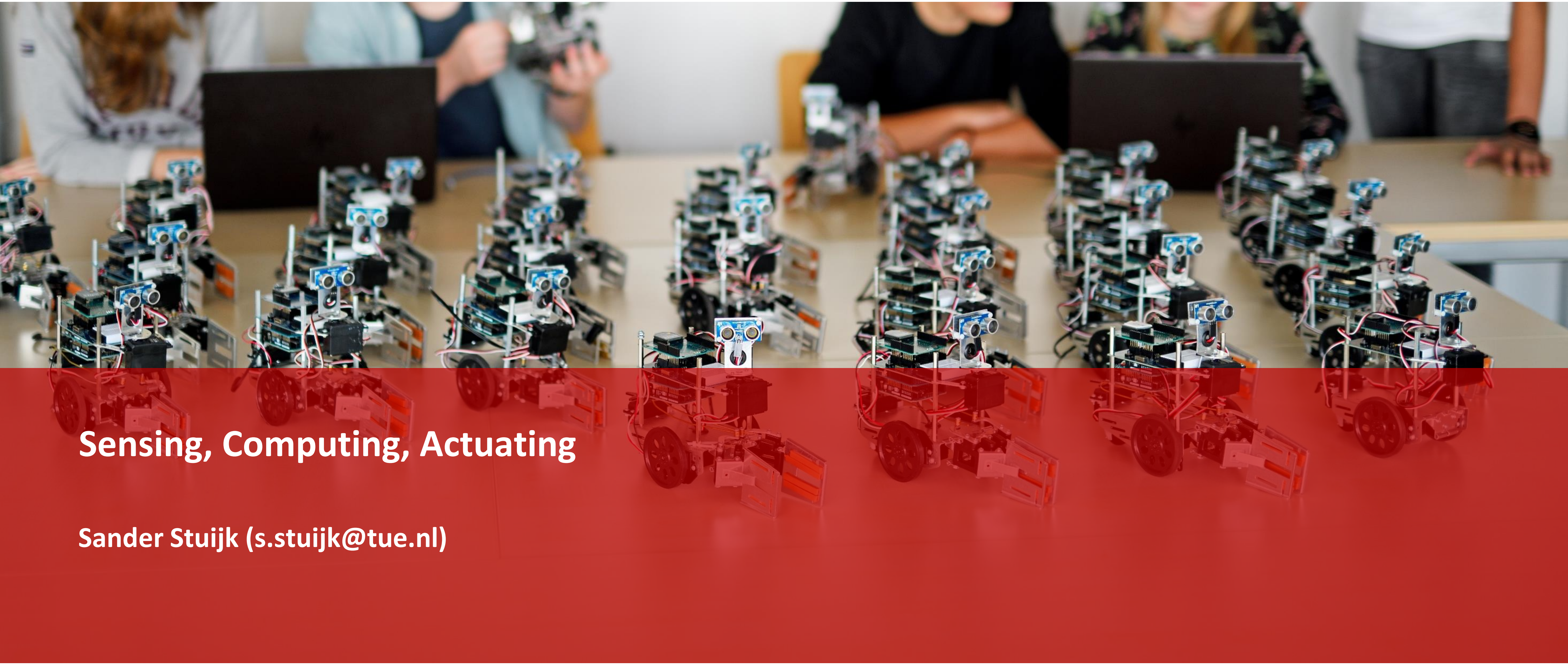


Sensing, Computing, Actuating

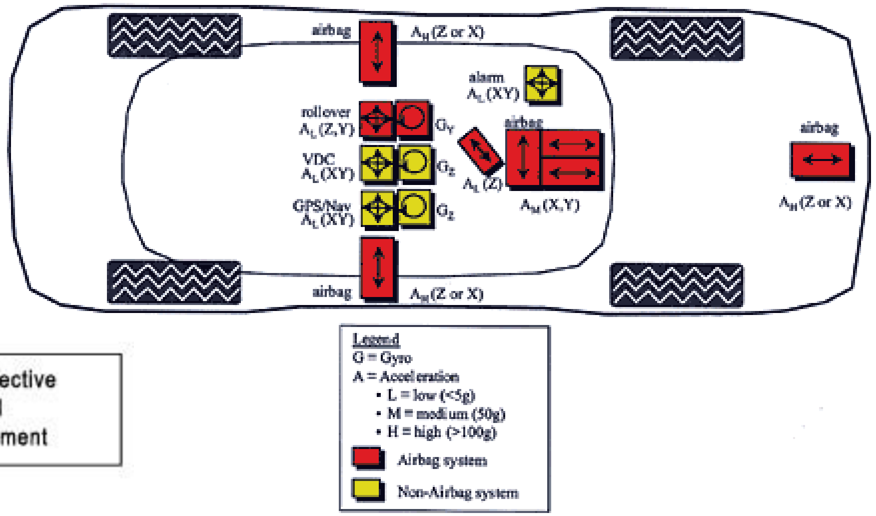
Sander Stuijk (s.stuijk@tue.nl)

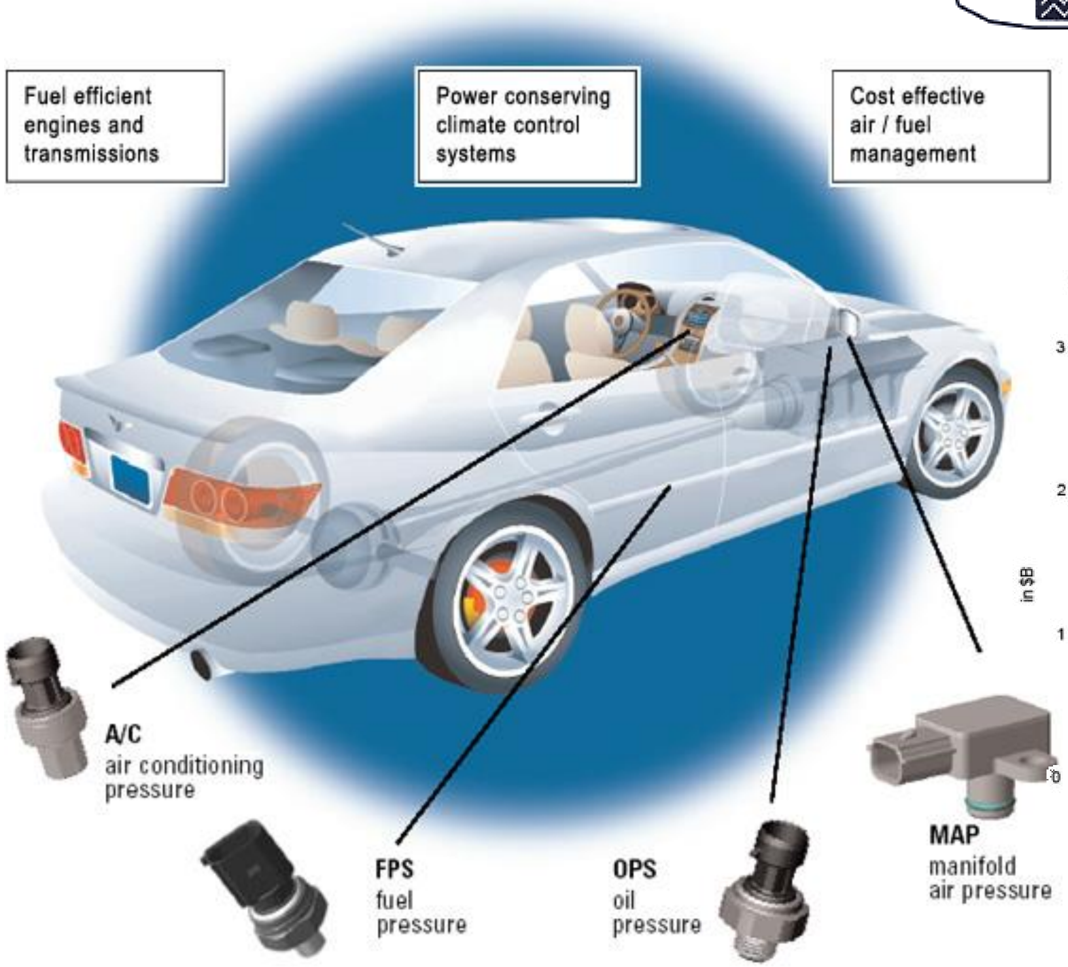
RESISTIVE STRAIN SENSORS

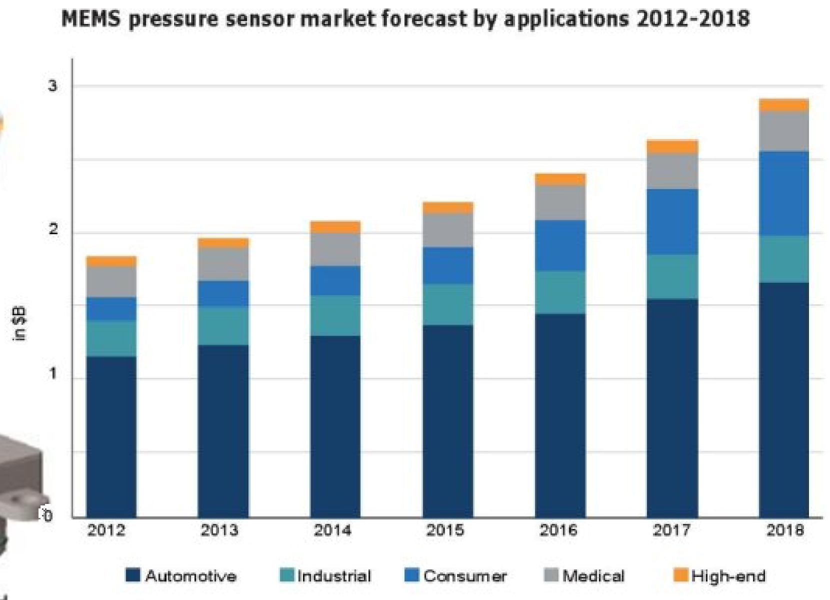
(Chapter 5.8)

Applications of resistive strain sensors

TU/e

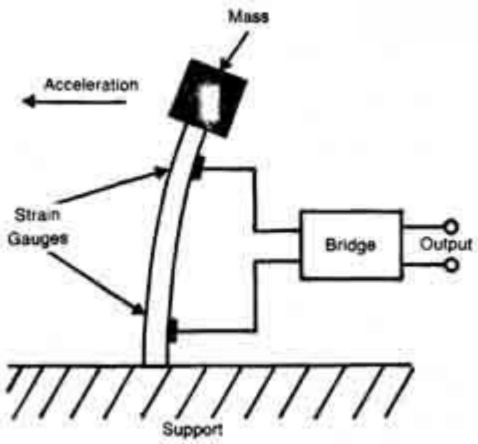


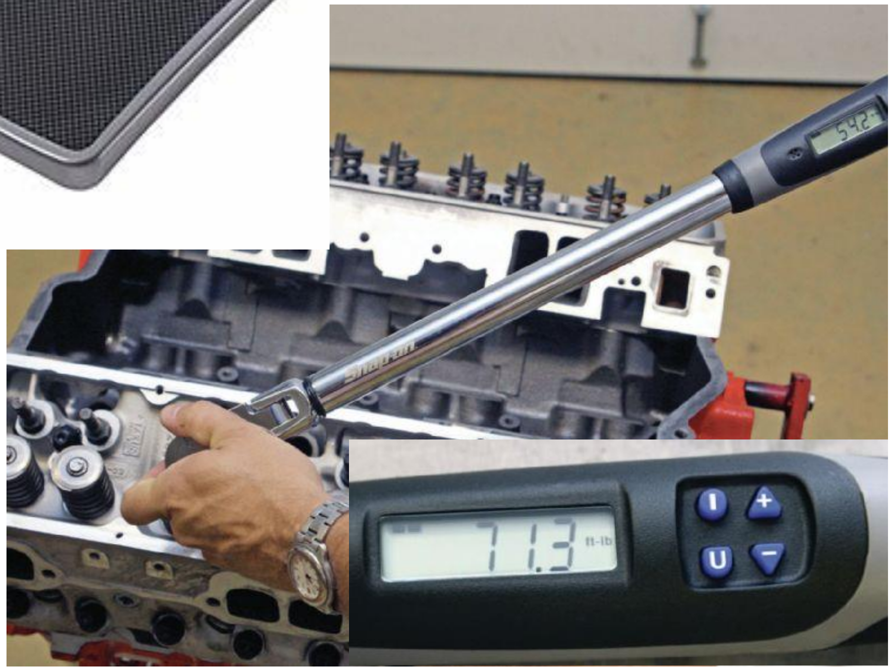




(Yole Développement, April 2013)

Applications of resistive strain sensors





Strain, force, pressure sensors

quantity	excitation	physical effect	sensor
strain	**active**	**resistive effect**	**strain gauge**
acceleration	active	capacitance	capacitive accelerometer
acceleration	active	inductance	inductive accelerometer
acceleration	active	resistive effect	piezoresistive accelerometer
pressure	active	capacitance	capacitive pressure sensor
pressure	active	resistive effect	piezoresistive sensor

resistive effect in strain gauge plays an important role in many sensors for different quantities

Strain gauge

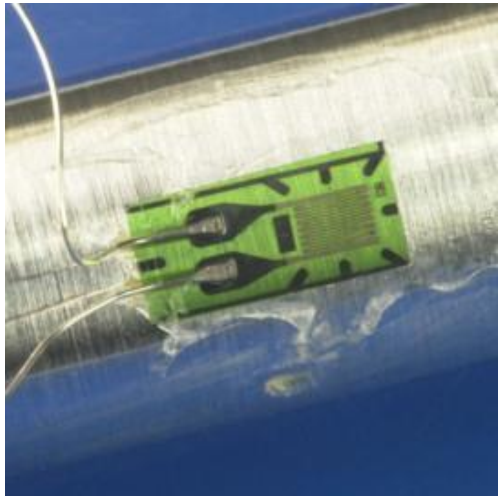
- resistance of a wire

$$R = \rho \frac{l}{a} = \frac{m}{ne^2\tau}\frac{l}{a}$$

- changing temperature affects resistance (thermoresistive effect)
- changing dimensions affects resistance (piezoresistive effect)

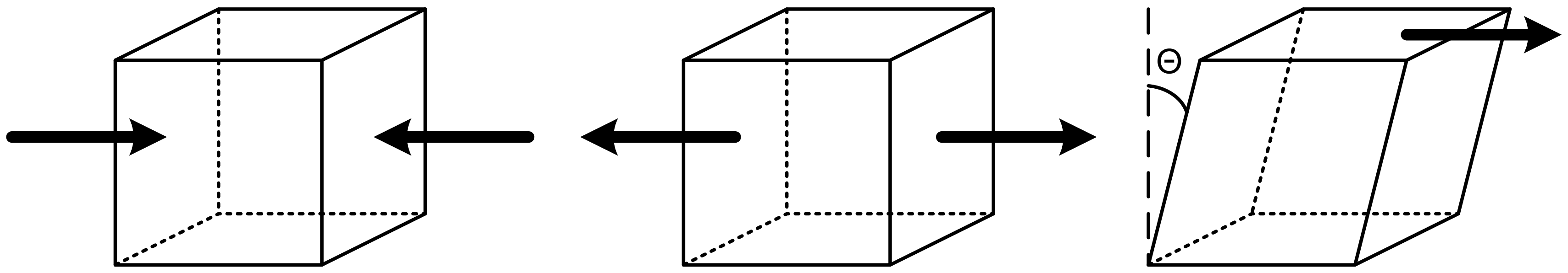
- strain gauges use piezoresistive effect to sense mechanical stress
- sensor based on strain gauges convert mechanical energy to electrical energy

- thermoresistive effect is an error source



Force

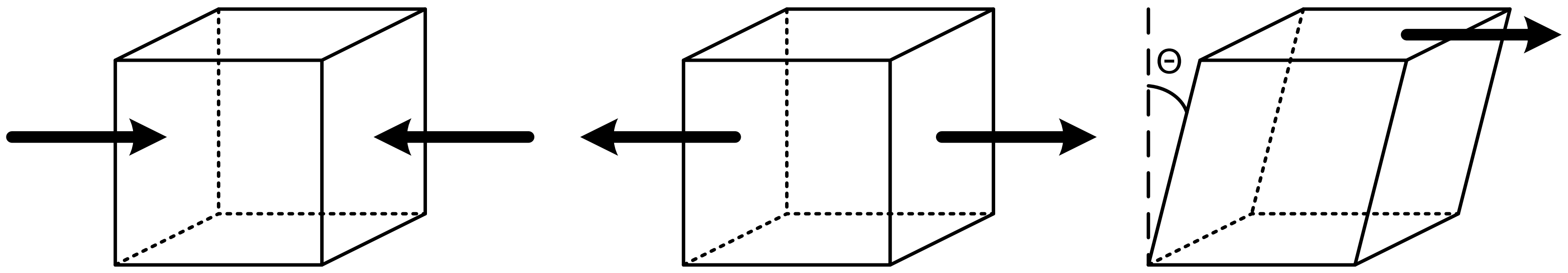
- force leads to deformation of object



- measuring deformation provides opportunity to sense mechanical force, which in turn is related to
  - torque
  - pressure
  - acceleration
  - mass
  - ...

Stress and strain

- force leads to deformation of object



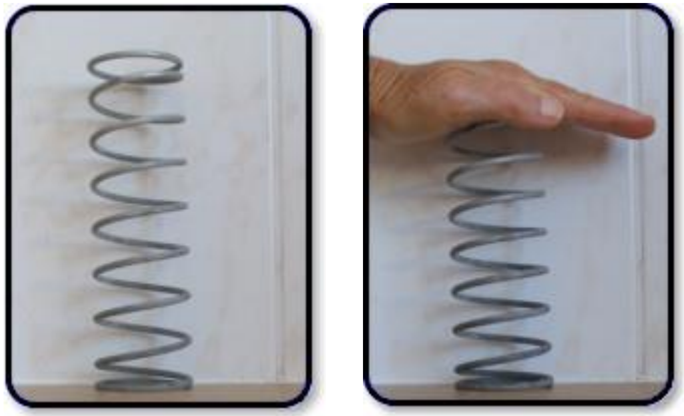
- deformation depends on force per area which is called stress (σ)

$$\sigma = \frac{F}{a}$$

- deformation also depends on
  - material properties
  - length or volume of object
- deformation per unit length (or volume) is called strain (ε)

$$\varepsilon = \frac{dl}{l}$$

Stress and strain

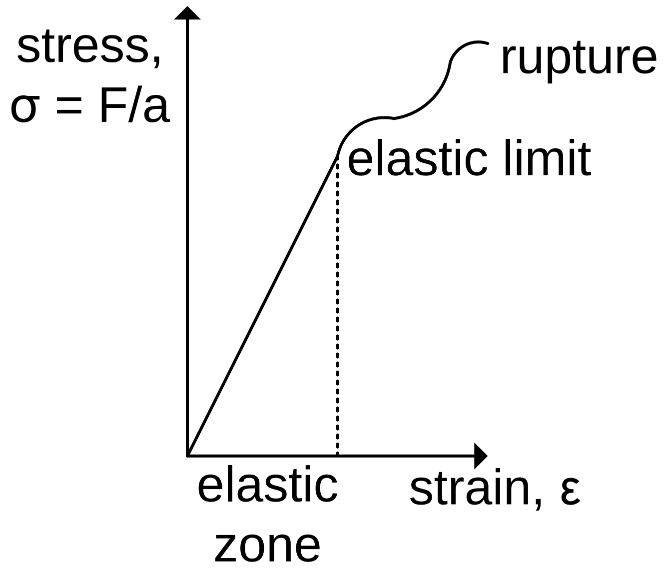


- object deform under force and restores to original state when force is removed (elasticity)
- materials resist deformation (rigidity)

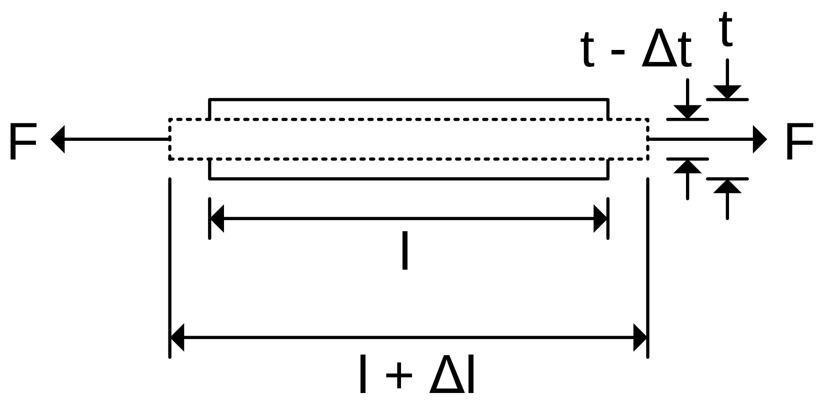
- change in length due to force F given by Hooke's law

$$\sigma = \frac{F}{a} = E\varepsilon = E\frac{dl}{l}$$

  - E – Young's modulus, which depends on
    - material
    - temperature
  - ε – strain (unit deformation, dimensionless)
- strain and stress are proportional in elastic zone



Strain gauge



- resistance of a wire

$$R = \rho \frac{l}{a}$$

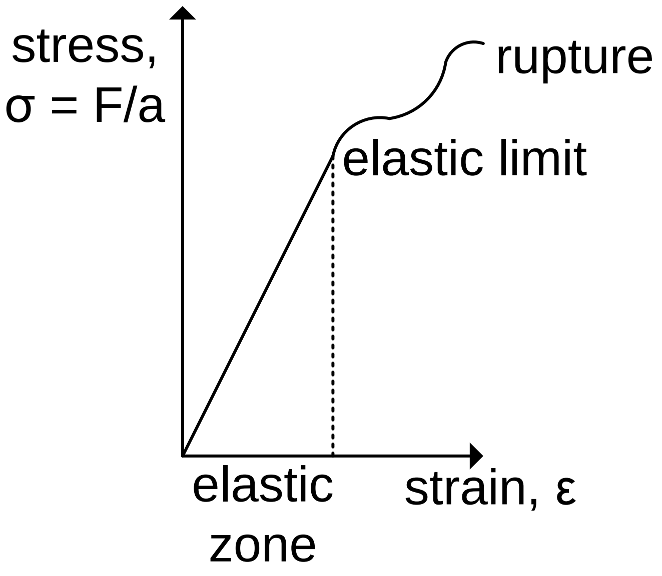
- stretching wire longitudinally changes resistance

$$\frac{dR}{R} = \frac{\Delta\rho}{\rho} + \frac{\Delta l}{l} - \frac{\Delta a}{a}$$

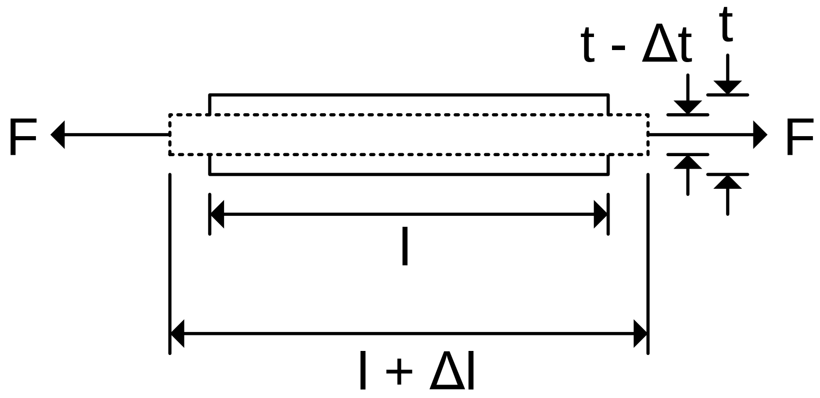
  - use -Δa since area decreases when wire is stretched (+Δl)
- change in length due to force F given by Hooke's law

$$\sigma = \frac{F}{a} = E\varepsilon = E\frac{dl}{l}$$

  - E – Young's modulus, which depends on
    - material
    - temperature
  - ε – strain (unit deformation, dimensionless)
- strain and stress are proportional in elastic zone



Strain gauge



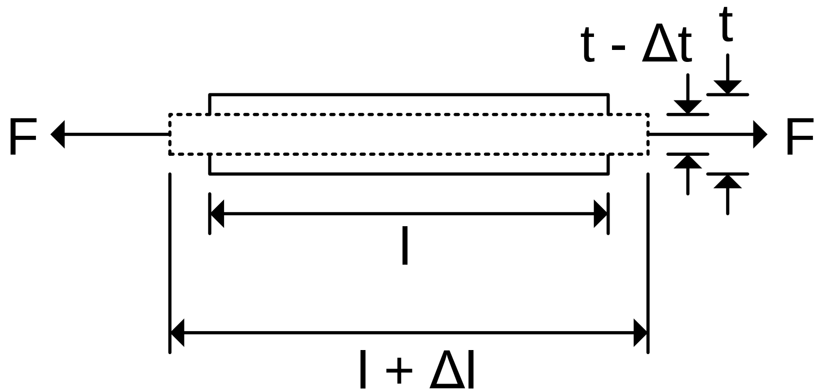
- longitudinal stress changes
  - length of wire (l)
  - thickness of wire (t)

- Poisson ratio gives relation between change in length and thickness

$$\nu = -\frac{dt/t}{dl/l}$$

  - Poisson ratio of perfectly compressible material: 0.0 (e.g. cork)
    - deformation in one direction does not change other direction
  - Poisson ratio of incompressible material: 0.5 (e.g. rubber)
    - volume of this material is constant when stress is applied

Strain gauge



- longitudinal stress changes
  - length of wire (l)
  - thickness of wire (t)

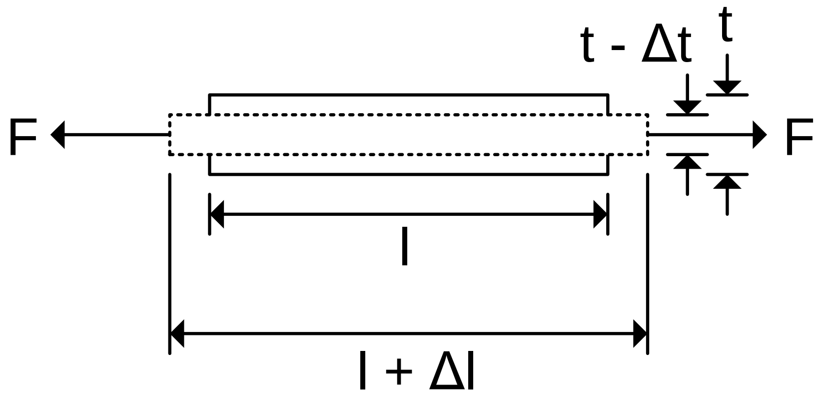
- Poisson ratio gives relation between change in length and thickness

$$\nu = -\frac{dt/t}{dl/l}$$

  - Poisson ratio of metals: 0 < ν < 0.5
    - volume of metal changes when deformed
    - cross-sectional area changes when metals are deformed
    - using same approach as used for volume we can show

$$\left.\begin{array}{l} a = \pi \cdot \left(\frac{t}{2}\right)^2 \Rightarrow \frac{da}{a} = \frac{2dt}{t} \\ \nu = -\frac{dt/t}{dl/l} \quad \Rightarrow \frac{dt}{t} = -\nu\frac{dl}{l} \end{array}\right\} \Rightarrow \frac{da}{a} = -2\nu\frac{dl}{l}$$

Strain gauge



- longitudinal stress changes
  - length of wire (l)
  - thickness of wire (t)

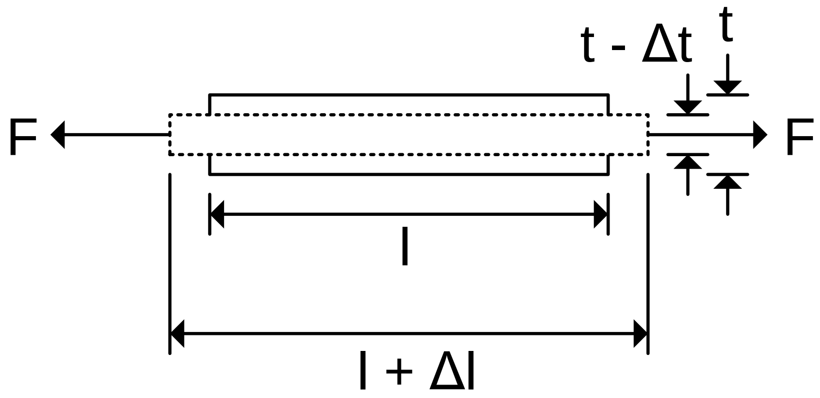
- Poisson ratio gives relation between change in length and thickness

$$\nu = -\frac{dt/t}{dl/l}$$

  - Poisson ratio of metals: $0 < \nu < 0.5$
    - volume of metal changes when deformed
    - consider a circular wire with diameter t (and thus radius: t/2)
    - change in volume per unit volume is then equal to

$$\left.\begin{aligned} V = \pi \cdot \left(\frac{t}{2}\right)^2 \cdot l \;&\Rightarrow \frac{dV}{V} = \frac{dl}{l} + \frac{2dt}{t} \\ \nu = -\frac{dt/t}{dl/l} \;&\Rightarrow \frac{dt}{t} = -\nu \frac{dl}{l} \end{aligned}\right\} \Rightarrow \frac{dV}{V} = \frac{dl}{l} - 2\nu \frac{dl}{l} = \frac{dl}{l}(1 - 2\nu)$$

Strain gauge



- longitudinal stress changes
  - length of wire (l)
  - thickness of wire (t)

- Poisson ratio gives relation between change in length and thickness

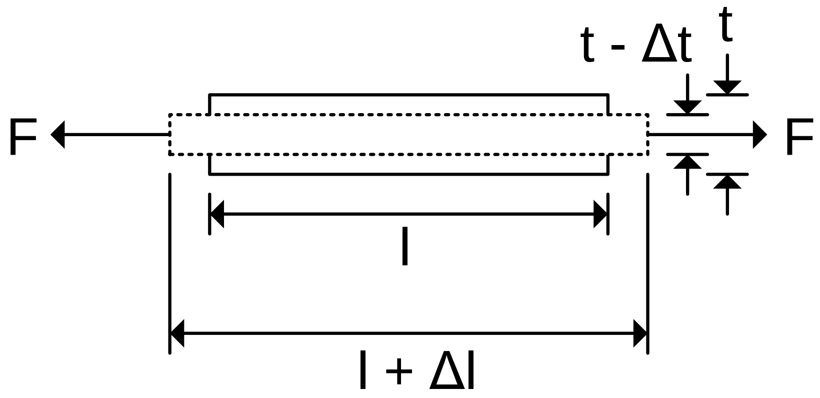
$$\nu = -\frac{dt/t}{dl/l}$$

  - Poisson ratio of metals: $0 < \nu < 0.5$
    - volume of metal changes when deformed
    - because of volume change
      - amplitude of vibrations in metal lattice changes
    - results in change of specific resistivity (for metals)

$$\frac{d\rho}{\rho} = C\frac{dV}{V}$$

      - C – Bridgman's constant

Strain gauge



- longitudinal stress changes
  - length of wire (l)
  - thickness of wire (t)

- stretching wire longitudinally changes resistance

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} - \frac{da}{a}$$

- using results found so far we find

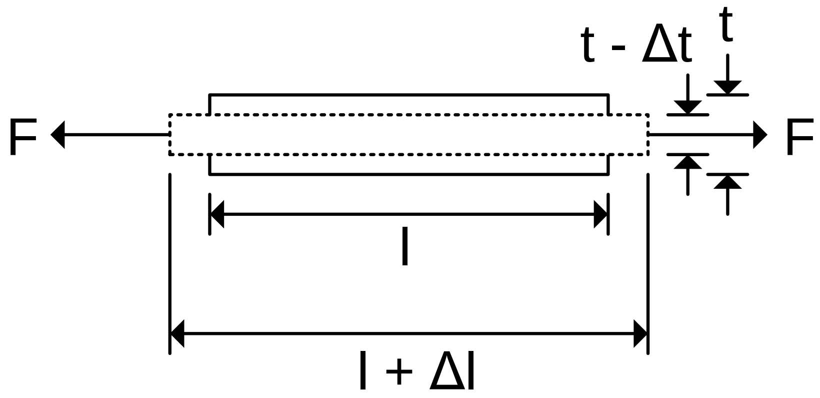
$$\left.\begin{array}{l} \dfrac{dR}{R} = \dfrac{d\rho}{\rho} + \dfrac{dl}{l} - \dfrac{da}{a} \\ \dfrac{d\rho}{\rho} = C\dfrac{dV}{V} \end{array}\right\} \Rightarrow \frac{dR}{R} = C\frac{dV}{V} + \frac{dl}{l} - \frac{da}{a}$$

$$\left.\begin{array}{l} \dfrac{dR}{R} = C\dfrac{dV}{V} + \dfrac{dl}{l} - \dfrac{da}{a} \\ \dfrac{dV}{V} = \dfrac{dl}{l}(1-2\nu), \quad \dfrac{da}{a} = -2\nu\dfrac{dl}{l} \end{array}\right\} \Rightarrow \frac{dR}{R} = \frac{dl}{l}[C(1-2\nu) + 1 + 2\nu]$$

$$\Leftrightarrow \frac{dR}{R} = G\frac{dl}{l}$$

  - G – gauge factor

Strain gauge



- change in resistance related to change in length

$$\frac{dR}{R} = G\frac{dl}{l}$$

- remember Hooke's law (relates stress σ to strain ε)

$$\sigma = \frac{F}{a} = E\varepsilon = E\frac{dl}{l}$$

- change in resistance related to force (per unit area) and strain

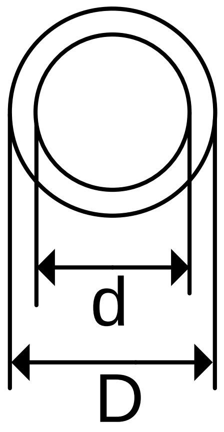
$$\frac{dR}{R} = G\varepsilon = G\frac{F}{aE}$$

- strain gauge can be used to sensor force and its derived quantities
- gauge factor is constant for metals, hence

$$R = R_0 + dR = R_0\left(1 + \frac{dR}{R_0}\right) = R_0(1 + G\varepsilon) = R_0(1 + x)$$

- typically $x < 0.002$

Strain gauge



example – strain gauge attached to aluminum strut

- strain gauge with R = 350 Ω and G = 2.1
- aluminum strut has E = 73 GPa
- outer diameter of the strut: D = 50 mm
- inner diameter of the strut: d = 47.5 mm

what is the change in resistance when the strut supports a load of 1000 kg?

- area supporting the force

$$a = \frac{\pi(D^2 - d^2)}{4} = \frac{\pi\left((50mm)^2 - (47.5mm)^2\right)}{4} = 191mm^2$$

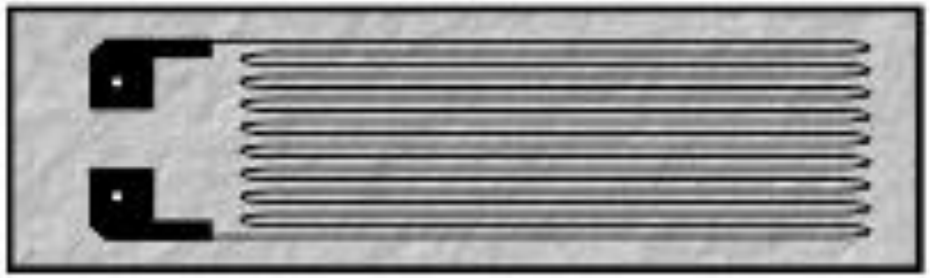
- change in resistance

$$\Delta R = RG\varepsilon = RG\frac{F}{aE} = (350\Omega)(2.1)\frac{9800N}{(191 \cdot 10^{-6}m^2) \cdot (73 \cdot 10^9 Pa)} = 0.5\Omega$$

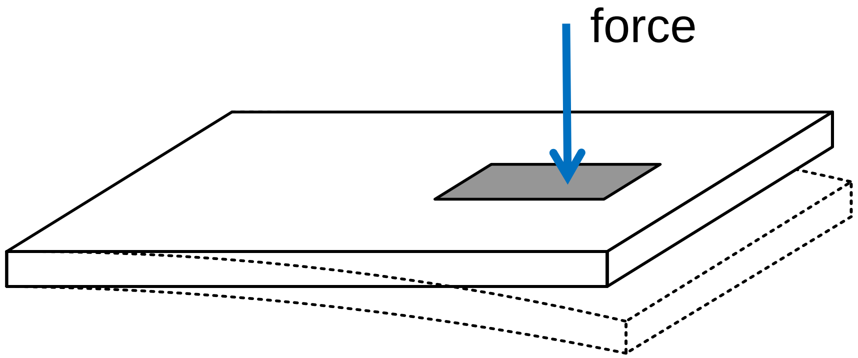
- change in resistance is less than 0.15% of the initial resistance

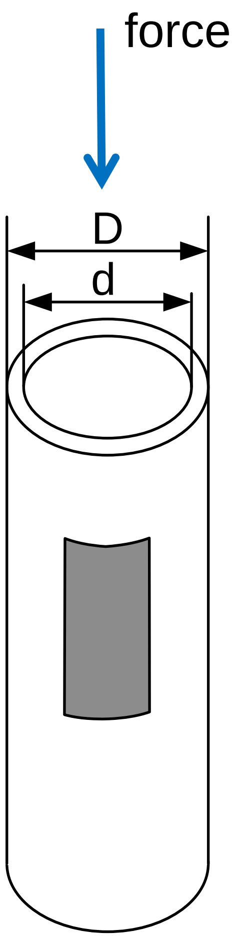
Construction

- bonded strain gauges
  - wires cemented onto a backing or
  - thin film resistor deposited on a substrate
  - resistor forms a long, meandering wire



- strain gauge connected to test object





Interface circuit

- strain gauge in resistive divider
- stress applied to gauge (resistive change ‘x’)
- **what is the output voltage v_o? (assume k = R_1/R_0 = 1)**

$$v_o = \left(\frac{R_2}{R_1 + R_2}\right) V_r = \left(\frac{R_0(1+x)}{kR_0 + R_0(1+x)}\right) V_r = \left(\frac{1+x}{k+1+x}\right) V_r$$

non-linearity

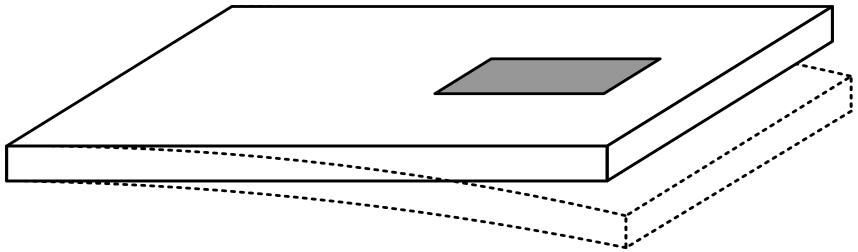
  - for strain gauges holds that x << k (typically x < 0.002)

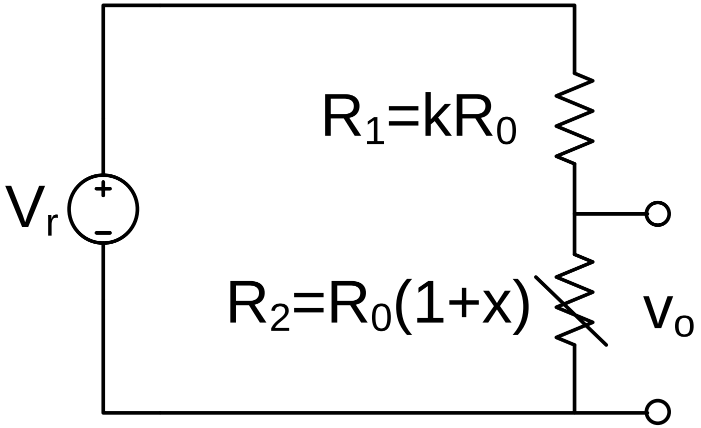
$$v_o \approx \left(\frac{1+x}{k+1}\right) V_r = \frac{1}{k+1} V_r + \frac{x}{k+1} V_r$$

  - maximal sensitivity when k = 1

$$v_o \approx \frac{1}{2} V_r + \frac{x}{2} V_r$$

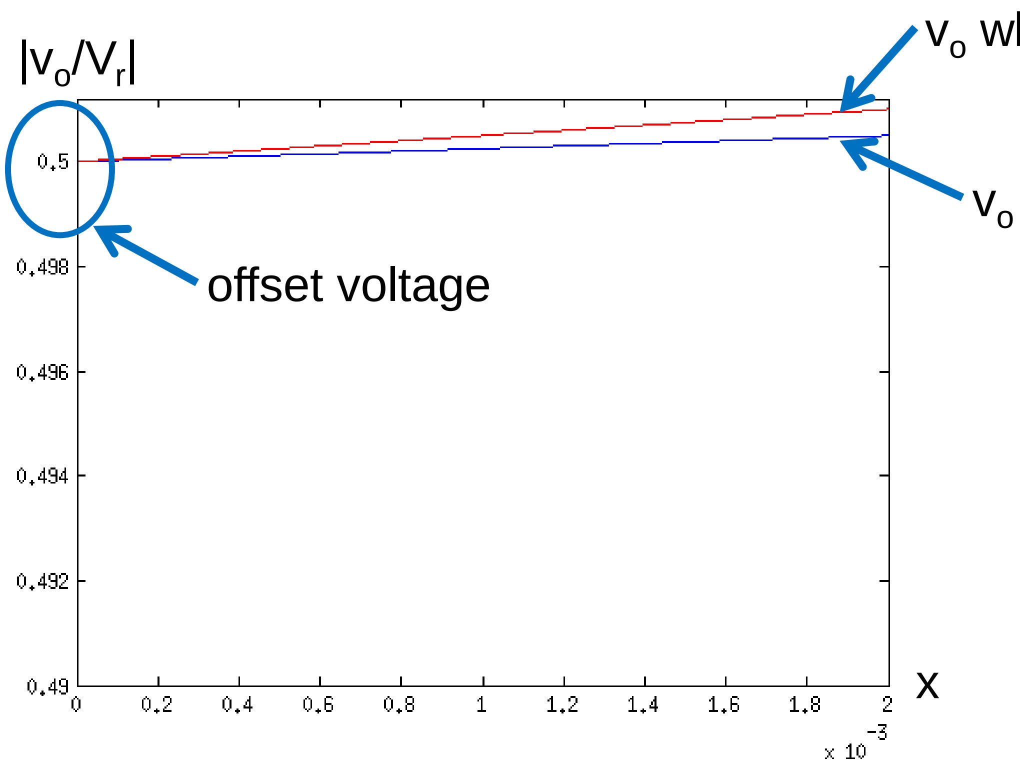
offset voltage

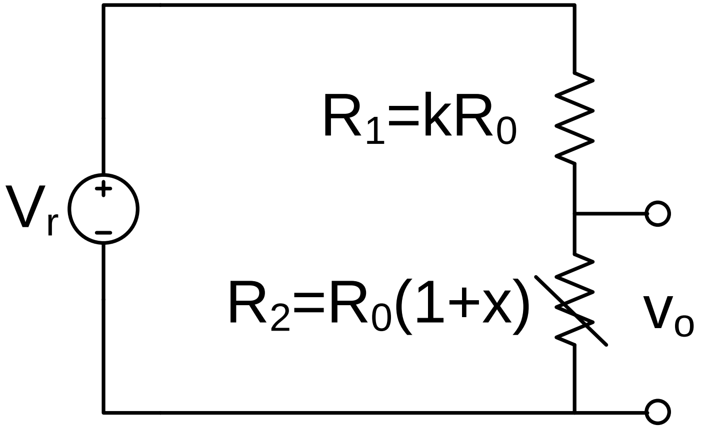




Interface circuit

- strain gauge in resistive divider
- stress applied to gauge (resistive change ‘x’)
- two problems when measuring output voltage
  - non-linearity in response
  - offset voltage present





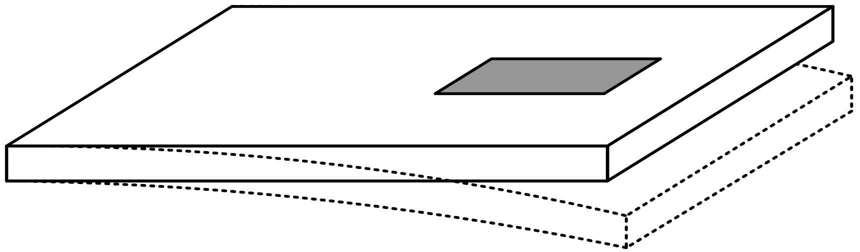
Interface circuit

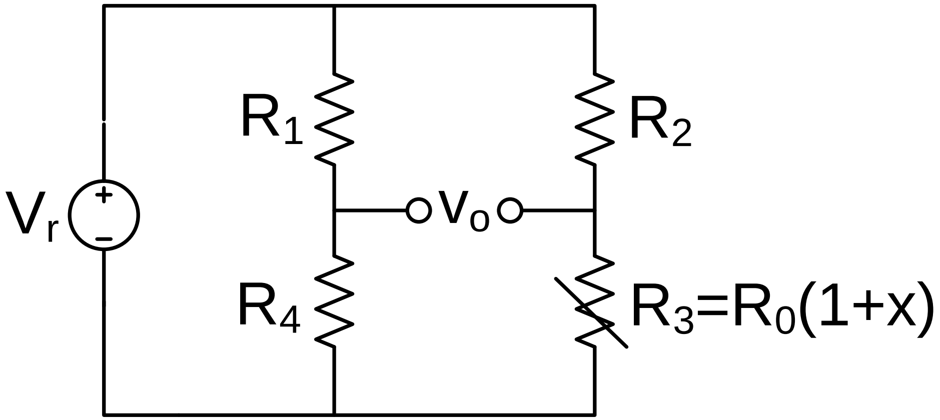
- remove offset voltage by placing strain gauge in bridge
- stress applied to gauge (resistive change ‘x’)
- **what is the output voltage v_o? (assume $k = R_1/R_4 = R_2/R_0 = 1$)**

$$v_o = \left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3}\right) V_r = \left(\frac{R_0}{2R_0} - \frac{R_0(1+x)}{R_0(2+x)}\right) V_r = \left(\frac{1}{2} - \frac{1+x}{2+x}\right) V_r = \left(\frac{-x}{4+2x}\right) V_r \approx -\frac{x}{4} V_r$$

non-linearity

- intermezzo: two sources of non-linearity
  - strain gauge itself does not adhere to $R = R_0(1+x)$
  - interface circuit causes non-linear resistance – voltage relation





Interface circuit

- remove offset voltage by placing strain gauge in bridge
- stress applied to gauge (resistive change ‘x’)
- **what is the output voltage v_o? (assume $k = R_1/R_4 = R_2/R_0 = 1$)**

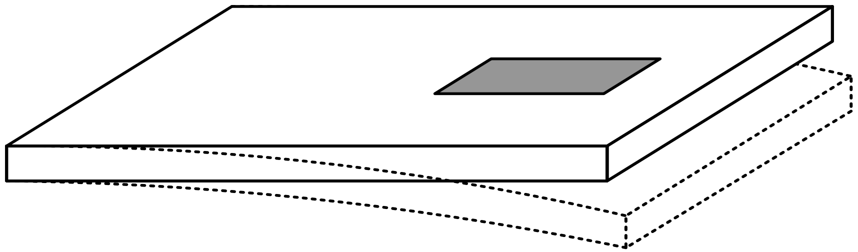
$$v_o = \left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3}\right)V_r = \left(\frac{R_0}{2R_0} - \frac{R_0(1+x)}{R_0(2+x)}\right)V_r = \left(\frac{1}{2} - \frac{1+x}{2+x}\right)V_r = \left(\frac{-x}{4+2x}\right)V_r \approx -\frac{x}{4}V_r$$

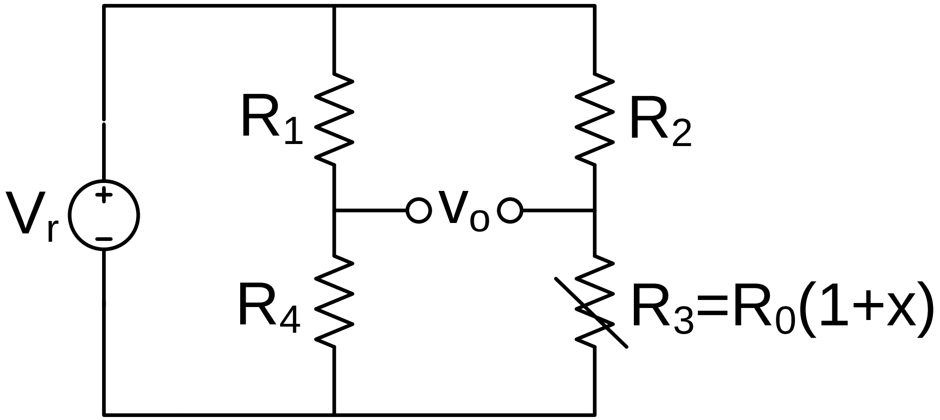
non-linearity

- compare output voltage bridge and divider

$$v_{o,divider} \approx \frac{1}{2}V_r + \frac{x}{2}V_r$$

- bridge removes offset
- bridge reduces sensitivity





Interface circuit

- increase sensitivity by adding extra strain gauge to bridge
- stress applied to gauge (resistive change 'x')
- **what is the output voltage v_o? (assume $k = R_0/R_4 = R_2/R_0 = 1$)**

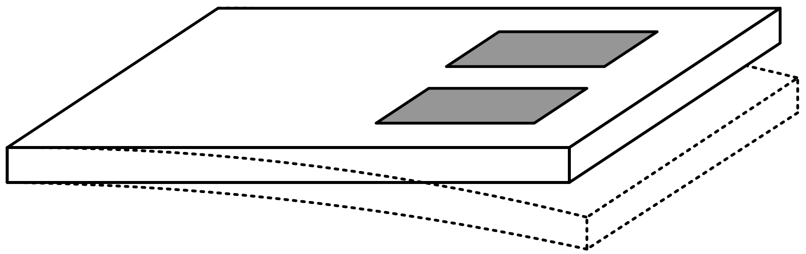
$$v_o = \left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3}\right)V_r = \left(\frac{R_0}{R_0(2+x)} - \frac{R_0(1+x)}{R_0(2+x)}\right)V_r = \left(\frac{-x}{2+x}\right)V_r \approx -\frac{x}{2}V_r$$

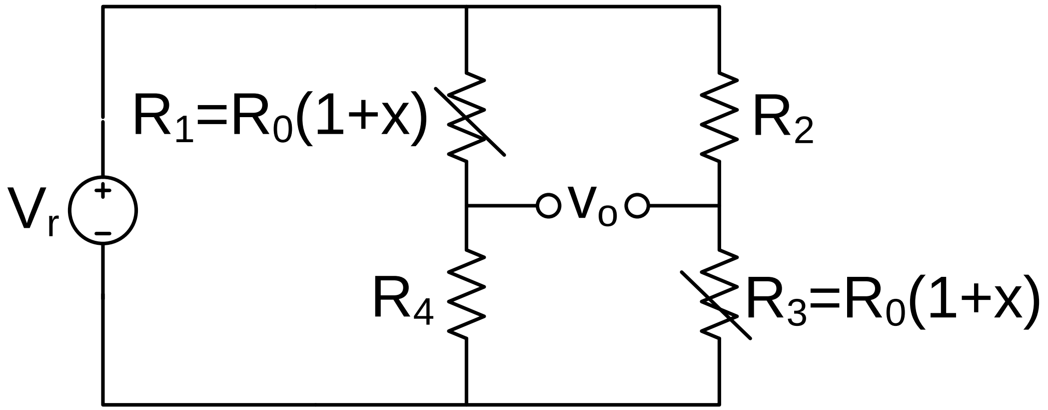
non-linearity

- compare output voltage to single sensor solution

$$v_{o,single} = \frac{-x}{4+2x}V_r \approx \frac{-x}{4}V_r$$

- extra sensor increases sensitivity



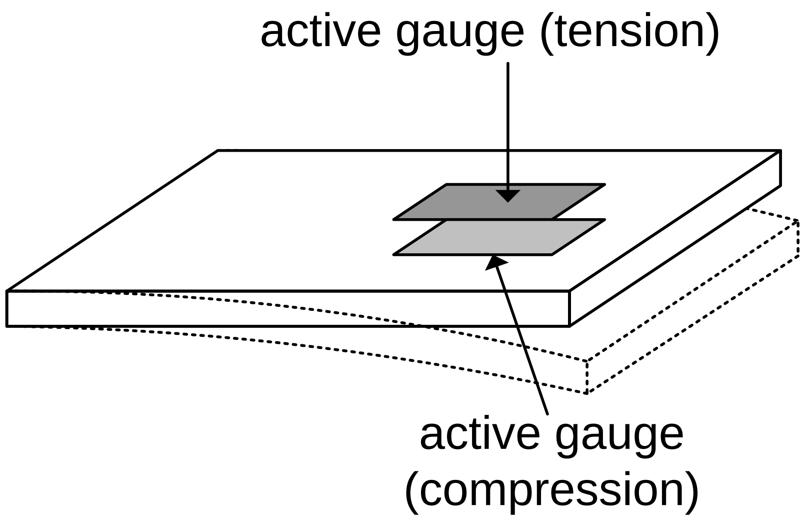


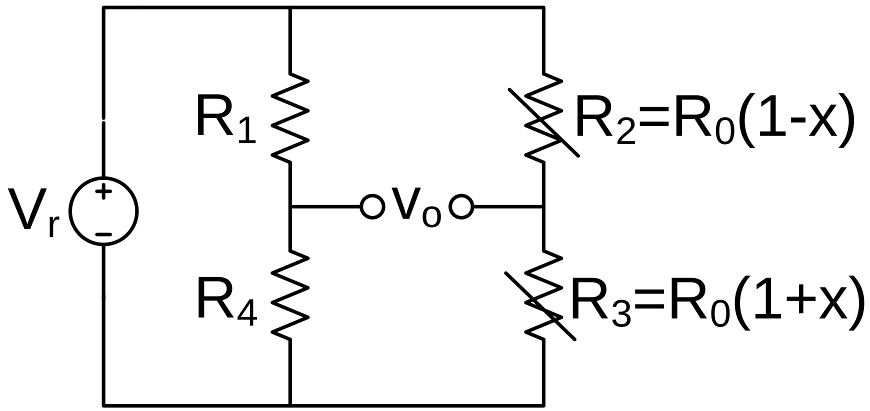
Interface circuit

- remove non-linearity by adding applying opposing signal to gauge
- stress applied to gauge (resistive change 'x')
- **what is the output voltage v_o? (assume $k = R_0/R_4 = R_1/R_0 = 1$)**

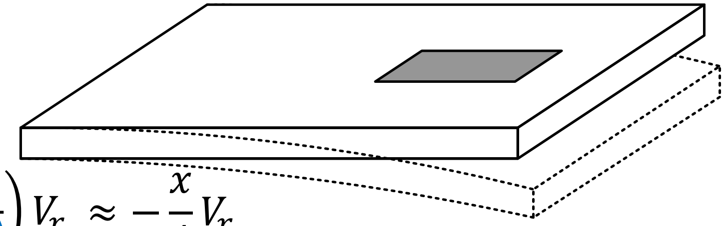
$$v_o = \left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3}\right) V_r = \left(\frac{R_0}{2R_0} - \frac{R_0(1+x)}{R_0(1-x) + R_0(1+x)}\right) V_r = \left(\frac{R_0}{2R_0} - \frac{R_0(1+x)}{2R_0}\right) V_r = \frac{-x}{2} V_r$$

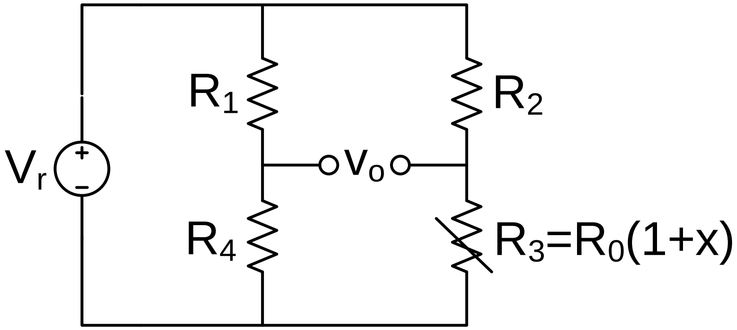
- sensitivity equal to previous arrangement
- non-linearity removed





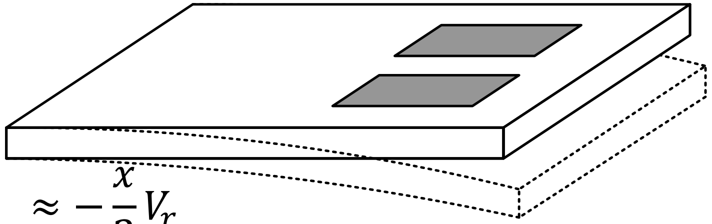
Different configurations

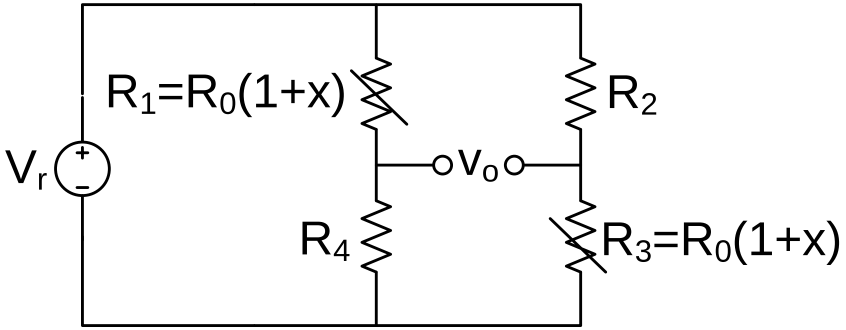




$$v_o = \left(\frac{-x}{4+2x}\right)V_r \approx -\frac{x}{4}V_r$$

non-linearity



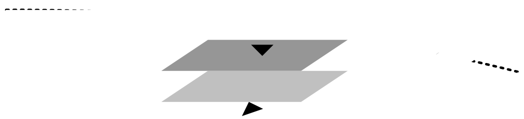


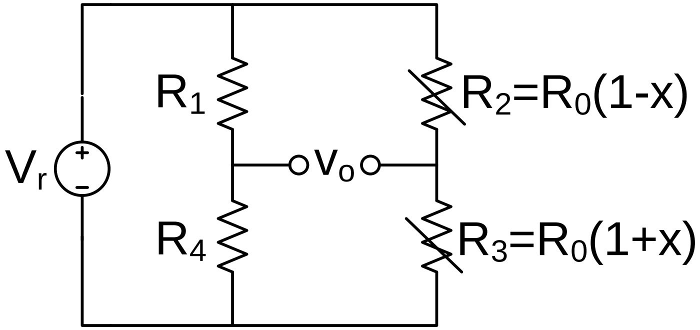
$$v_o = \left(\frac{-x}{2+x}\right)V_r \approx -\frac{x}{2}V_r$$

non-linearity

increased sensitivity

active gauge (tension)





$$v_o = \frac{-x}{2}V_r$$

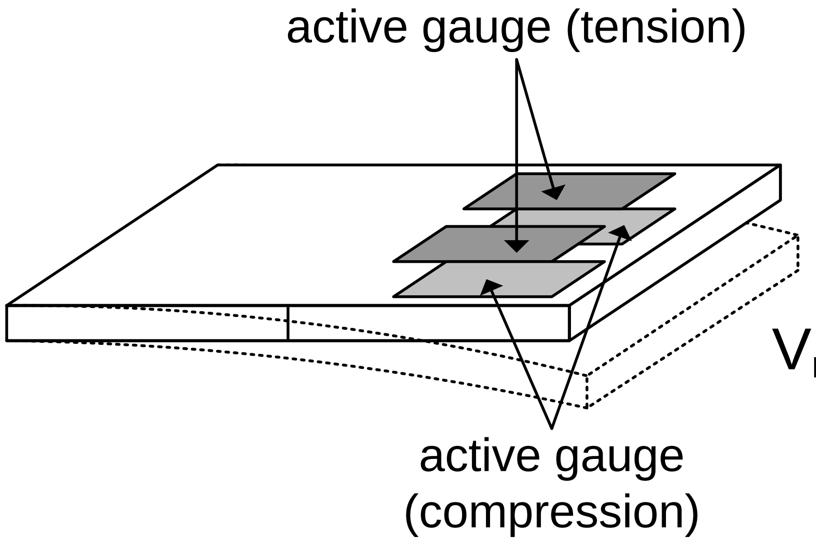
active gauge (compression)

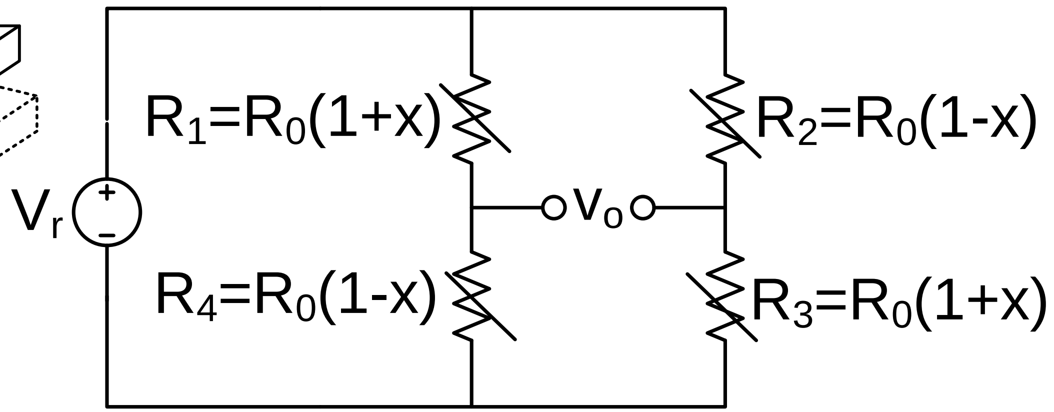
Interface circuit

- add more strain gauges to increase sensitivity
- stress applied to gauge (resistive change ‘x’)
- **what is the output voltage v_o?**

$$v_o = \left(\frac{R_4}{R_1+R_4} - \frac{R_3}{R_2+R_3}\right)V_r = \left(\frac{R_0(1-x)}{R_0(1+x)+R_0(1-x)} - \frac{R_0(1+x)}{R_0(1-x)+R_0(1+x)}\right)V_r$$

$$= \left(\frac{1-x}{2} - \frac{1+x}{2}\right)V_r = \frac{-2x}{2}V_r = -xV_r$$

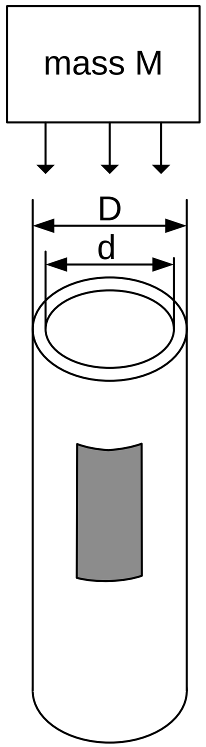


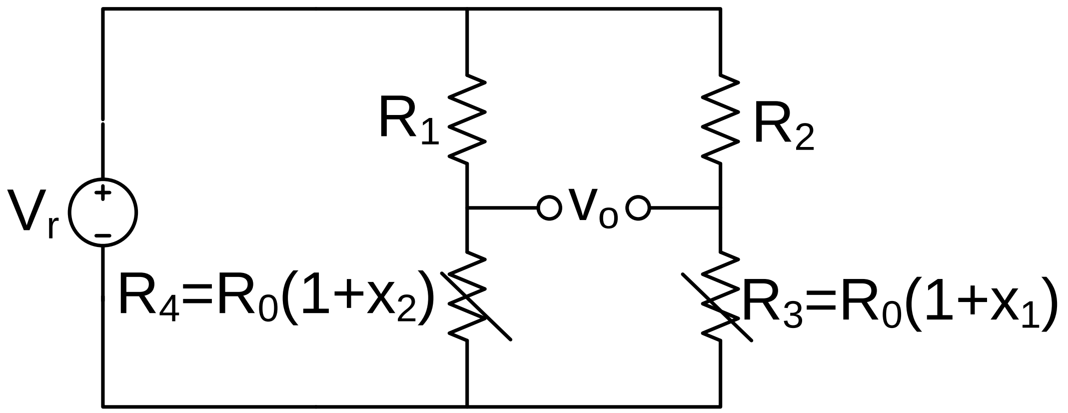


Example – electronic two armed balance

Example – electronic two armed balance

- solution: place to two load cells in bridge circuit
- specification
  - D = 50.0 mm, d = 47.5 mm
  - E = 73.0 GPa = 73.0 · 109 Pa
  - G = 2.1, R0 = 350 Ω
  - R1 = R2 = 350 Ω
  - Vr = 10 V
- **what is the output voltage vo when load of 1000 kg is placed on R4 and a load of 2000 kg on R3?**





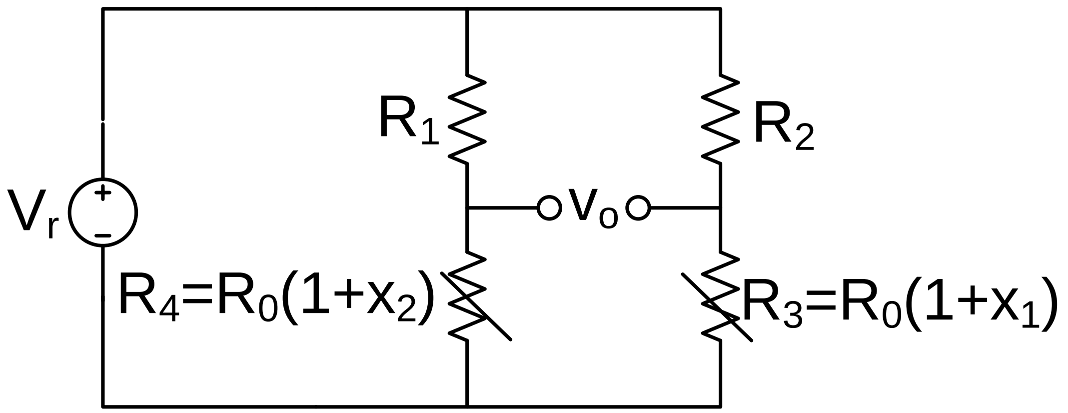
Example – electronic two armed balance

- step 1: compute output voltage of bridge circuit

$$v_o = \left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3}\right) V_r = \left(\frac{R_0(1 + x_2)}{R_0 + R_0(1 + x_2)} - \frac{R_0(1 + x_1)}{R_0 + R_0(1 + x_1)}\right) V_r$$

$$= \left(\frac{1 + x_2}{2 + x_2} - \frac{1 + x_1}{2 + x_1}\right) V_r = \left(\frac{(1 + x_2)(2 + x_1)}{(2 + x_1)(2 + x_2)} - \frac{(1 + x_1)(2 + x_2)}{(2 + x_1)(2 + x_2)}\right) V_r$$

$$= \frac{x_2 - x_1}{(2 + x_1)(2 + x_2)} V_r$$



Example – electronic two armed balance

- step 2: compute change in resistance due to load of 1000 kg

$$dR = RG\varepsilon = RG\frac{dl}{l} = RG\frac{F}{AE}$$

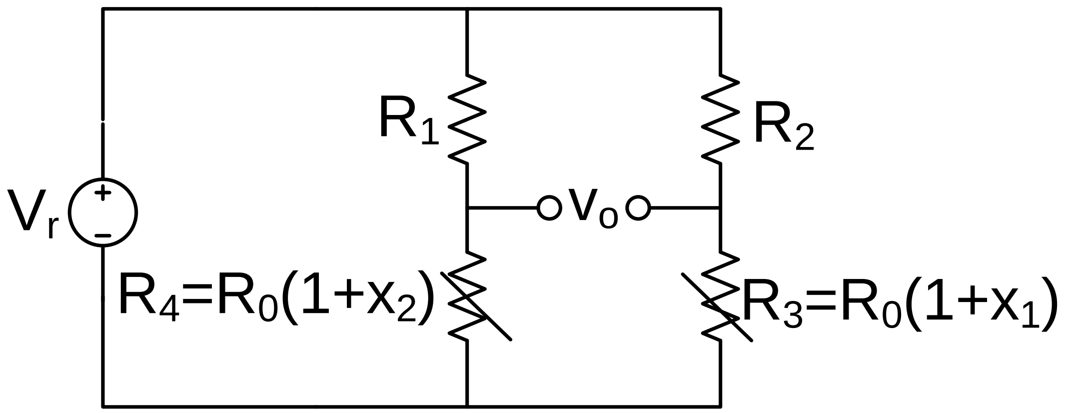
  - area on which load is applied

$$A = \frac{\pi(D^2 - d^2)}{4} = \frac{\pi\left((50.0mm)^2 - (47.5mm)^2\right)}{4} = 191.4mm^2$$

  - change in resistance

$$dR = 350\Omega \cdot 2.1 \cdot \frac{9800N}{(191.4 \cdot 10^{-6}m^2)(73.0 \cdot 10^9 Pa)} = 0.5\Omega$$

$$dR = \frac{0.5\Omega}{350\Omega} \cdot 100\% \approx 0.15\%$$



Example – electronic two armed balance

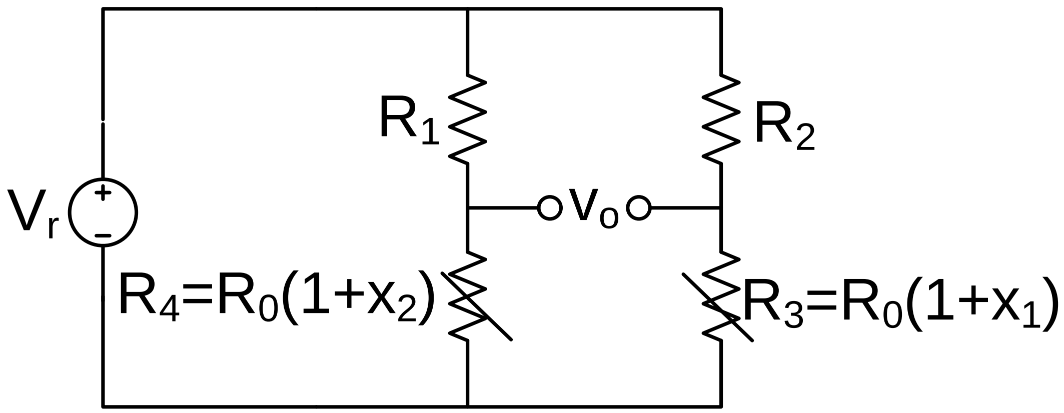
- step 3: compute change in x_2 due to load of 1000 kg

$$R_4 = R_0(1 + x_2) \Rightarrow 350.5\Omega = 350\Omega(1 + x_2)$$

$$\Leftrightarrow x_2 = 0.0014$$

- step 4: compute change in x1 due to load of 2000 kg
  - relation between change in R and weight is linear

$$\Rightarrow x_1 = 2x_2$$



Example – electronic two armed balance

- step 5: compute output voltage of circuit for given loads

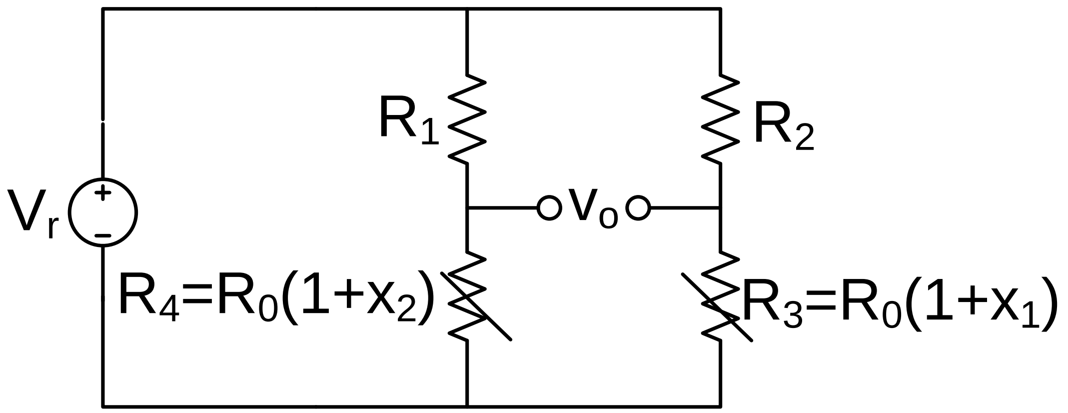
$$v_o = \frac{x_2 - x_1}{(2+x_1)(2+x_2)}V_r = \frac{x_2 - 2x_2}{(2+2x_2)(2+x_2)}V_r = \frac{-x_2}{(2+2x_2)(2+x_2)}V_r$$

  - when ignoring non-linearity (assume $x_2 << 2$)

$$v_o \approx \frac{-x_2}{4}V_r = \frac{-0.0014}{4}10V = -3.5mV$$

  - taking non-linearity into account

$$v_o = -3.5mV$$



ERROR SOURCES

(Chapter 4.8)

Strain gauge

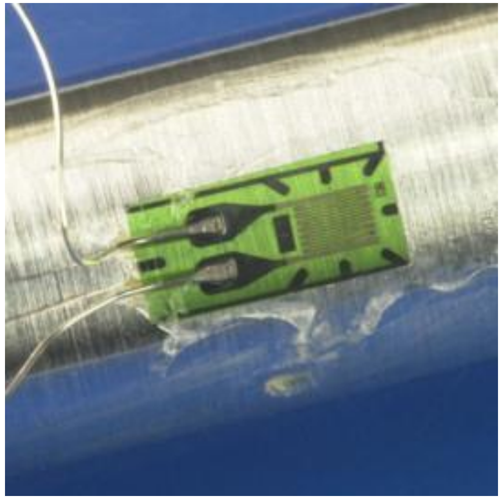
- resistance of a wire

$$R = \rho \frac{l}{a} = \frac{m}{ne^2\tau} \frac{l}{a}$$

- changing temperature affects resistance (thermoresistive effect)
- changing dimensions affects resistance (piezoresistive effect)

- strain gauges use piezoresistive effect to sense mechanical stress
- sensor based on strain gauges convert mechanical energy to electrical energy

- thermoresistive effect is an error source



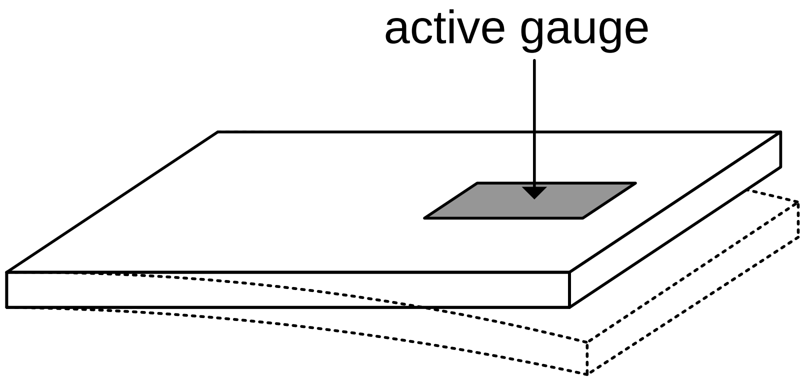
Interface circuit

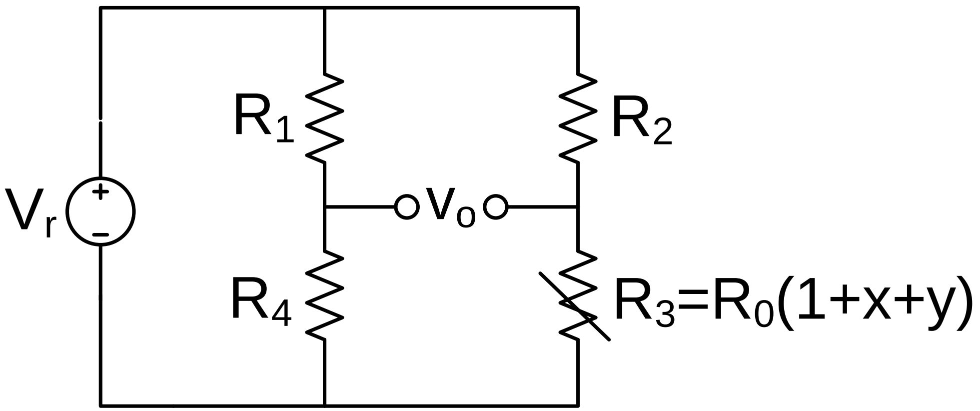
- strain gauge in bridge circuit
- stress applied to active gauge (resistive change ‘x’)
- temperature change applied to strain gauge (resistive change ‘y’)
- **what is the output voltage v_o? (assume $k = R_1/R_4 = R_2/R_0 = 1$)**

$$v_o = \left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3}\right)V_r = \left(\frac{R_0}{2R_0} - \frac{R_0(1 + x + y)}{R_0 + R_0(1 + x + y)}\right)V_r = \left(\frac{1}{2} - \frac{1 + x + y}{2 + x + y}\right)V_r$$

$$= \frac{-x - y}{4 + 2x + 2y}V_r \approx \frac{-x - y}{4}V_r$$

- change in temperature leads to temperature error





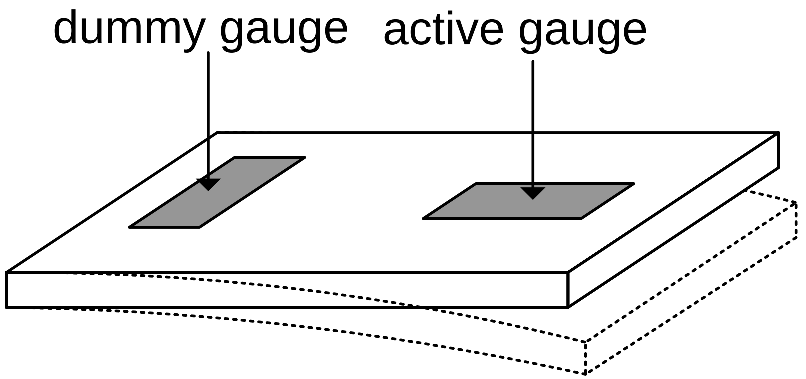
Interface circuit

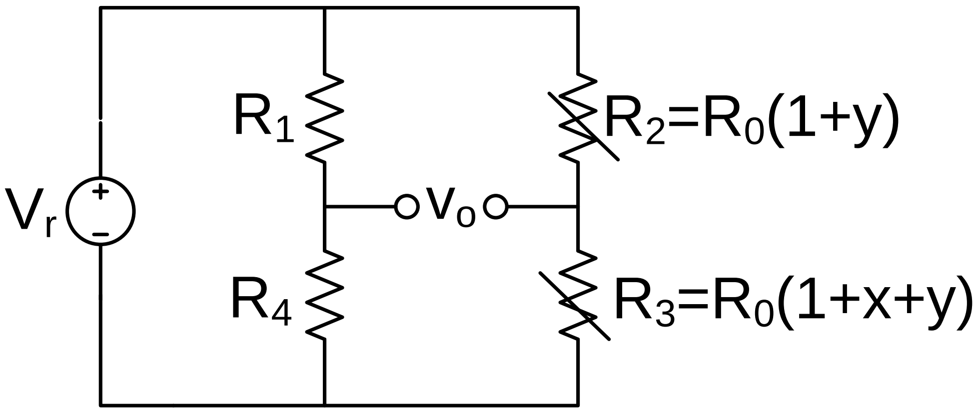
- add passive strain gauge (only 'y' applied)
- stress applied to active gauge (resistive change 'x')
- temperature change applied to strain gauge (resistive change 'y')
- **what is the output voltage v_o? (assume $k = R_1/R_4 = 1$)**

$$v_o = \left(\frac{R_4}{R_1+R_4} - \frac{R_3}{R_2+R_3}\right)V_r = \left(\frac{R_0}{2R_0} - \frac{R_0(1+x+y)}{R_0(1+y)+R_0(1+x+y)}\right)V_r = \left(\frac{1}{2} - \frac{1+x+y}{2+x+2y}\right)V_r$$

$$= \frac{-x}{4+2x+4y}V_r \approx \frac{-x}{4}V_r$$

- dummy gauge removes temperature error

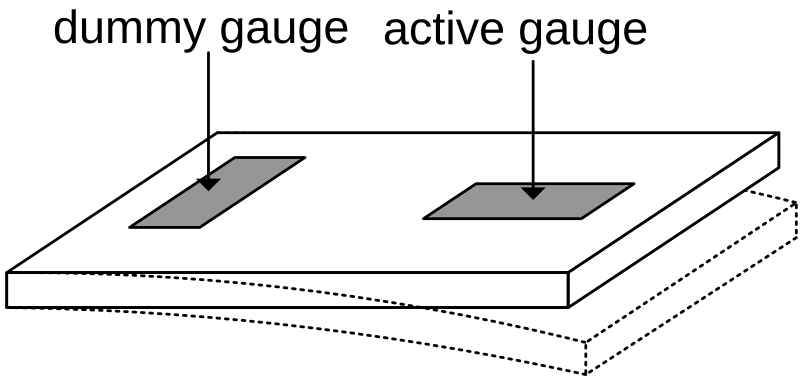


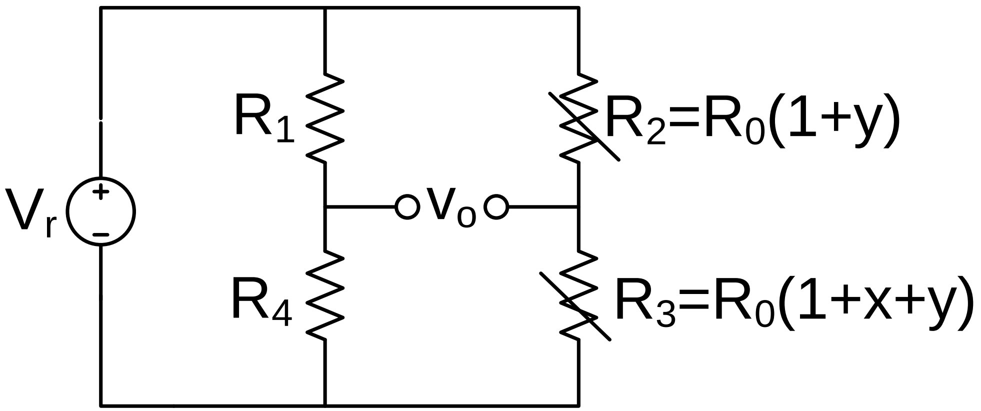


Interface circuit (summary)

- error sources (discussed)
  - non-linearity in strain gauge
  - non-linearity due to interface circuit
  - temperature dependency

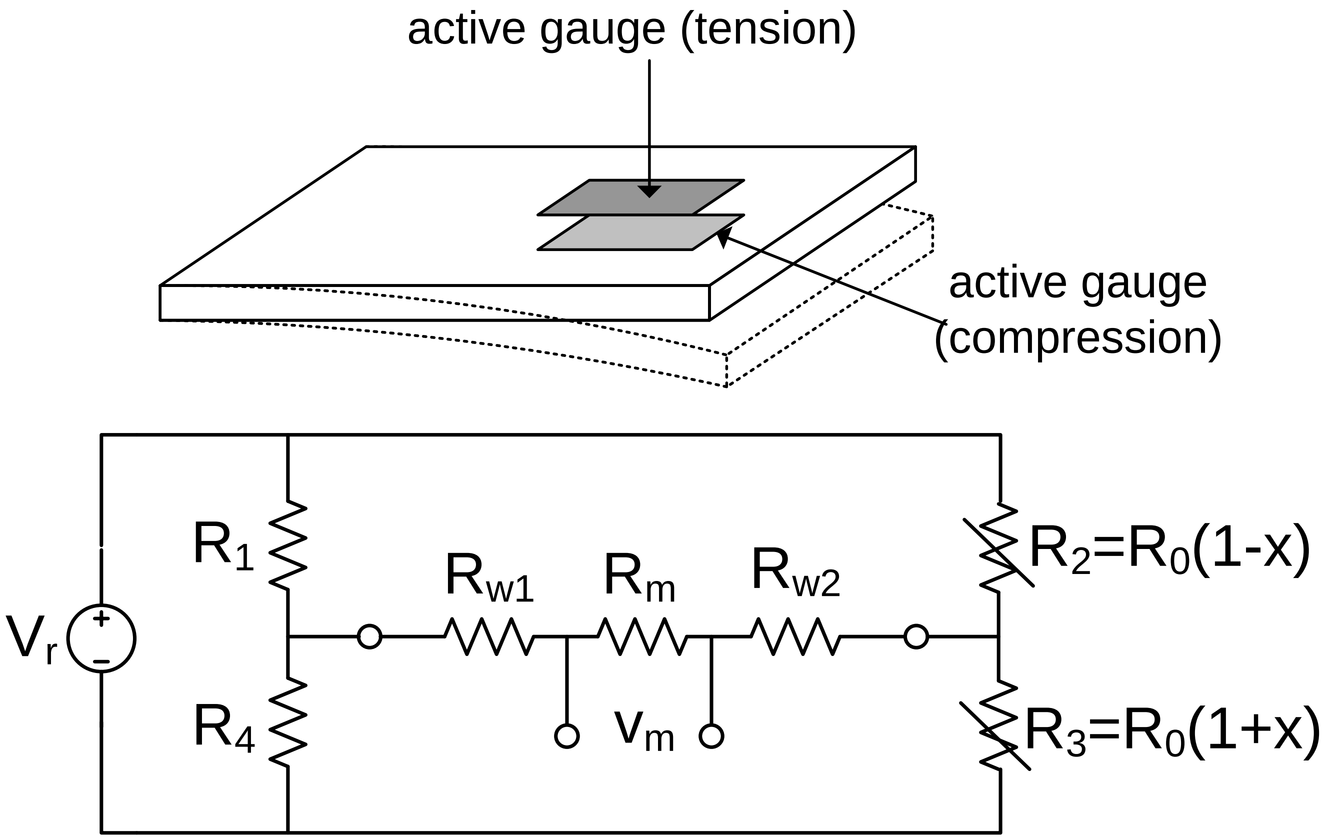
- additional error sources
  - lead-wire resistance
  - loading effect





Loading effect and lead-wire resistance

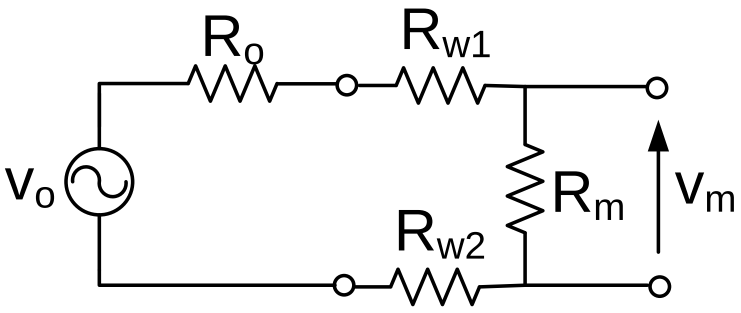
- sensor circuit (bridge) connected to
  - measurement device (with resistance R_m)
  - using two wires (with resistance R_{w1} and R_{w2})
- **what is the measured voltage v_m? (assume $k = R_1/R_4 = 1$)**

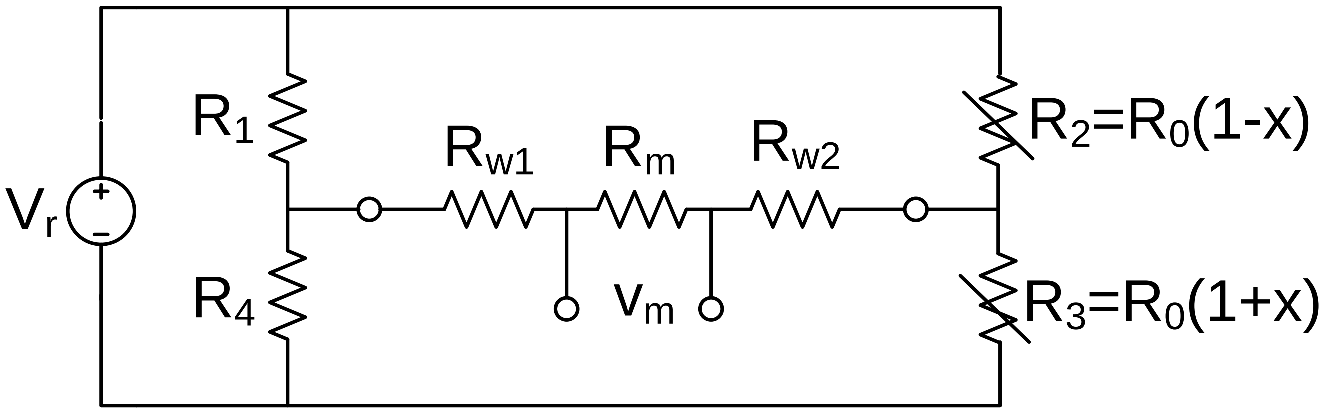


Loading effect and lead-wire resistance

- **what is the measured voltage v_m? (assume $k = R_1/R_4 = 1$)**
- step 1: Thevenin equivalent circuit (of bridge)
  - open circuit voltage

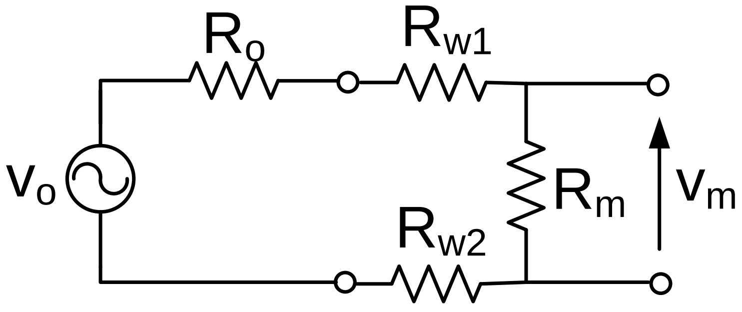
$$v_o = \frac{-x}{2} V_r$$





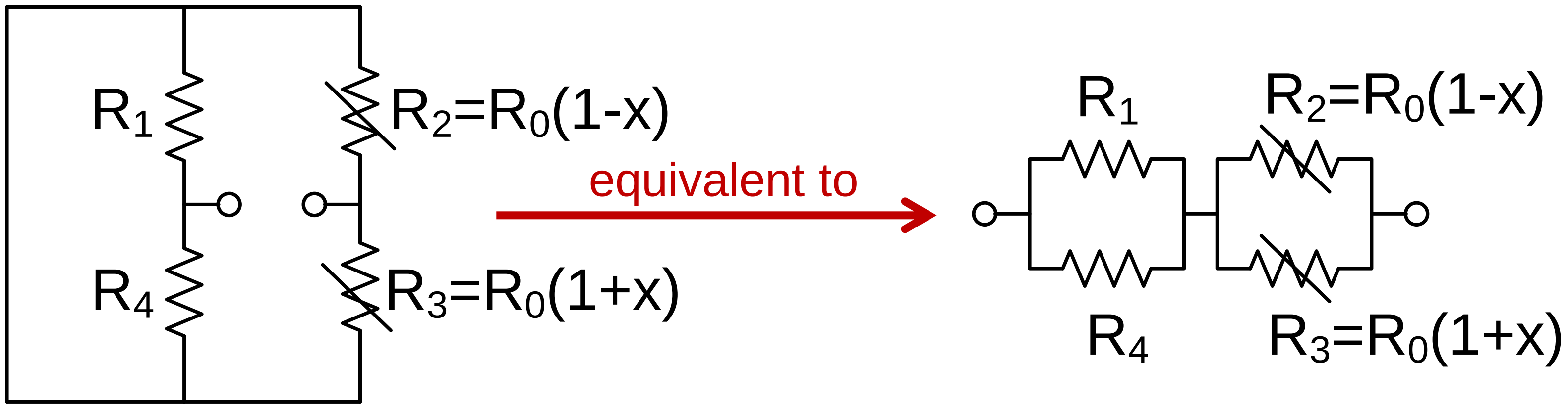
Loading effect and lead-wire resistance

- **what is the measured voltage v_m? (assume $k = R_1/R_4 = 1$)**
- step 1: Thevenin equivalent circuit (of bridge)
  - open circuit voltage



$$v_o = \frac{-x}{2} V_r$$

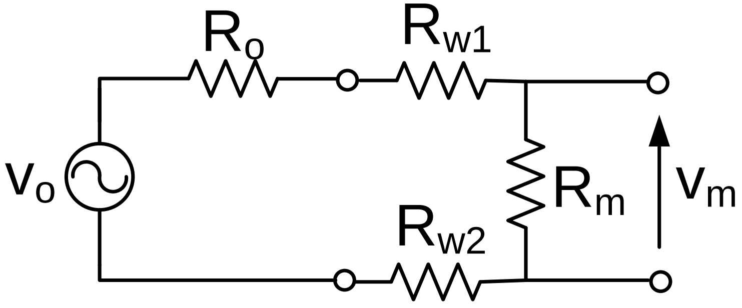
  - output resistance (assume V_r short circuit)



$$R_o = \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_3}{R_2 + R_3} = \frac{R_0}{2} + \frac{R_0(1 - x^2)}{2} = \frac{R_0}{2}(2 - x^2)$$

Loading effect and lead-wire resistance

- **what is the measured voltage v_m? (assume $k = R_1/R_4 = 1$)**
- step 2: output voltage of resistor divider



$$v_m = \frac{R_m}{R_o + R_{w1} + R_m + R_{w2}} v_o$$

$$= \frac{R_m}{\frac{R_0}{2}(2 - x^2) + R_{w1} + R_m + R_{w2}} \frac{-x}{2} V_r$$

reduced sensitivity

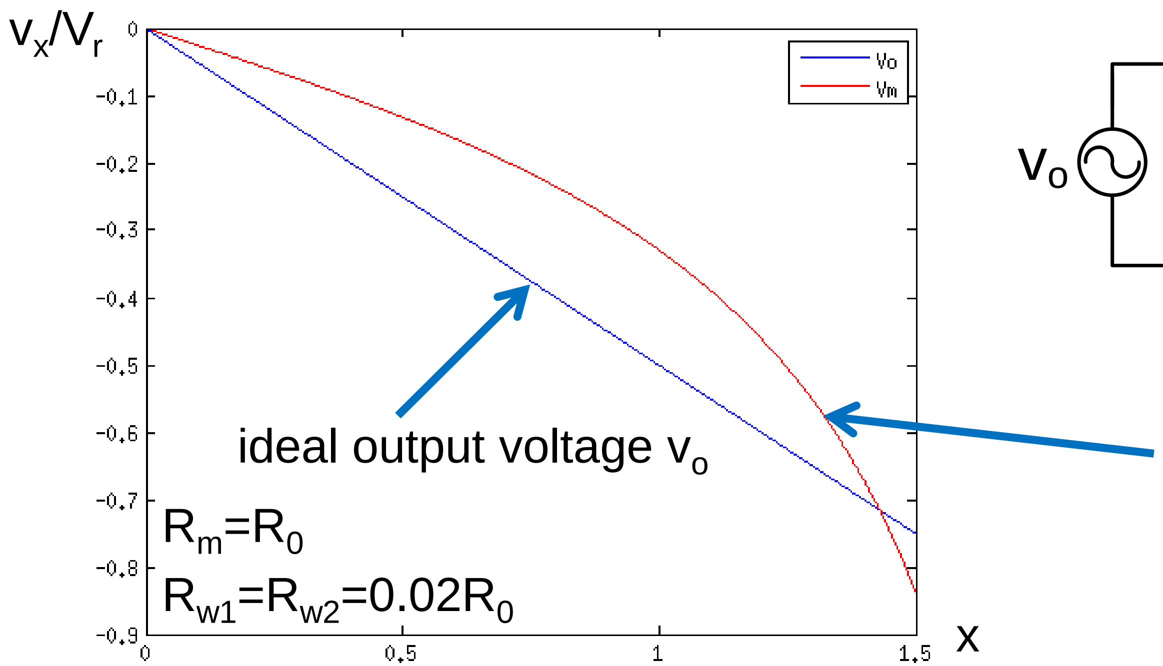
$$= \frac{-xR_m}{R_0(2 - x^2) + 2R_{w1} + 2R_m + 2R_{w2}} V_r$$

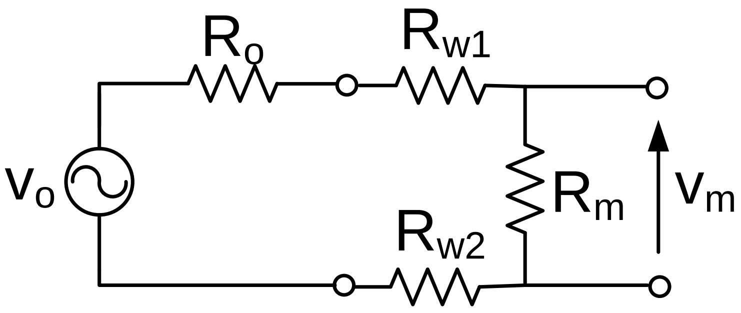
non-linearity

- two errors in measured voltage
  - non-linearity
  - reduced sensitivity

Loading effect and lead-wire resistance

- **what is the measured voltage v_m? (assume $k = R_1/R_4 = 1$)**





- two errors in measured voltage
  - non-linearity
  - reduced sensitivity

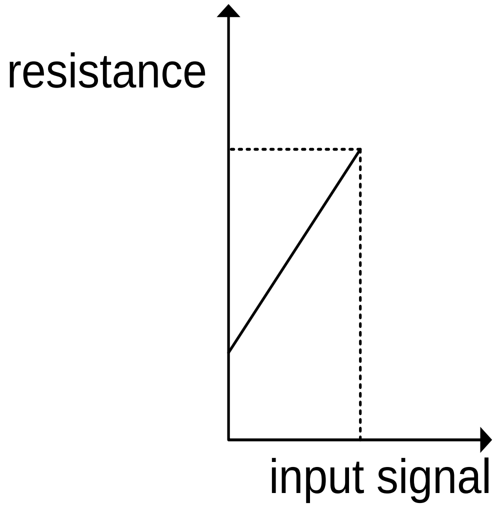
SUMMARY

(on all resistive sensors and interface circuits seen so far)

Resistive sensors

- resistance of resistive sensor $R = R_0 f(x)$
  - f(x) – fractional change in resistance (with f(0) = 1)

- resistance of linear resistive sensor $R = R_0(1 + x)$
  - range of x depends on type of sensor
    - [-1, 0] – linear potentiometer
    - [1, 10] – RTDs
    - [0.00001 , 0.002] – strain gauges



- requirements on signal conditioners for resistive sensors
  electric voltage or current must be applied
  - supply and output voltage/current are limited by error sources
  - several error sources need to be considered when using sensor

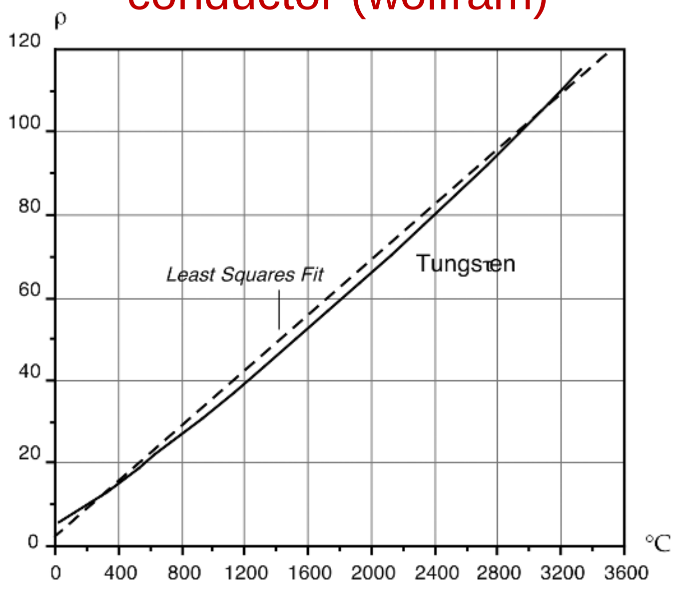
Resistive sensors

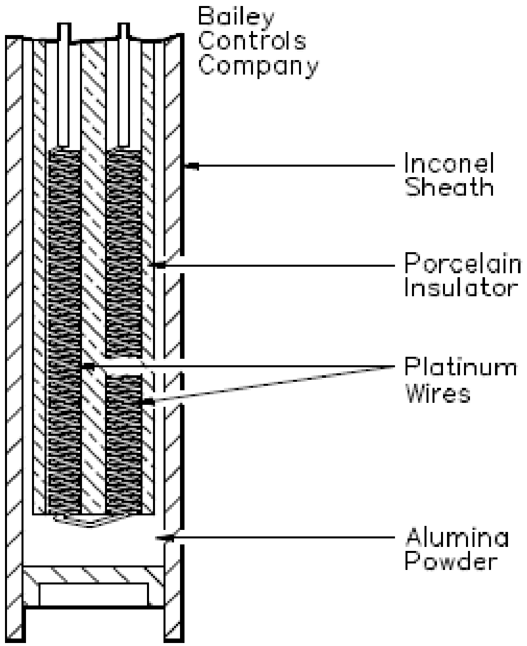
- error sources
  - inherent non-linearity in sensor
  - resistance depends on temperature and strain

$$R = \rho \frac{l}{a} = \frac{m}{ne^2\tau}\frac{l}{a}$$

    - strain or temperature is signal
    - other is error source
  - self-heating effect
    - current passed through sensor causes heat production

conductor (wolfram)





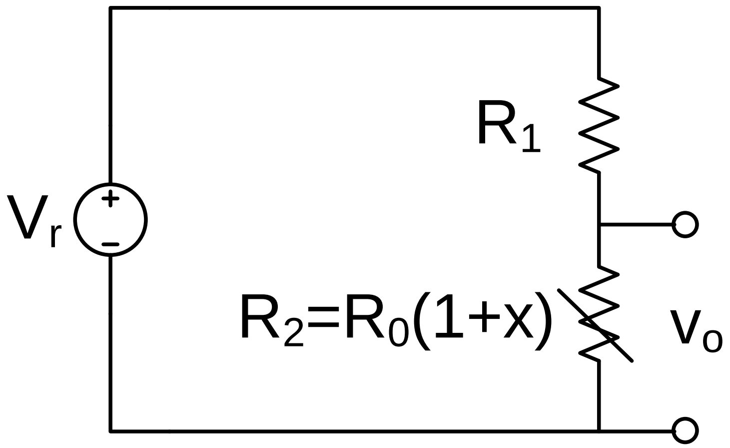
$P_D = I^2R$

$$\Delta T = \frac{P_D}{\delta} = \frac{I^2R}{\delta}$$

Resistive sensors

- error sources
  - loading effect ($R_1=R_0$)

$$v_o = \frac{1+x}{2+x} V_r \approx \frac{1}{2} V_r + \frac{x}{2} V_r$$

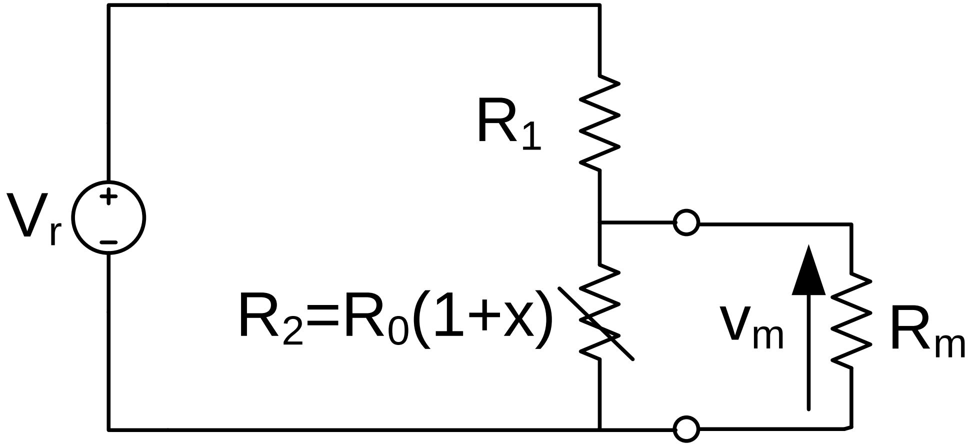


    - measurement adds load R_m
    - measured voltage v_m
    - (use Thévenin circuit, R_o,v_o)

$$v_m = \frac{R_m}{R_m + R_o} v_o$$

with

$$R_o = R_1 // R_2 = \frac{R_0 R_0 (1+x)}{R_0 + R_0(1+x)}$$



$$\Rightarrow v_m = \frac{R_m}{R_m + \frac{R_0(1+x)}{2+x}} v_o = \frac{(2+x)R_m}{(2+x)R_m + R_0(1+x)} v_o$$

$$k = R_m / R_0$$

$$\Rightarrow v_m = \frac{(2+x)}{(2+x) + \frac{(1+x)}{k}} v_o$$

Resistive sensors

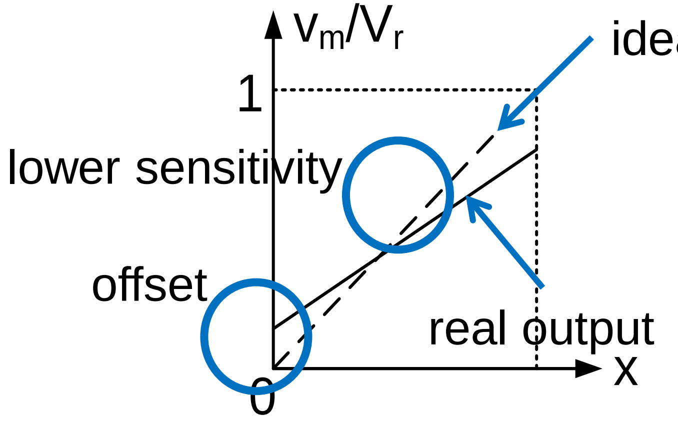
- error sources
  - loading effect (R_1=R_0)

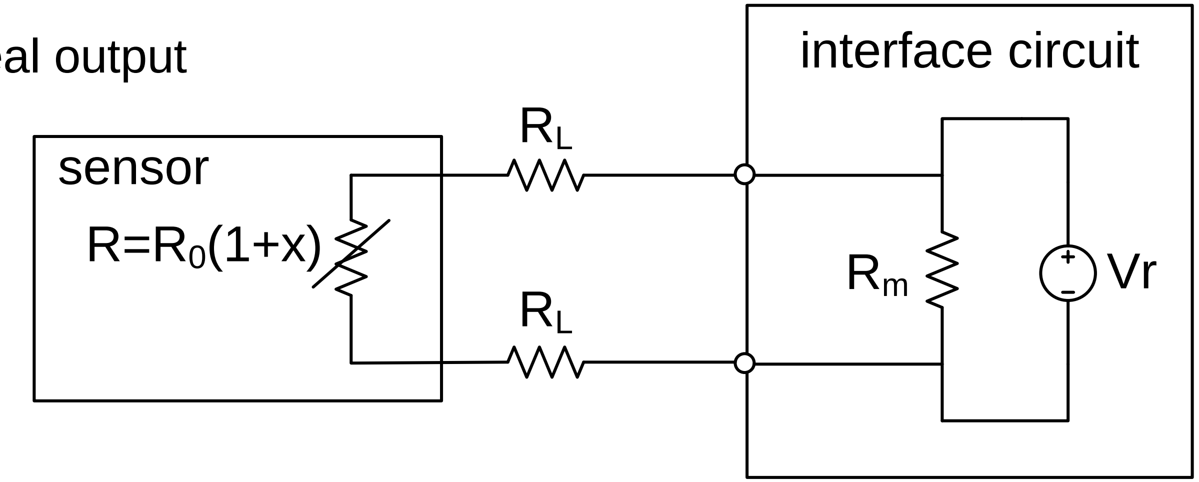
absolute error

$$\varepsilon = \left|\frac{v_m - v_o}{v_o}\right| = \left|\frac{\frac{(2+x)k}{(2+x)k+(1+x)}v_o - v_o}{v_o}\right| = \left|\frac{\frac{(2+x)k-(2+x)k-(1+x)}{(2+x)k+(1+x)}v_o}{v_o}\right| = \left|\frac{-(1+x)}{(2+x)k+(1+x)}\right|$$

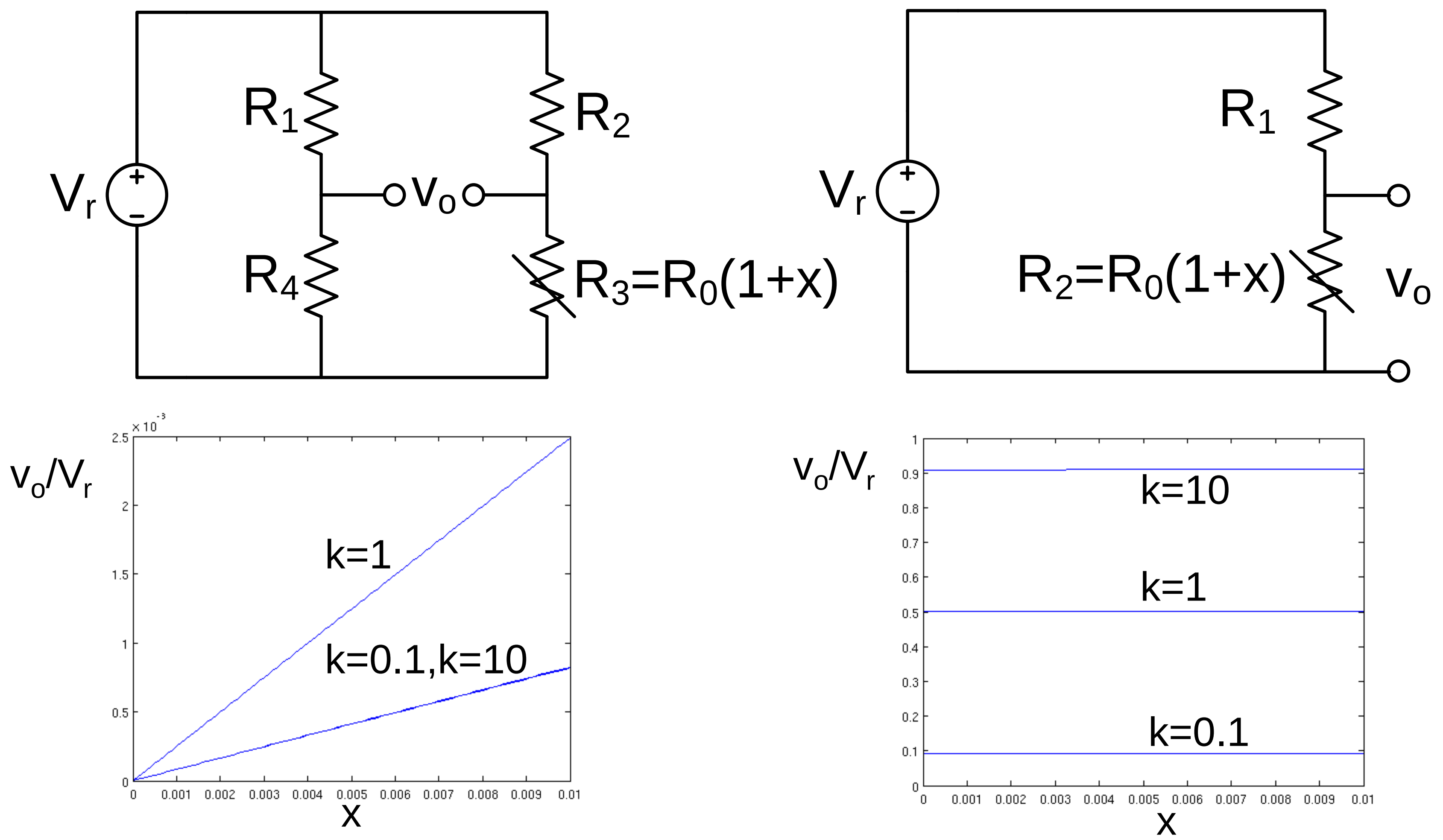
relative error

    - larger k means smaller error (closer to open circuit)
  - lead-wire resistance





Interface circuits



- sensitivity is equal, but DC offset makes response look “flat”