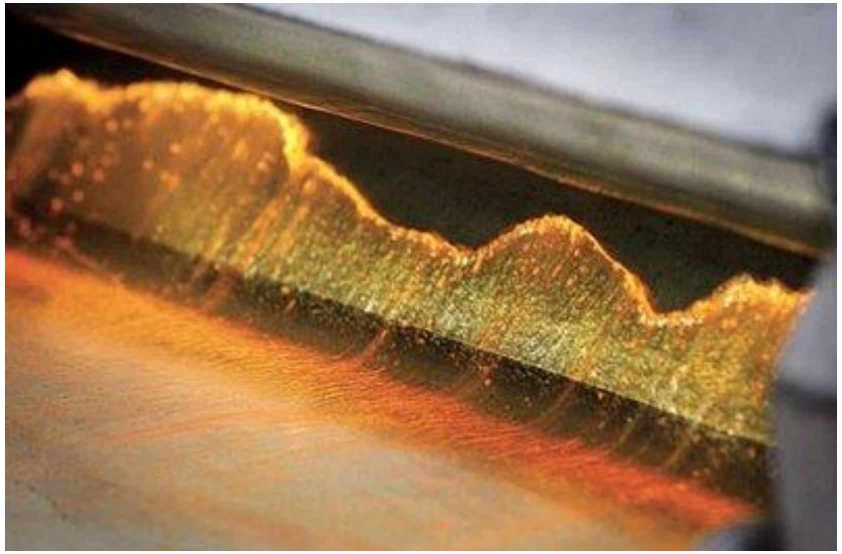
cylinder head temperature
(thermocouple)

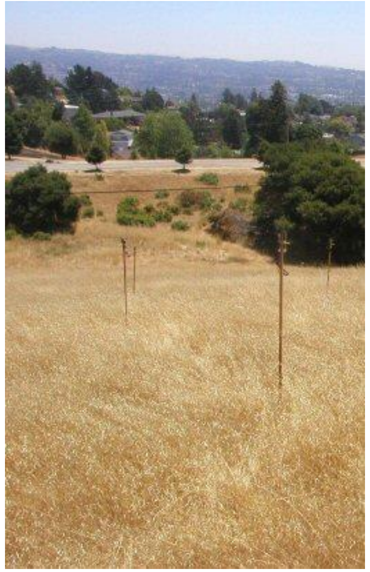
intake air temperature (NTC
thermistor)



exhaust gas recycle Temperature (RTD)

Applications





Temperature sensors

placement	excitation	physical effect	material	thermal sensor
contact	passive	thermal expansion	metal	bimetal
contact	active	resistive effect	metal	RTD
contact	active	resistive effect	semiconductor	silicon resistive
contact	active	resistive effect	polymer or ceramic	thermistor
contact	passive	thermoelectric effect	conductor	thermocouple
contact	active	PN junction	semiconductor	
non-contact	passive	pyroelectric effect	pyroelectric	pyroelectric
non-contact	active	ultrasound	piezoelectric	acoustic

- there are many other classification criteria: construction, linearity, reference point, ...

Temperature sensors

- temperature sensors are deceptively simple
  - resistive sensor – a conductor connected to a voltage source
  - thermocouple – any two dissimilar materials welded together at one end and connected to a micro-voltmeter

- temperature sensors can be used to measure other quantities, e.g.,

Sensor	Quantity		
	Acceleration / Vibration	**Flow rate / Point velocity**	**Force**
Resistive	Mass-spring + strain gage	Thermistor	Strain gage
Self-generating	Mass-spring + piezoelectric sensor	Thermal transport + thermocouple	Piezoelectric sensor

- complex sensors for radiation, pressure, position, level, and chemical reactions can be constructed on the basis of temperature or temperature-difference sensors

Temperature sensors

placement	excitation	physical effect	material	thermal sensor
contact	passive	thermal expansion	metal	bimetal
contact	**active**	**resistive effect**	**metal**	**RTD**
contact	active	resistive effect	semiconductor	silicon resistive
contact	active	resistive effect	polymer or ceramic	thermistor
contact	passive	thermoelectric effect	conductor	thermocouple
contact	active	PN junction	semiconductor	
non-contact	passive	pyroelectric effect	pyroelectric	pyroelectric
non-contact	active	ultrasound	-	acoustic

Resistance

- resistance of a material is defined as $R = \frac{V}{i}$
- resistance depends on geometrical factors $R = \rho \frac{l}{a}$
  - length of wire (l)
  - cross-sectional area (a)
- resistance depends on temperature $R = \rho \frac{l}{a} = \frac{m}{ne^2\tau} \frac{l}{a}$
  - number of free electrons (n)
  - mean time between collisions (τ)
- changing dimensions affect resistance (piezoresistive effect)
- changing temperature affect resistance (thermoresistive effect)
- resistive sensor can be used to sense changes in these quantities
  - only one quantity should be measured
  - other quantity should be kept constant or corrected for

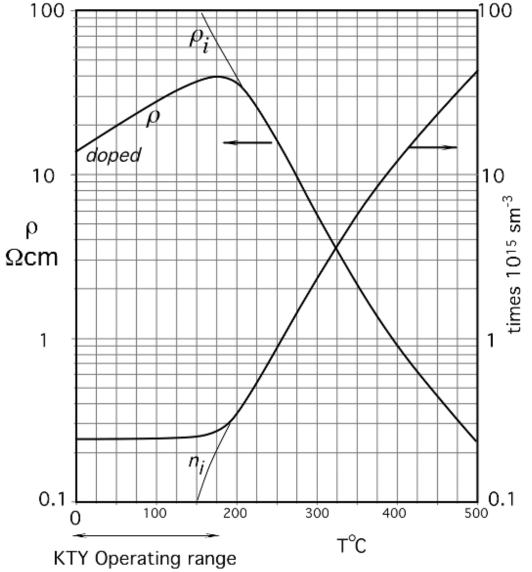
Thermoresistive effect

- specific resistivity of a material

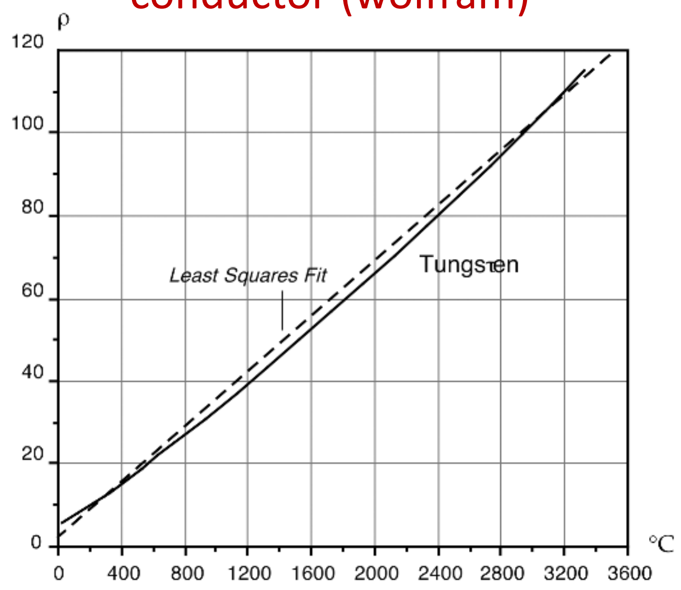
$$\rho = \frac{m}{ne^2\tau}$$

  - temperature dependency through
    - number of free electrons (n) in semiconductors
      - NTC behavior: ↑T → ↑n → ρ↓
    - mean time between collisions (τ) in conductors
      - PTC behavior: ↑T → ↓τ → ρ↑

semiconductor (silicon)



conductor (wolfram)



Resistance temperature detectors (RTDs)

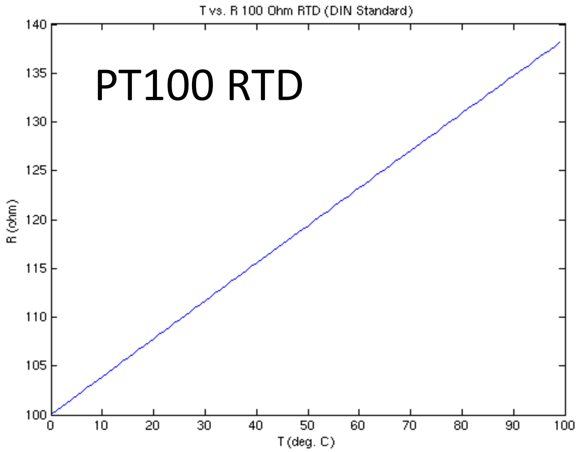
- RTD is a temperature sensor build from metal (conductor)
  - specific resistivity of metal depends on time between collisions
  - positive temperature coefficient (PTC)
    - increasing temperature leads to increasing resistivity

- relation between temperature and resistance

$$R_T = R_0[1 + \alpha_1(T - T_0) + \alpha_2(T - T_0)^2 + \ldots + \alpha_n(T - T_0)^n]$$

  - T_0 – reference temperature
  - R_0 – resistance at T_0

- coefficients can be found using calibration
- example – PT100 RTD
  - $\alpha_1 \approx 3.89 \cdot 10^{-3}/K$, $\alpha_2 \approx -5.83 \cdot 10^{-7}/K^2$, $\alpha_3 \approx 1.92 \cdot 10^{-7}/K^3$
  - almost linear relation between temperature and resistance



Temperature coefficient of resistance

- assume linear temperature/resistance relation for an RTD sensor

$$R_T = R_0[1 + \alpha(T - T_0)]$$

- α_T is called the temperature coefficient of resistance (TCR)

$$\alpha_T = \frac{R_T - R_0}{(T - T_0)R_T}$$

  - TCR indicates relative change in resistance per unit temperature (between temperature T and a reference temperature T_0)
  - TCR is often called relative sensitivity

- PTC devices: TCR decreases when temperature increase (**why?**)
  - fractional increase in resistance decreases with increasing temperature
  - limits usability of PTC devices at higher temperatures

Temperature coefficient of resistance

example – PT100 sensor

- R_0 = 100Ω, α_0 = 0.00389 (Ω/Ω)/K at 0°C, $R_T = R_0[1 + \alpha_0(T - T_0)]$
- **what is the sensitivity of this sensor?**
- **what are TCR at 25°C and 50°C? (use 0°C as reference)**
  - A) 0.389 Ω/K, 0.00389 /K, 0.00389 /K
  - B) 0.389 Ω/K, 0.00355 /K, 0.00326 /K
  - C) 0.389 Ω/K, 0.00326 /K, 0.00355 /K
  - D) 0.355 Ω/K, 0.00355 /K, 0.00326 /K

Temperature coefficient of resistance

example – PT100 sensor

- R_0 = 100Ω, α_0 = 0.00389 (Ω/Ω)/K at 0°C, $R_T=R_0[1+\alpha_0(T-T_0)]$
- **what is the sensitivity of this sensor?**
- sensitivity is the slope of the resistance-temperature curve
- resistance-temperature curve is straight line
- sensitivity
- **what are TCR at 25°C and 50°C? (use 0°C as reference)**
  - TCR (at 25°C)

$$S = \alpha_0 R_0 = \alpha_{25} R_{25} = \alpha_{50} R_{50} = 0.00389(\Omega/\Omega)/K \cdot 100\Omega = 0.389\Omega/K$$

$$\Rightarrow \quad \alpha_{25} = \frac{\alpha_0 R_0}{R_{25}} = \frac{\alpha_0 R_0}{R_0[1+\alpha_0(25^0C - 0^0C)]} = \frac{\alpha_0}{1+\alpha_0(25^0C)}$$

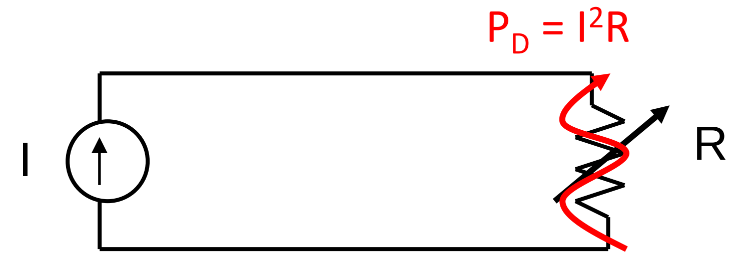
$$\alpha_0 R_0 = \alpha_{25} R_{25} \quad = \frac{0.00389(\Omega/\Omega)/K}{1+0.00389/K(25^0C)} = 0.00355(\Omega/\Omega)/K$$

  - TCR (at 50°C) $\alpha_{50} = \dfrac{\alpha_0}{1+\alpha_0(50^oC)} = 0.00326(\Omega/\Omega)/K$

  - TCR decreases for increasing temperature

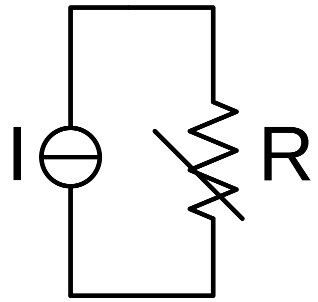
Self-heating in RTDs

- current must be passed through sensor to measure resistance
- power will be dissipated in the RTD creating heat (self-heating)



- effect of self-heating reduced by thermal dissipation to environment
  - heat dissipation factor δ (W/K) depends on
    - surrounding fluid
    - velocity of the fluid
  - temperature error given by $\Delta T = \frac{P_D}{\delta} = \frac{I^2 R}{\delta}$
- self-heating error can be limited by dimensioning the current I

Self-heating in RTDs

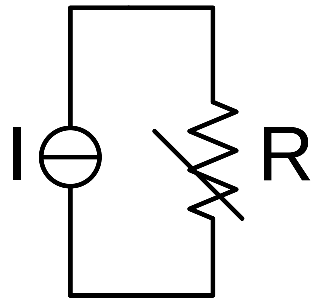


example – PT100 sensor $R(T)=R_0[1+ \alpha_0(T-T_0)]$

- R_0 = 100Ω, α_0 = 0.00389 (Ω/Ω)/K at 0°C
- δ = 6mW/K (in air), δ = 100mW/K (in still water)
- sensor used in range [0°C, +100°C]

- **what is the maximal current through the sensor to keep the self-heating error below 0.1°C when emerged in air?**
  - provide a numerical answer

Self-heating in RTDs



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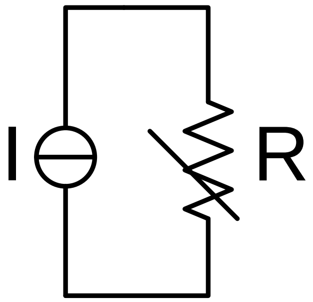
- sensor used in range [0°C, +100°C]

- **what is the maximal current through the sensor to keep the self-heating error below 0.1°C when emerged in air?**
  - temperature PT100 above environment $\Delta T = \frac{P_D}{\delta} = \frac{I^2 R}{\delta}$
  - self-heating error maximal when resistance is maximal
    - maximal self-heating at +100°C
    - resistance $R(100°C) = 100\Omega[1 + 0.00389/°C \cdot 100°C] = 139\ \Omega$

Self-heating in RTDs



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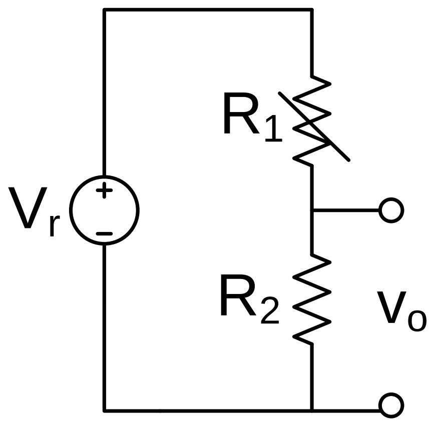
- sensor used in range [0°C, +100°C]

- **what is the maximal current through the sensor to keep the self-heating error below 0.1°C when emerged in air?**
  - temperature PT100 above environment $\Delta T = \frac{P_D}{\delta} = \frac{I^2 R}{\delta}$
  - relation between current and temperature $I = \sqrt{\frac{\Delta T \cdot \delta}{R}}$
  - max current in air $I = \sqrt{\frac{(0.1^o C) \cdot (0.006W/K)}{139\Omega}} = 2.1mA$
  - max current in water $I = \sqrt{\frac{(0.1^o C) \cdot (0.1W/K)}{139\Omega}} = 8.5mA$

Self-heating in RTDs



example – PT100 sensor $R(T)=R_0[1+\alpha_0(T-T_0)]$

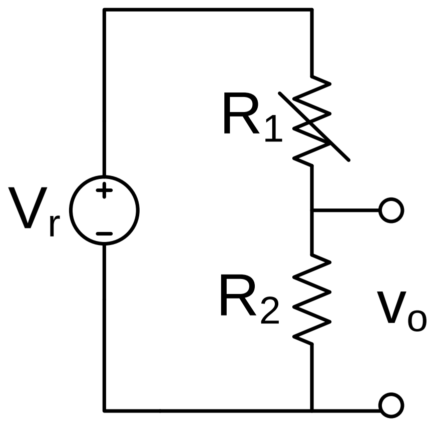
- R_0 = 100 Ω, α_0 = 0.00389 (Ω/Ω)/K at 0°C
- sensor used in range [0°C, +100°C]
- δ = 6 mW/K (in air), V_r = 5 V, R_2 = 1 kΩ

- **what is the maximal self-heating error (resolution) of this sensor?**
  - temperature PT100 above environment $\Delta T = \frac{P_D}{\delta} = \frac{I^2 R_1}{\delta}$
  - current I depends on resistance R_1 and temperature T

$$I = \frac{V_r}{R_1 + R_2} = \frac{V_r}{R_0\left(1 + \alpha_0(T - T_0)\right) + R_2}$$

    - maximal current when T = 0°C, but minimal resistance
  - temperature error depends on power dissipation
    - maximal power dissipation when T = 100°C
    - maximal self-heating error occurs when T = 100°C

Self-heating in RTDs



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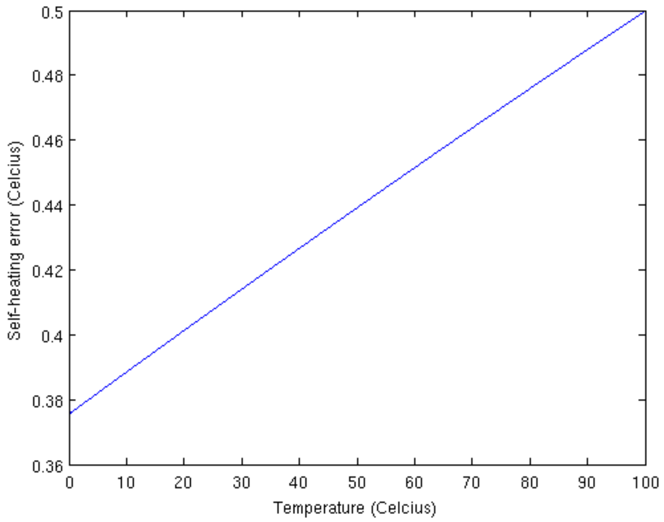
- **what is the maximal self-heating error (resolution) of this sensor?**
  - temperature PT100 above environment $\Delta T = \frac{P_D}{\delta} = \frac{I^2 R_1}{\delta}$
  - maximal self-heating error occurs when T = 100°C

$$R(100^\circ C) = 139\Omega$$

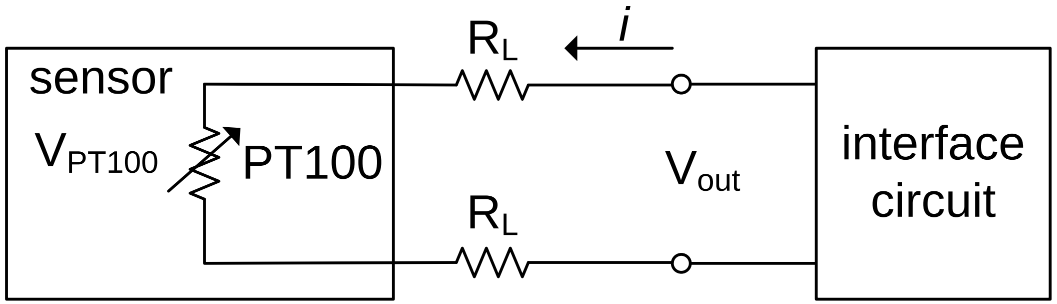
$$\Delta T = \frac{V_r^2}{R_1 + 2R_2 + \frac{R_2^2}{R_1}} \frac{1}{\delta} \approx \frac{V_r^2 R_1}{R_2^2} \frac{1}{\delta} \quad \Rightarrow \Delta T = 0.5^\circ C$$

dominates when $R_1 << R_2$



Lead-wire resistance

- lead wires are not perfect conductors (lead-wire resistance)
- resistance of the wires will affect measured voltage



- temperature error due to lead-wire resistance when interface circuit provides constant current i

$$\left.\begin{array}{r} R = 2R_L + PT100 \Rightarrow \Delta R = 2R_L \\ \Delta T = \dfrac{\Delta R}{S} \end{array}\right\} \Rightarrow \Delta T = \frac{2R_L}{S}$$

- example – PT100
  - PT100 has resistance of 107.8Ω at 20°C and S = 0.389Ω/K
  - assume R_L = 1Ω
  - ΔT = +5.1°C → interface circuit measures temperature of 25°C
  - measured temperature 25% above actual temperature

Lead-wire resistance

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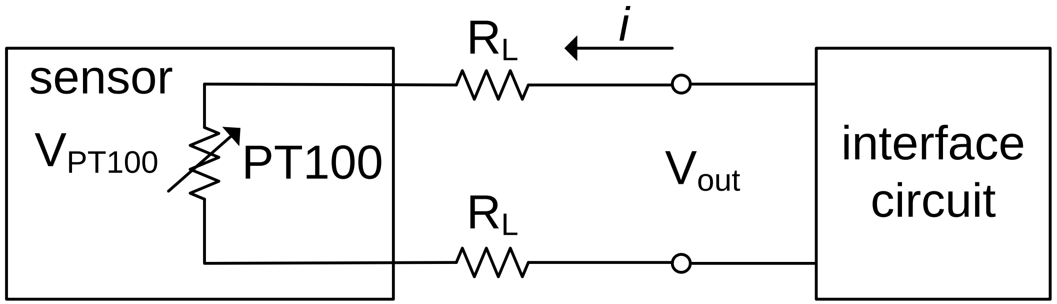
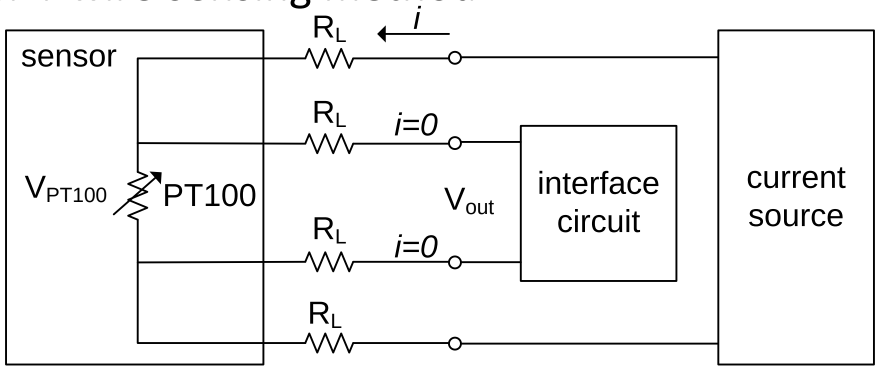
- resistance of the wires will affect measured voltage



$$V_{out} = V_{PT100} + 2R_L i$$

- lead wire resistance can be cancelled with 4-wire sensing method
  - interface circuit has high impedance



$$V_{out} = V_{PT100}$$

- 4-wire sensing method requires stable current source
- 6-wire sensing method can be used with stable voltage source

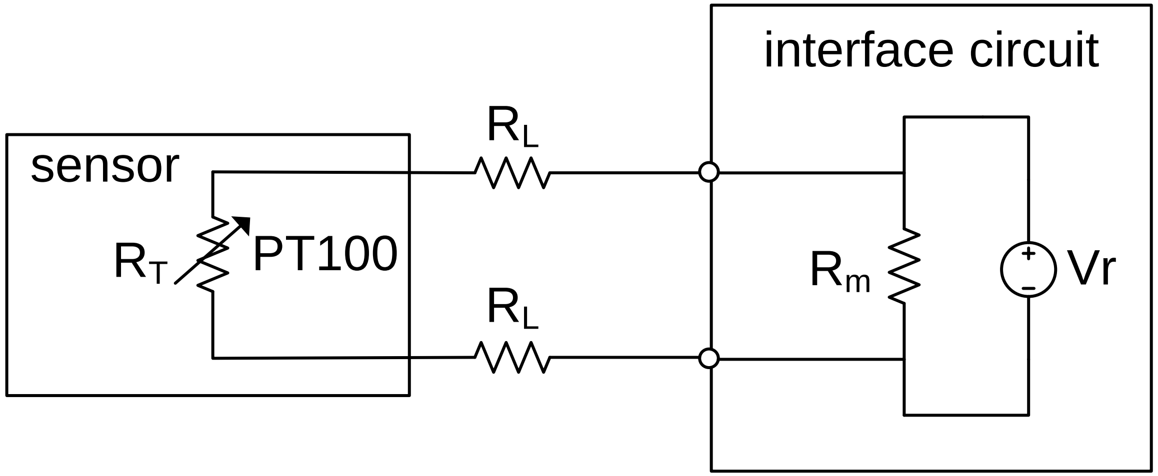
Summary - Resistive temperature detectors (RTDs)

- temperature-resistance relation $R_T = R_0[1 + \alpha(T - T_0)]$
- α_T is called the temperature coefficient of resistance (TCR)

$$\alpha_T = \frac{dR_T/dT}{R_T} = \frac{R_T - R_0}{(T - T_0)R_T}$$

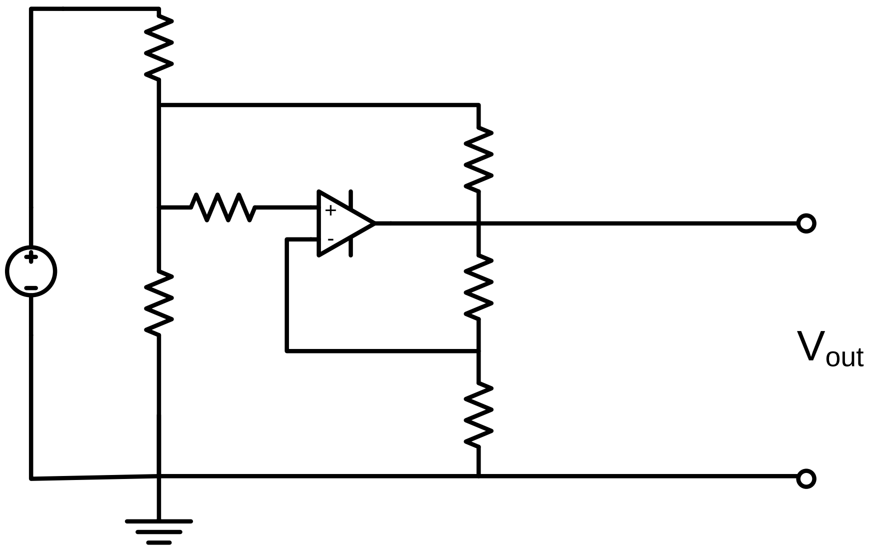
  - TCR indicates relative change in resistance per unit temperature (between two reference temperatures)
  - TCR is not equal to the sensitivity

- several error sources influencing accuracy
  - lead-wire resistance (R_L)
  - self-heating (R_T)
  - non-linearity (R_T)
  - loading effect (R_m)



Interface circuits

- interface circuits can be used to
  - increase sensor sensitivity
  - cancel lead-wire resistance
  - linearization of the sensor output
  - limit self-heating
  - compensate for error sources (e.g. strain or temperature)
  - set output voltage at reference point
  - amplify sensor output for use with AD-converter
  - ...



RESISTIVE SENSORS

(Ch 2.8)

Resistive sensors

- resistance of resistive sensor $R = R_0 f(x)$
  - f(x) – fractional change in resistance (with f(0) = 1)

- resistance of linear resistive sensor $R = R_0(1 + x)$
  - range of x depends on type of sensor
    - [-1, 0] – linear potentiometer
    - [1, 10] – RTDs
    - [0.00001 , 0.001] – strain gauges
    - [1, 100] – NTC thermistors
    - [1, 10000] – switching PTC thermistors

- requirements on signal conditioners for resistive sensors
  - electric voltage or current must be applied
  - supply and output voltage/current are limited by self-heating

Resistive sensors – current excitation

- sensor driven by current source
- deflection measurement with current source
  - feedback loop enforces constant current

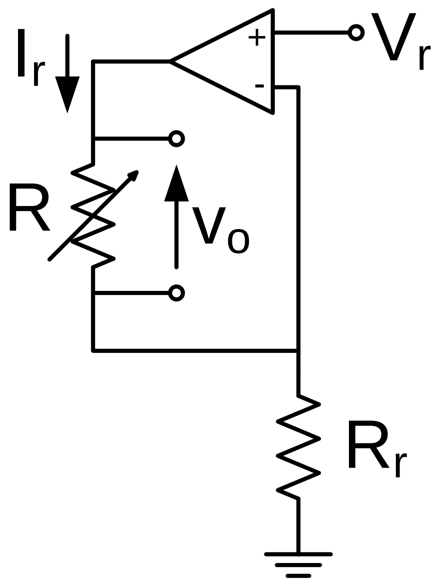
$$I_r = \frac{V_r}{R_r}$$

  - output of a linear sensor

$$v_o = I_r R = \frac{V_r}{R_r} R_0(1 + x)$$

  - choose $R_0 = R_r$ then
  - output consists of offset and (small) fluctuation around offset

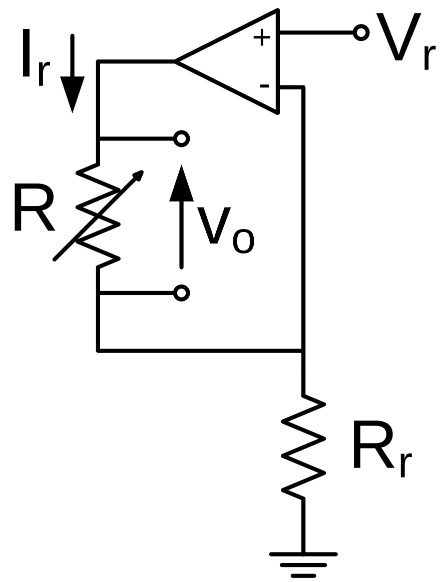
$$v_o = V_r(1 + x) = V_r + xV_r$$



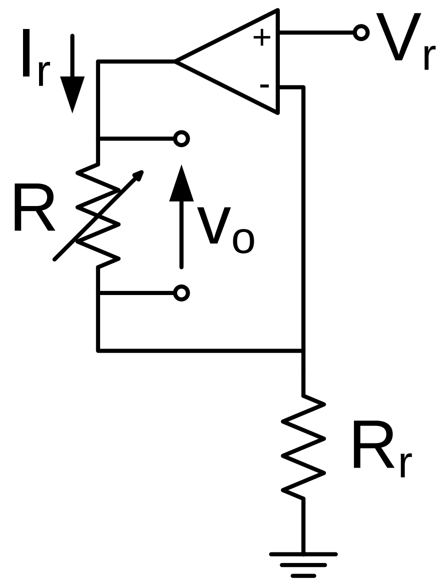
Resistive sensors – current excitation

example – circuit for temperature measurement [20°C,100°C]

- measure temperature with 0.1°C resolution (self-heating < 0.1°C)
- PT 100 sensor (R_0=100Ω and α=0.00389Ω/Ω/K at 0°C)
- dissipation factor δ = 40mW/K in 0.4m/s water
- reference voltage V_r = 5V
- **what resistance should R_r have to get a sensitivity of 1mV/°C?**
    - provide a numerical answer



Resistive sensors – current excitation



example – circuit for temperature measurement [20°C,100°C]

- measure temperature with 0.1°C resolution (self-heating < 0.1°C)
- PT 100 sensor (R_0=100Ω and α=0.00389Ω/Ω/K at 0°C)
- dissipation factor δ = 40mW/K in 0.4m/s water
- reference voltage V_r = 5V
- **what resistance should R_r have to get a sensitivity of 1mV/°C?**
- temperature resolution limited by self-heating

$$\frac{I_r^2 R}{\delta} = \left(\frac{V_r}{R_r}\right)^2 \frac{R}{\delta} = \Delta T < 0.1^\circ C$$

- maximal dissipation at 100°C, condition is thus

$$\left.\begin{array}{l} R_r > V_r \sqrt{\dfrac{R_{100}}{\delta \cdot (0.1^\circ C)}} \\ R_{100} = R_0\big(1 + \alpha(T - T_0)\big) \\ = 100\Omega(1 + 0.00389 \cdot 100^\circ C) \\ = 138.9\Omega \end{array}\right\} \Rightarrow R_r > (5V)\sqrt{\frac{138.9\Omega}{(40mW/K)\cdot(0.1^\circ C)}} = 932\Omega$$

Resistive sensors – current excitation

example – circuit for temperature measurement [20°C,100°C]

- measure temperature with 0.1°C resolution (self-heating < 0.1°C)
- PT 100 sensor (R_0=100Ω and α=0.00389Ω/Ω/K at 0°C)
- dissipation factor δ = 40mW/K in 0.4m/s water
- reference voltage V_r = 5V
- **what resistance should R_r have to get a sensitivity of 1mV/°C?**
- output voltage of the sensor

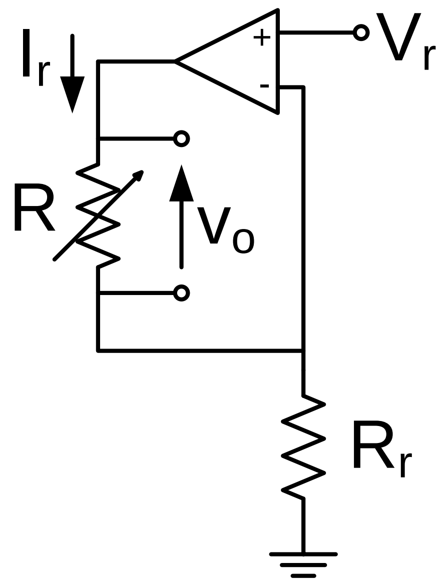
$$v_o = \frac{V_r}{R_r} R_0(1+x) = \frac{V_r}{R_r} R_0(1+\alpha T)$$

- to get sensitivity of 1 mV/°C, R_r should be

$$S = \frac{dv_o}{dT} = \frac{V_r}{R_r} R_0 \alpha \Rightarrow R_r = \frac{V_r R_0 \alpha}{S} = \frac{(5V)(100\Omega)(0.00389/K)}{1mV/K} = 1945\Omega$$

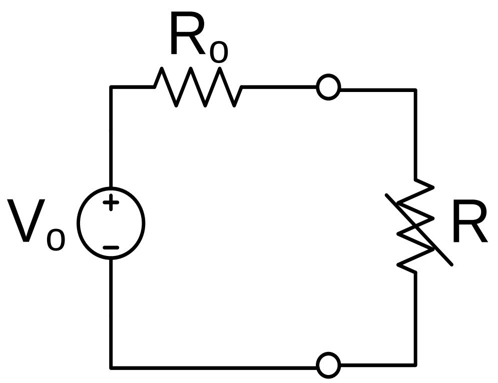
- sensor output has also an offset (output not 0V at 20°C)

$$R_{20} = 107.8\Omega \Rightarrow V_{offset} = \frac{V_r}{R_r} R_{20} \approx 277mV$$



Interface circuits

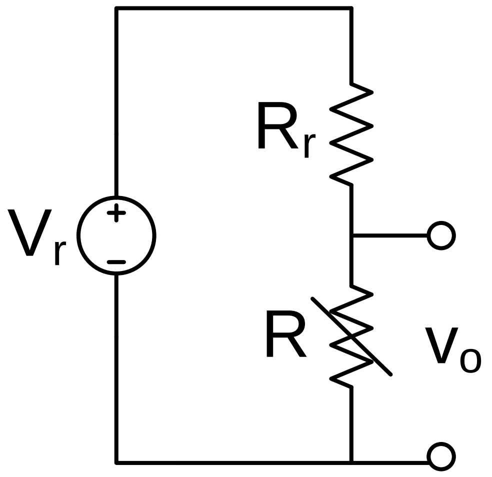
- resistance of linear resistive sensor: $R(x) = R_0(1+x)$
  - range of x depends on type of sensor



- requirements on signal conditioners for resistive sensors
  - electric voltage or current must be applied
  - supply and output voltage/current are limited by self-heating

- voltage excitation
  - **when does maximal self-heating error occur?**
  - **when is sensitivity maximal?**
  - **when is non-linearity error minimized?**

Voltage divider – self-heating error



- sensor driven by voltage source
  - sensor: R
  - load resistance: R_r
- **when does maximal self-heating error occur?**
- power consumption by sensor

$$P = \left(\frac{V_r}{R + R_r}\right)^2 R$$

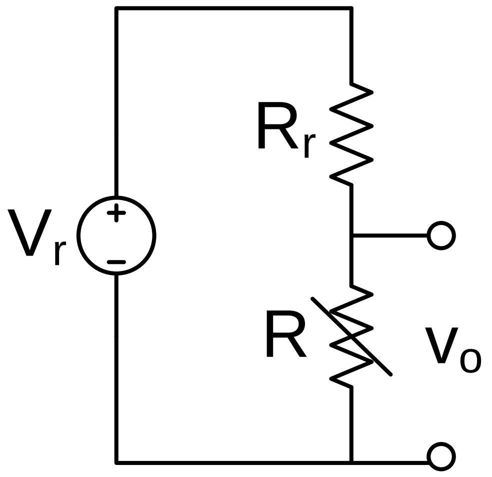
- maximal power consumption occurs when

$$\frac{dP}{dR} = 2\frac{V_r}{R + R_r}\frac{-V_r}{(R + R_r)^2}R + \left(\frac{V_r}{R + R_r}\right)^2 = \left(\frac{V_r}{R + R_r}\right)^2 \frac{R_r - R}{R + R_r} \equiv 0 \quad \Rightarrow R_r = R$$

- self-heating error is maximal when $R_r = R$
- power consumption is then equal to

$$P = \left(\frac{V_r}{R_r + R_r}\right)^2 R_r = \frac{V_r^2}{4R_r}$$

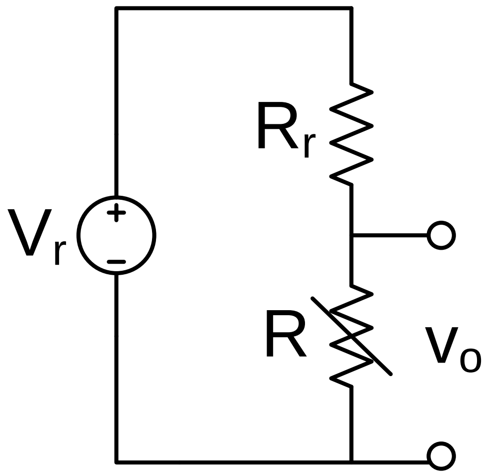
Voltage divider – self-heating error



- example – dimension voltage divider for temperature measurement
- measure temperature from 0°C to 100°C
- PT 100 sensor (R_0=100Ω and α=0.00389Ω/Ω/K at 0°C)
- maximal power dissipation in sensor is 1mW
- voltage source Vr = 5V
- **what resistance Rr must be used for this voltage divider?**

  - provide a numerical answer

Voltage divider – self-heating error



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- voltage source Vr = 5V
- **what resistance Rr must be used for this voltage divider?**
- power dissipation in sensor

$$\left(\frac{V_r}{R + R_r}\right)^2 R < 1mW$$

- maximal dissipation when R=Rr

$$\left(\frac{V_r}{R_r + R_r}\right)^2 R_r < 1mW \;\Rightarrow R_r > \frac{V_r^2}{4 \cdot 0.001W} = \frac{(5V)^2}{4 \cdot 0.001W} = 6.25k\Omega$$

- sensor range is from 100Ω to 139Ω
- always R < Rr, thus power dissipation always below limit

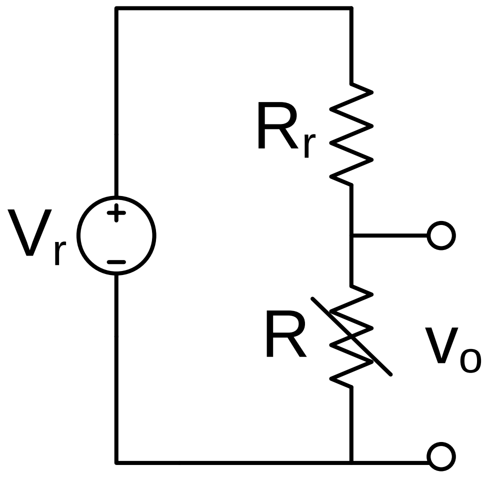
Voltage divider – linearity

- measure fractional change in resistance x
  - sensor: $R = R_0(1+x)$
  - load resistance: $R_r = R_0k$

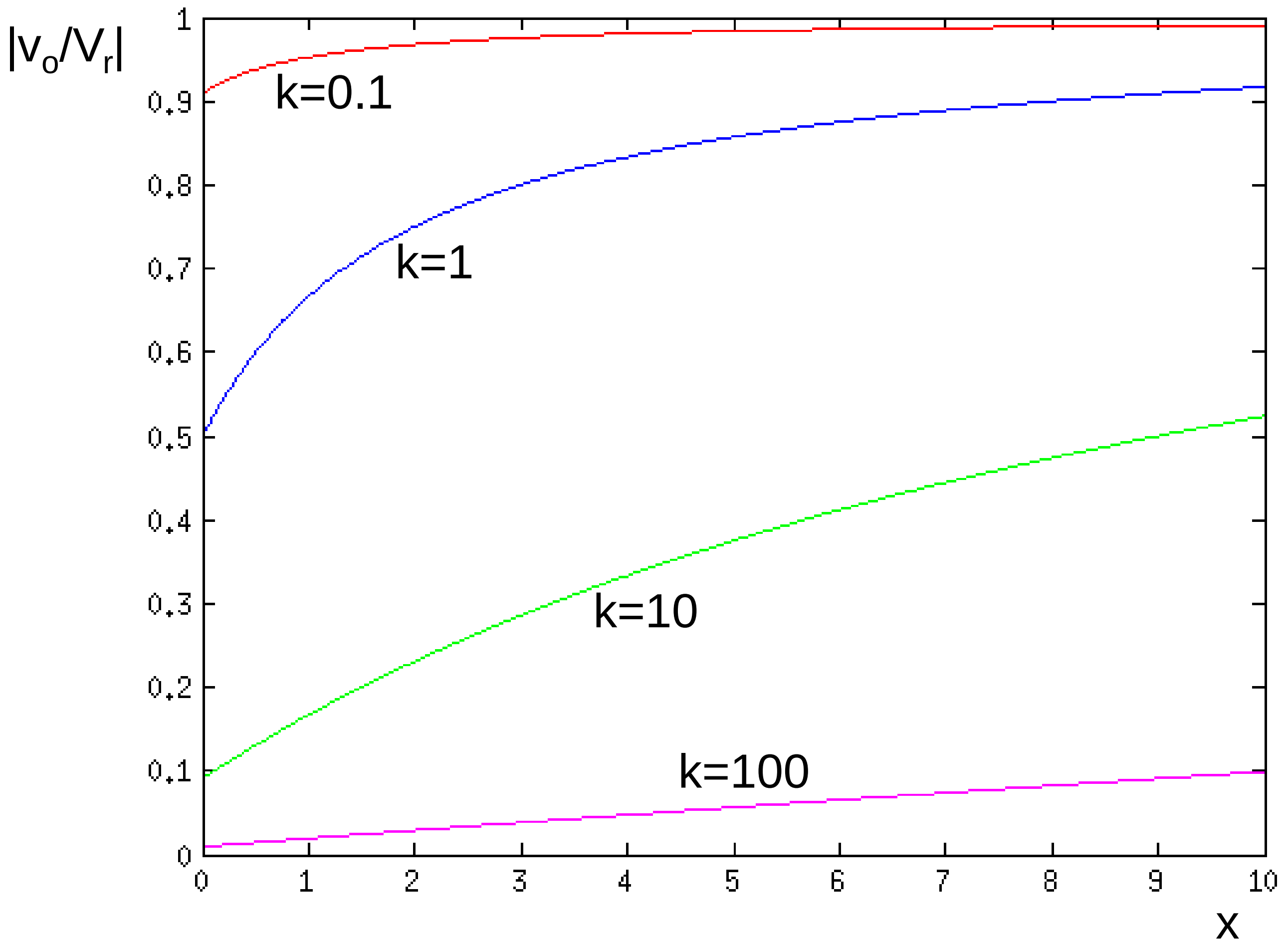
- output voltage of the circuit

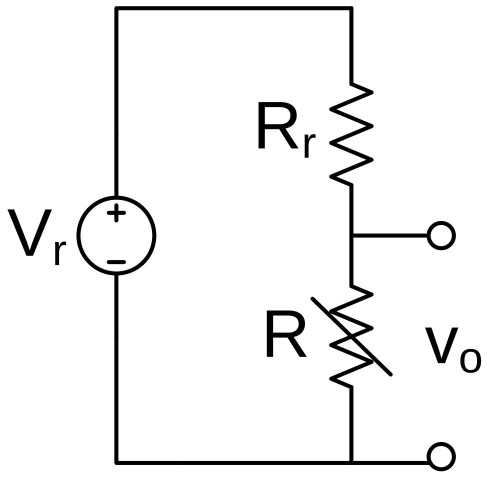
$$v_o = \frac{R}{R_r + R}V_r = \frac{R_0(1+x)}{R_0k + R_0(1+x)}V_r = \frac{1+x}{k+1+x}V_r$$

  - response becomes linear when $R_r >> R$ (i.e. $k >> 1+x$)



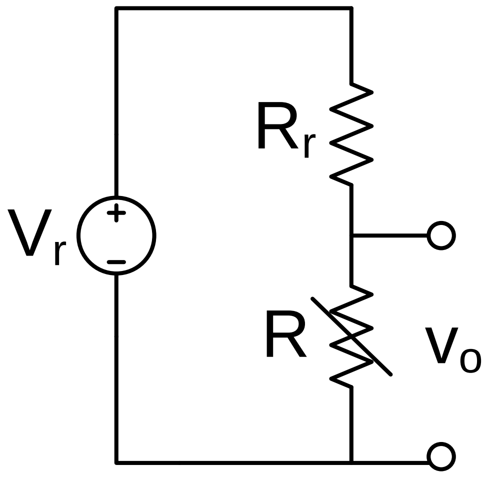
Voltage divider – linearity





- **increasing k is good for linearity, but what about sensitivity?**

Voltage divider – sensitivity



- measure fractional change in resistance x
  - sensor: $R = R_0(1+x)$
  - load resistance: $R_r = R_0k$

- sensitivity

$$S = \frac{dv_o}{dx} = \frac{d}{dx}\left(\frac{1+x}{k+1+x}V_r\right) = \frac{(k+1+x)-(1+x)}{(k+1+x)^2}V_r = \frac{k}{(k+1+x)^2}V_r$$

- maximal sensitivity

$$\frac{dS}{dk} = 0 \Rightarrow \frac{d}{dk}\left(\frac{k}{(k+1+x)^2}V_r\right) = 0$$

$$\Rightarrow \frac{(k+1+x)^2 - 2k(k+1+x)}{(k+1+x)^4} = \frac{x+1-k}{(k+1+x)^3} = 0 \Rightarrow k = x+1$$

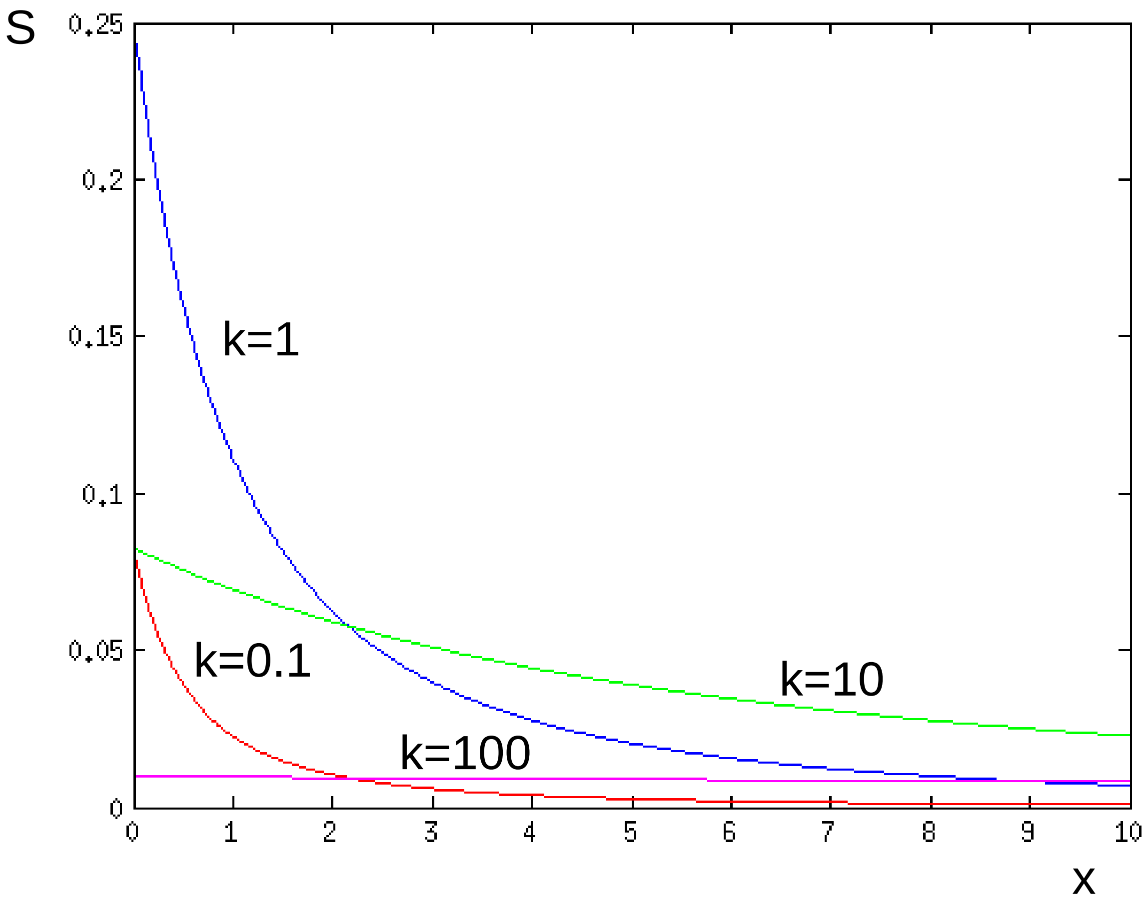
  - maximal sensitivity reached when $R = R_r$
  - same situation as when self-heating error is maximal
  - maximal transfer of power (at $R = R_r$) leads to
    - maximal sensitivity and maximal self-heating

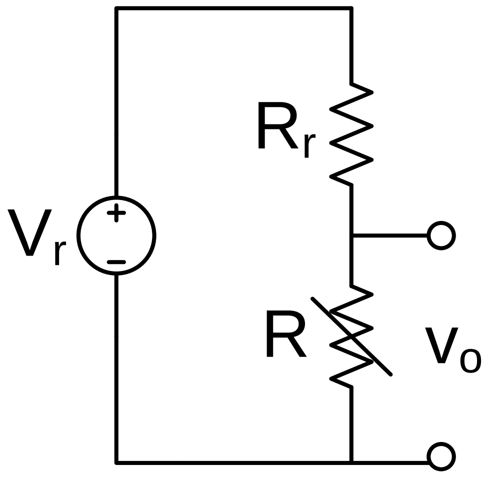
use quotient rule

$$\frac{d}{dx}\frac{j(x)}{h(x)} = \frac{j'(x)h(x) - j(x)h'(x)}{(h(x))^2}$$

Voltage divider – sensitivity and linearity

TU/e



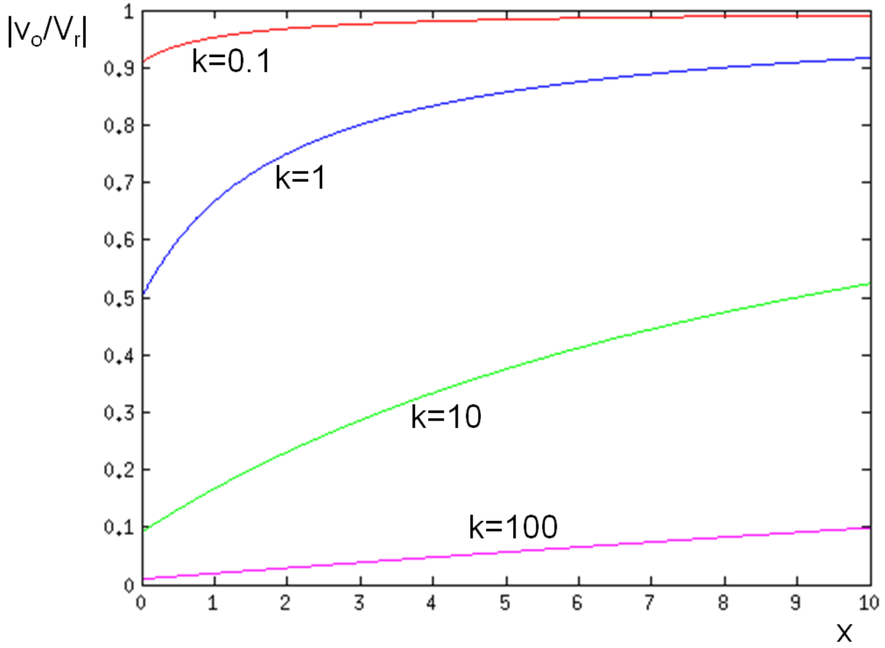


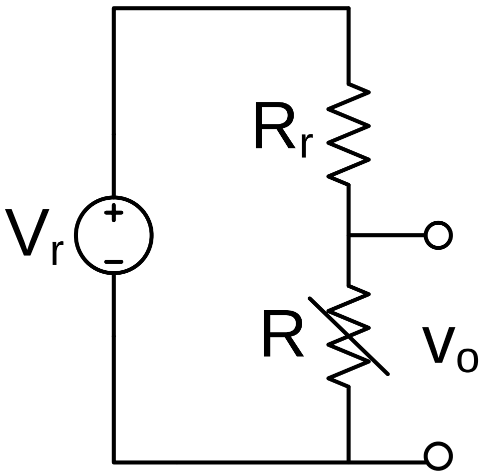
- for many sensors x < 1
  - sensitivity largest for k = 1
  - sensitivity may be considered constant if maximal value of x << 1

Voltage divider – output voltage

- maximal sensitivity when k = 1
- output voltage

$$v_o = \frac{1+x}{k+1+x} V_r = \frac{1+x}{2+x} V_r$$





- offset voltage present in output