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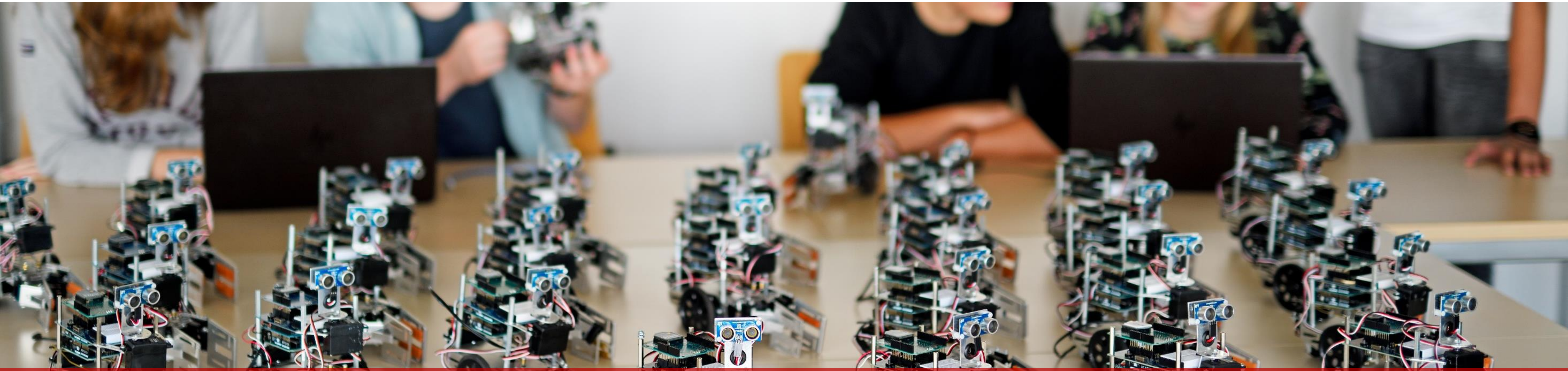


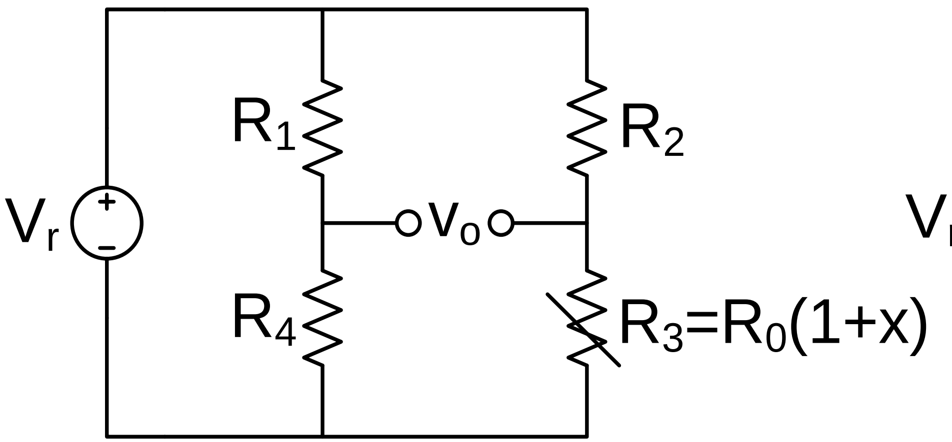
Sensing, Computing, Actuating

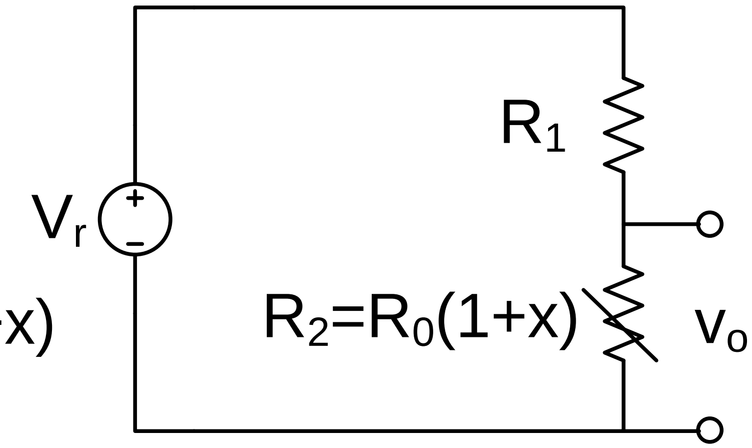
Sander Stuijk (s.stuijk@tue.nl)

AMPLIFIERS

(Chapter 2.4)

Interface circuits





- output voltage (k=1)

$$v_o = \frac{x}{4+2x} V_r \approx \frac{x}{4} V_r$$

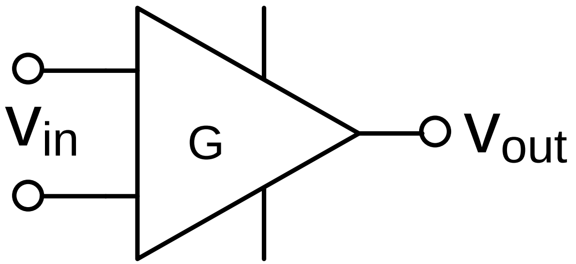
$$v_o = \frac{1+x}{2+x} V_r \approx \frac{1}{2} V_r + \frac{x}{2} V_r$$

- response of bridge output to change in x only half of response when using divider
- **can we change the bridge to get the same response?**
  - use an additional sensor
  - use operational amplifier (also amplifies non-linearity error)

Amplifier

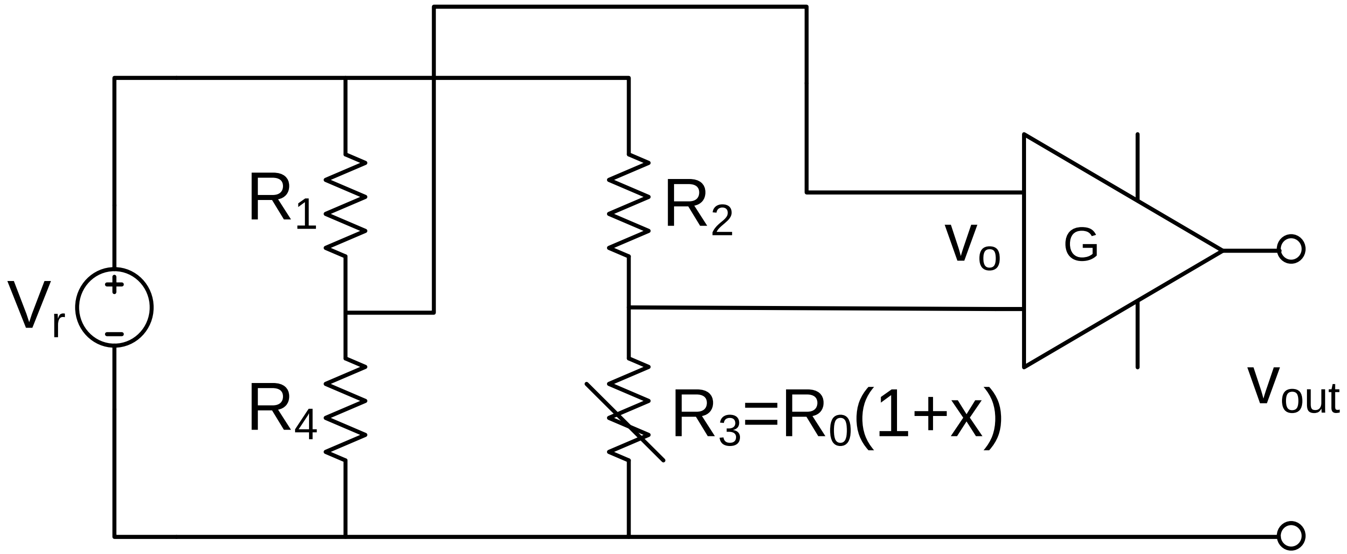
- increase sensitivity by adding amplifier to output of bridge
- adjust voltage for digitization
- amplifier with gain G

$$v_{out} = G \cdot v_{in}$$



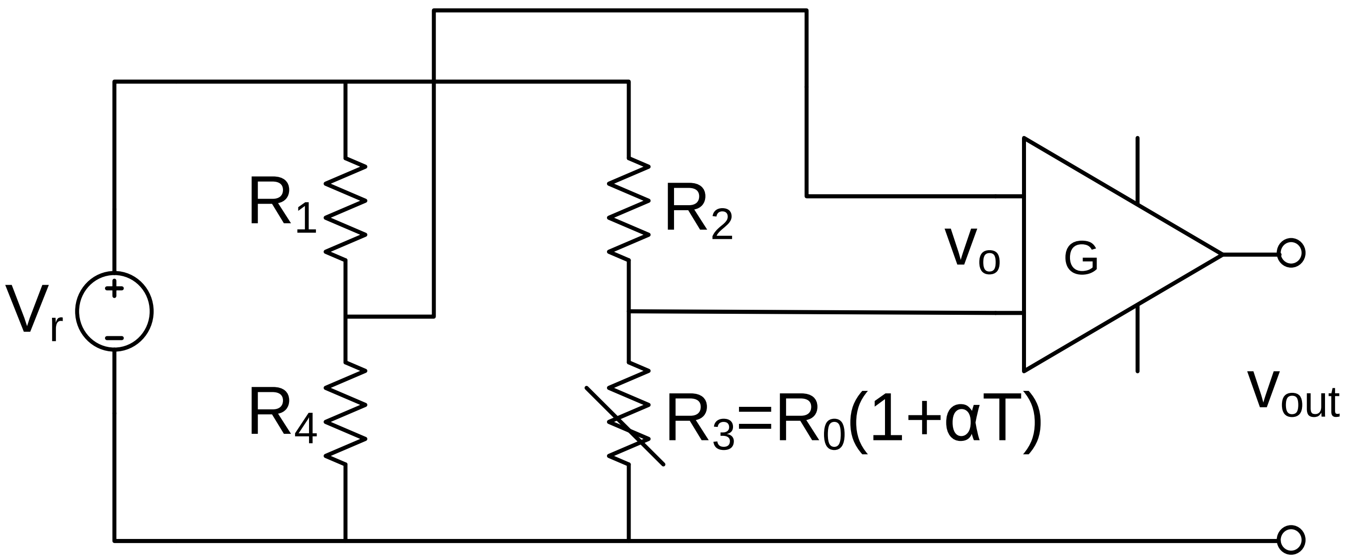
- circuit output voltage

$$v_{out} = G \cdot v_o = G \cdot \frac{x}{4+2x} V_r \approx G \cdot \frac{x}{4} V_r$$



Amplifier

- example – PT100 temperature sensor
- PT100 (R_0=100Ω and α=0.004Ω/Ω/K at 0°C)
- measure temp from -10°C to +50°C in environment with δ=5mW/K
- output must range from -1V to +5V with error < 0.5% of the reading (due to non-linearity) plus 0.2% FSO (due to self-heating)
- **what values should be used for the resistors and voltage supply, and what gain must the amplifier have?**



Amplifier

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- output must range from -1V to +5V with error < 0.5% of the reading (due to non-linearity) plus 0.2% FSO (due to self-heating)
- **what values should be used for the resistors and voltage supply, and what gain must the amplifier have?**

$$v_o = \frac{k\alpha T}{(k+1)(k+1+\alpha T)} V_r$$

- assume αT << k+1, the response is then equal to

$$v_i = \frac{k\alpha T}{(k+1)^2} V_r$$

- introduces an error (due to non-linearity); requirements is relative to reading, thus look at relative error; gain plays no role since its both v_o and v_i are multiplied with same gain G

$$\varepsilon = \left|\frac{v_o - v_i}{v_i}\right| = \left|\frac{-\alpha T}{k+1+\alpha T}\right|$$

Amplifier

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- output must range from -1V to +5V with error < 0.5% of the reading (due to non-linearity) plus 0.2% FSO (due to self-heating)
- **what values should be used for the resistors and voltage supply, and what gain must the amplifier have?**
- maximal error when T=50°C

$$\varepsilon = \left|\frac{-(0.004/°C)(50°C)}{k+1+(0.004/°C)(50°C)}\right| < 0.005$$

- this requires k > 39
- R_4 must be R_3=R_0=100Ω to get 0V at 0°C

$$k = \frac{R_1}{R_4} = \frac{R_2}{R_3}$$

- R_4 = 100Ω and R_1=R_2=3900Ω
  - larger values of R1 and R2 would decrease sensitivity!

Amplifier

- example – PT100 temperature sensor
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- measure temp from -10°C to +50°C in environment with δ=5mW/K
- output must range from -1V to +5V with error < 0.5% of the reading (due to non-linearity) plus 0.2% FSO (due to self-heating)
- **what values should be used for the resistors and voltage supply, and what gain must the amplifier have?**
- supply voltage limited by sensor self-heating

$$\Delta T = \frac{P_D}{\delta} \Rightarrow P_D = \left(\frac{V_r}{R_2 + R_3}\right)^2 R_3 < (0.002 \cdot 50°C) \cdot (5mW/°C) = 0.5mW$$

- maximal self-heating when R_2=R_3
- R_3 will however always be below R_2 in measurement range
  - maximal heating occurs at 50°C
  - R(50°C) = 120Ω

$$V_r < \sqrt{\frac{0.0005W}{120\Omega}} \cdot (3900\Omega + 120\Omega) = 8.2V$$

Amplifier

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- PT100 (R_0=100Ω and α=0.004Ω/Ω/K at 0°C)
- measure temp from -10°C to +50°C in environment with δ=5mW/K
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- **what values should be used for the resistors and voltage supply, and what gain must the amplifier have?**

$$V_r < \sqrt{\frac{0.0005W}{120\Omega}} \cdot (3900\Omega + 120\Omega) = 8.2V$$

- choose V_r = 8V
- output of bridge at 50°C

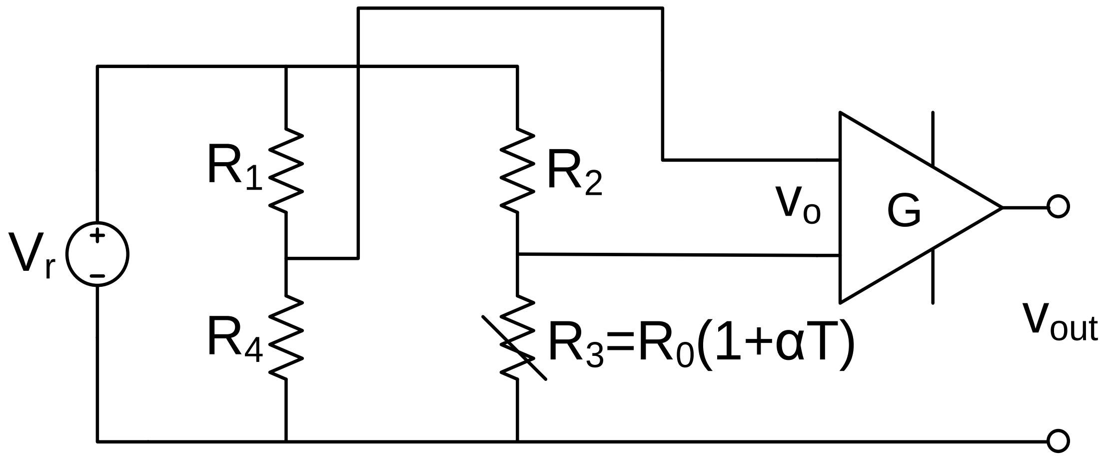
$$v_o = \frac{k\alpha T}{(k+1)(k+1+\alpha T)} V_r \approx \frac{39 \cdot 0.004/°C \cdot 50°C}{40^2} \cdot 8 = 39mV$$

- gain needed to get +5V output at 50°C

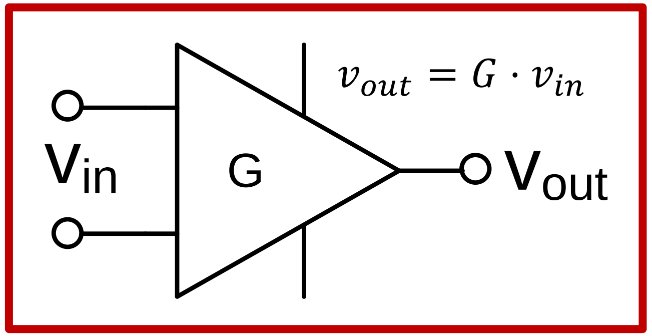
$$G = \frac{5V}{39mV} = 128.2$$

Amplifier

- output of voltage divider or bridge may be very small



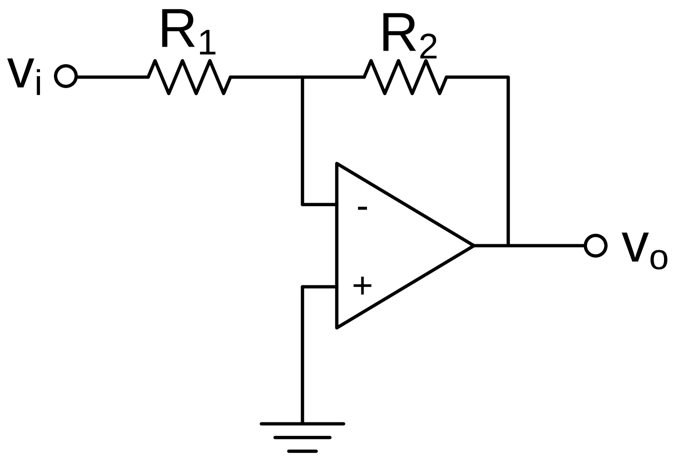
- (digital) processing circuits require higher voltage (0-5V)
- two types of amplifiers considered
  - differential amplifiers
  - instrumentation amplifiers



Operational amplifier

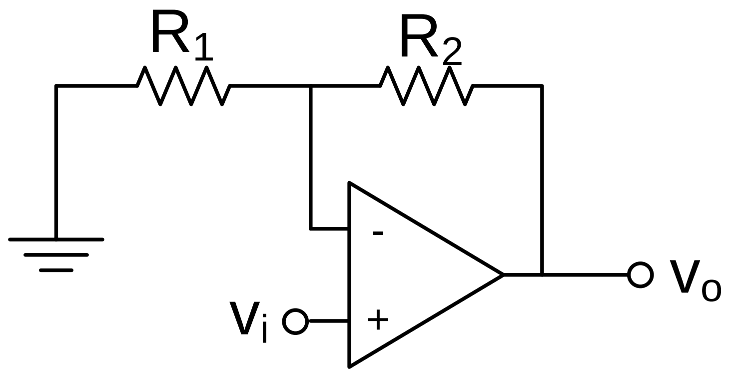
- inverting amplifier
  - amplifier draws no current
  - current through R_1 and R_2 are equal

$$\left.\begin{array}{c}\frac{v_i - v_-}{R_1} = \frac{v_- - v_o}{R_2} \\ v_- = v_+ = 0\end{array}\right\} \Rightarrow v_o = -\frac{R_2}{R_1} v_i$$



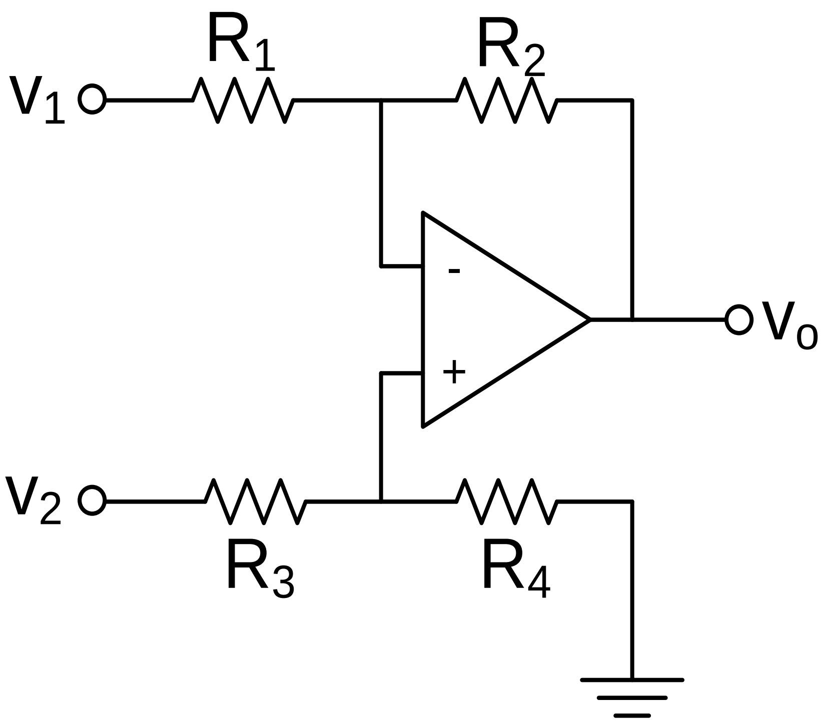
- non-inverting amplifier

$$\left.\begin{array}{c}\frac{-v_-}{R_1} = \frac{v_- - v_o}{R_2} \\ v_- = v_+ = v_i\end{array}\right\} \Rightarrow v_o = \left(1 + \frac{R_2}{R_1}\right) v_i$$



- these circuits cannot be used in bridge since they have only one input terminal and bridge has two output terminals

Differential amplifier



- assumptions about op-amp
  - negligible common mode gain (A_c=0)
  - considerable differential gain (A_d≠0)

- output voltage

$$v_o = \frac{R_4}{R_3+R_4}\left(1+\frac{R_2}{R_1}\right)v_2 - \frac{R_2}{R_1}v_1$$

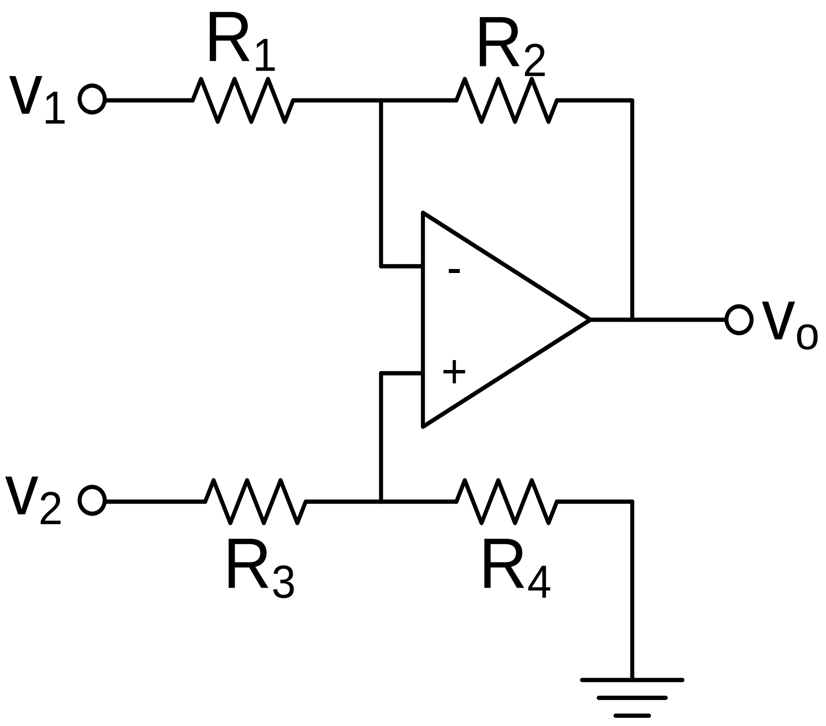
- inputs v_1 and v_2 have differential and common part

$$v_d = v_2 - v_1,\ \ v_c = \frac{v_1+v_2}{2}$$

- output voltage

$$v_o = \frac{1}{2}v_d\left[\frac{R_4}{R_3+R_4}\left(1+\frac{R_2}{R_1}\right)+\frac{R_2}{R_1}\right] + v_c\left[\frac{R_4R_1 - R_2R_3}{R_1(R_3+R_4)}\right]$$

Differential amplifier



- differential mode gain

$$G_d = \left.\frac{v_o}{v_d}\right|_{v_c=0} = \frac{1}{2}\left[\frac{R_4}{R_3+R_4}\left(1+\frac{R_2}{R_1}\right)+\frac{R_2}{R_1}\right]$$

- common mode gain

$$G_c = \left.\frac{v_o}{v_c}\right|_{v_d=0} = \frac{R_4R_1 - R_2R_3}{R_1(R_3+R_4)}$$

- ideal differential amplifier has G_c=0

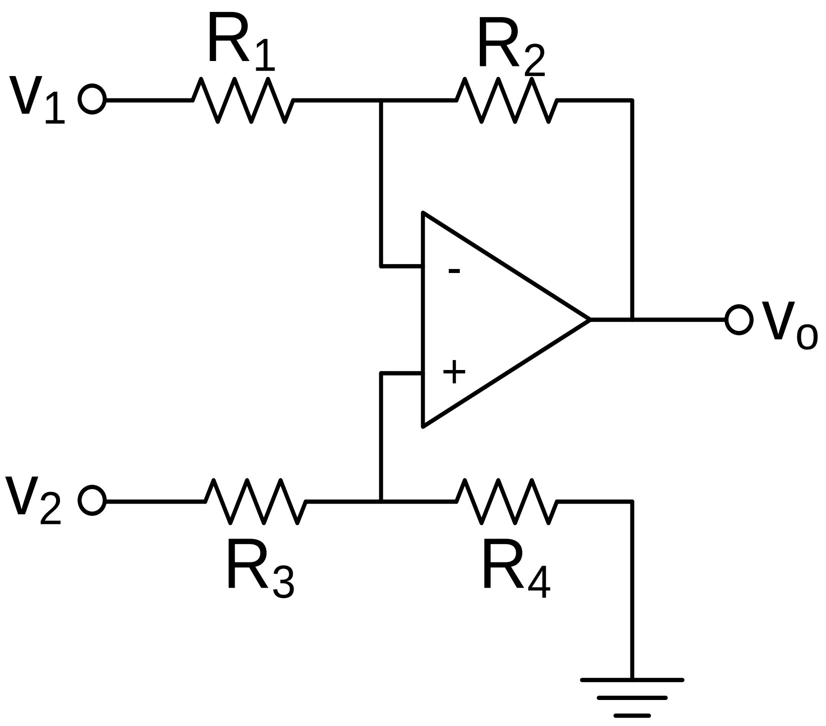
$$\frac{R_4}{R_3} = \frac{R_2}{R_1} = k$$

- differential gain of an ideal differential amplifier

$$G_d = \left.\frac{v_o}{v_d}\right|_{v_c=0} = \frac{1}{2}\left[\frac{k}{1+k}(1+k)+k\right] = \frac{1}{2}\left[\frac{k(1+k)+k(1+k)}{1+k}\right] = \frac{2k(1+k)}{2(1+k)} = k$$

- differential gain G_d depends on ratio k of resistors

Differential amplifier



- matching condition is hard to realize

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} = k$$

- common mode rejection ratio (CMRR)

$$CMRR_R = \frac{G_d}{G_c} = \frac{1}{2}\frac{R_1R_4 + R_2R_3 + 2R_2R_4}{R_1R_4 - R_2R_3}$$

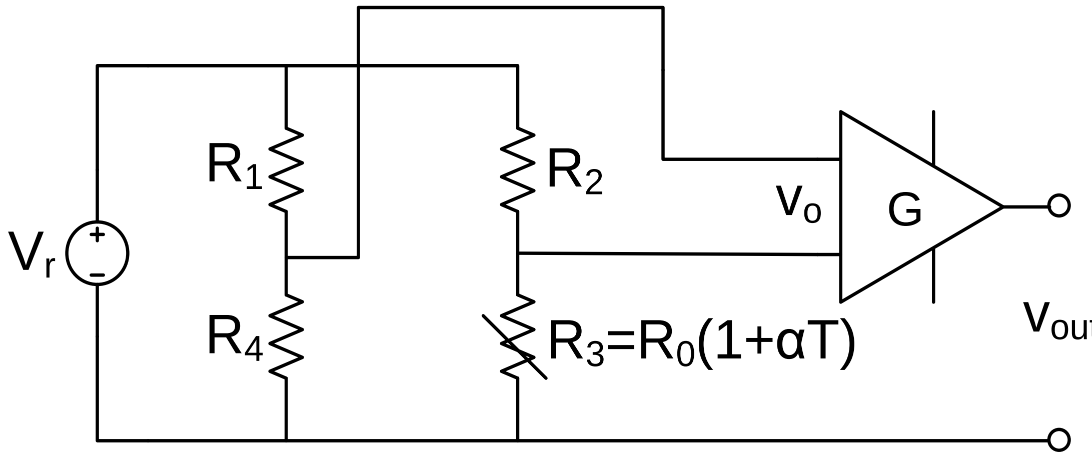
- $CMRR_R$ indicates mismatch only due to resistors
- expressed in decibel (dB), defined as $20 \cdot {}^{10}\log(CMRR_R)$

- amplification of common mode voltage is error source
- large CMRR implies small influence of common mode signal on output signal (small error)

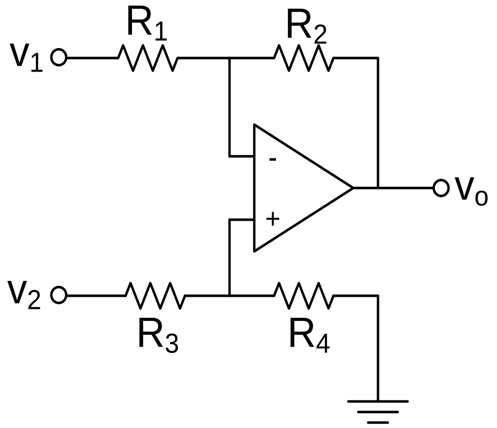
- differential and common mode gain are dependent on each other

Differential amplifier

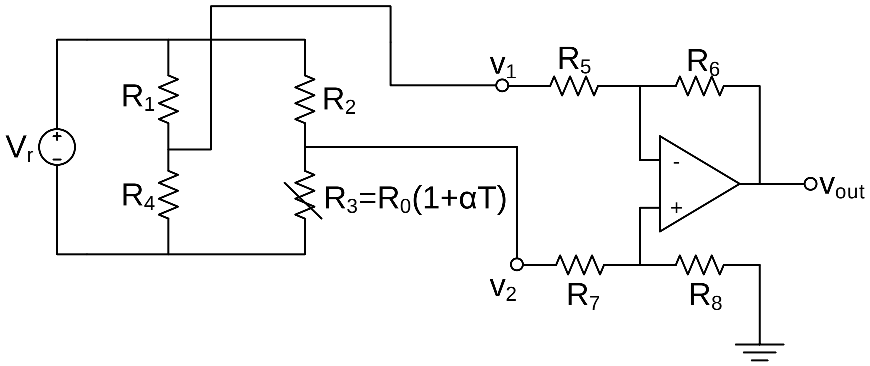
- sensing circuit with amplifier



- differential amplifier

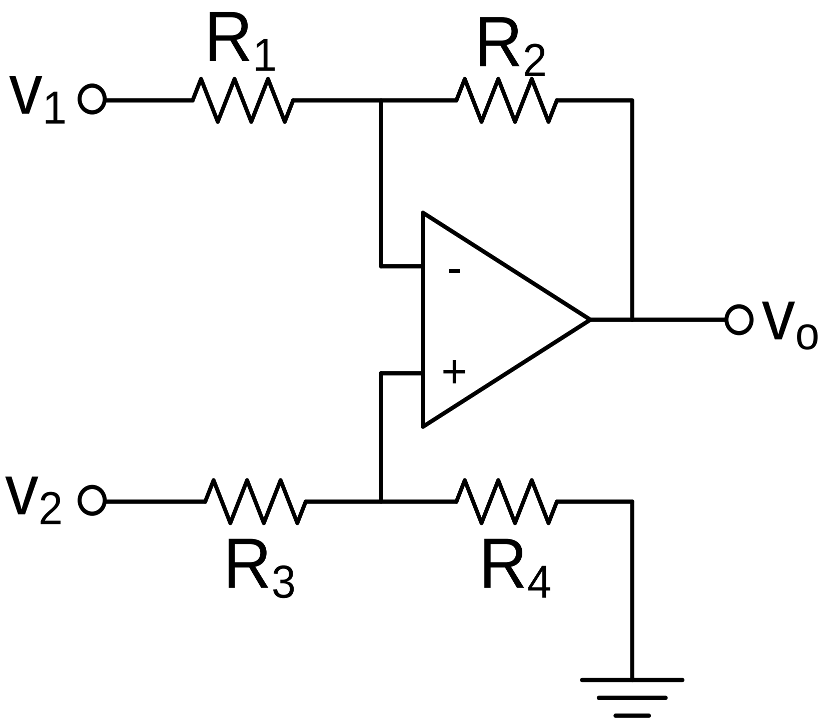


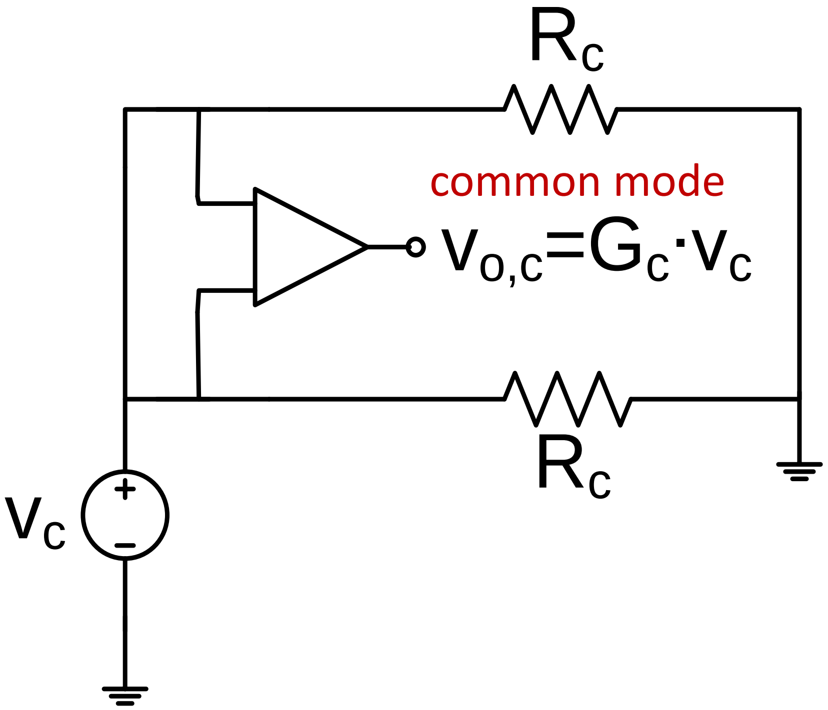
- sensing circuit with differential amplifier

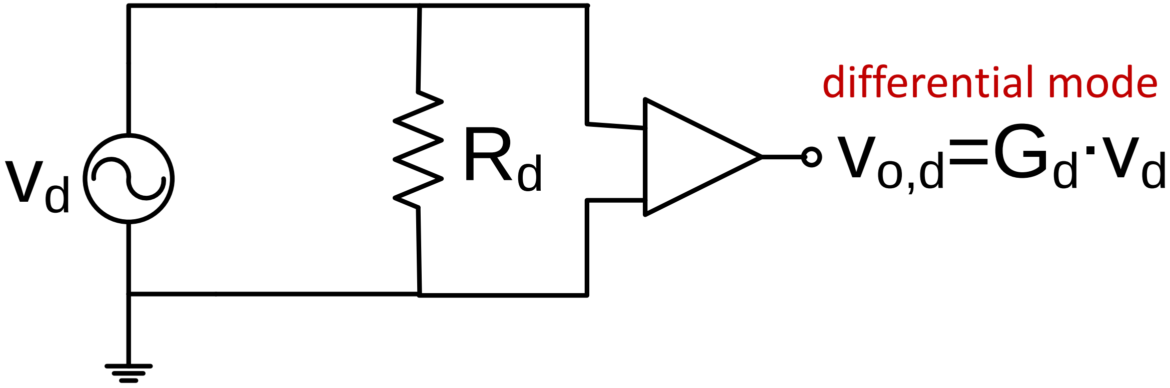


Differential amplifier

- differential and common mode analysis
- electrically equivalent circuits for both modes





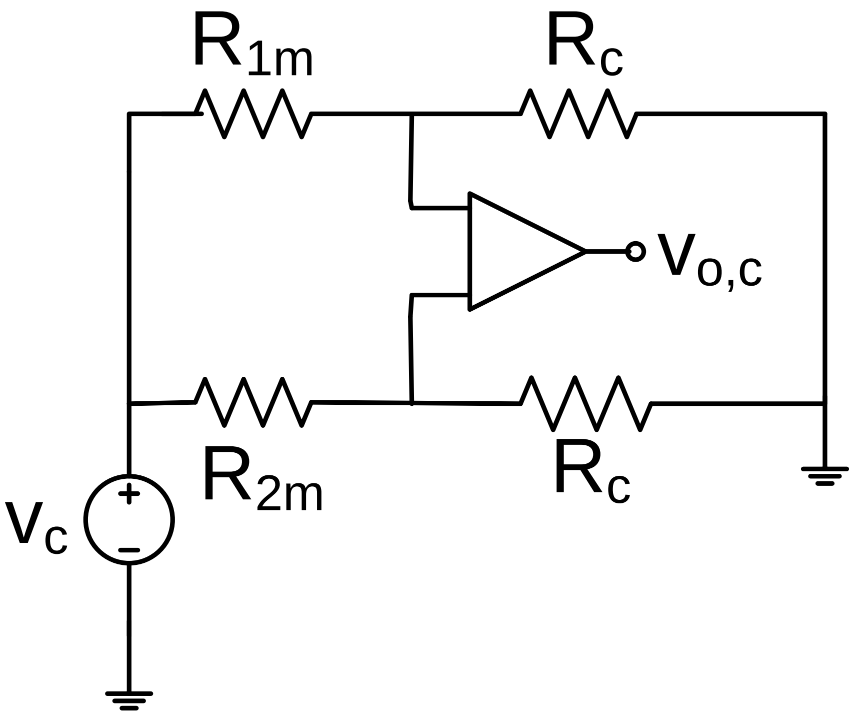


- loading effect is ignored in these electrically equivalent circuits

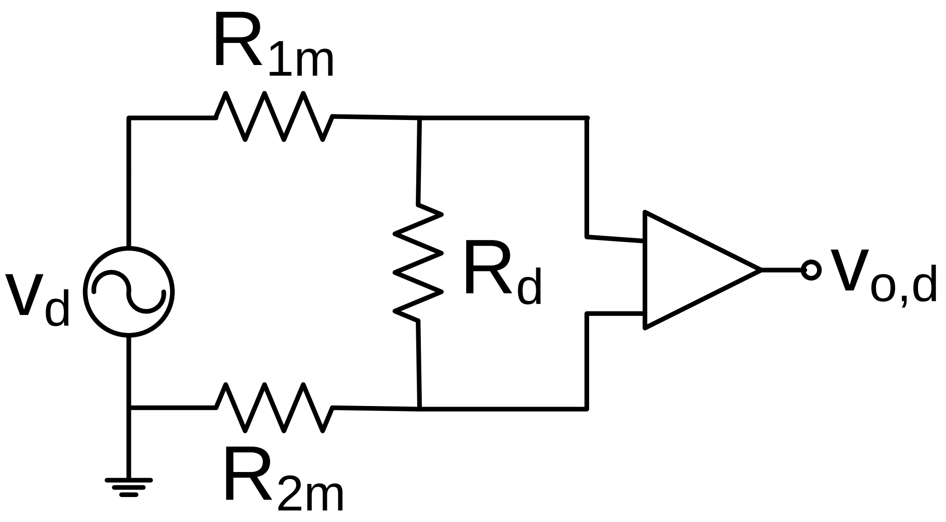
Differential amplifier

- loading effect changes common and differential output voltage

common mode

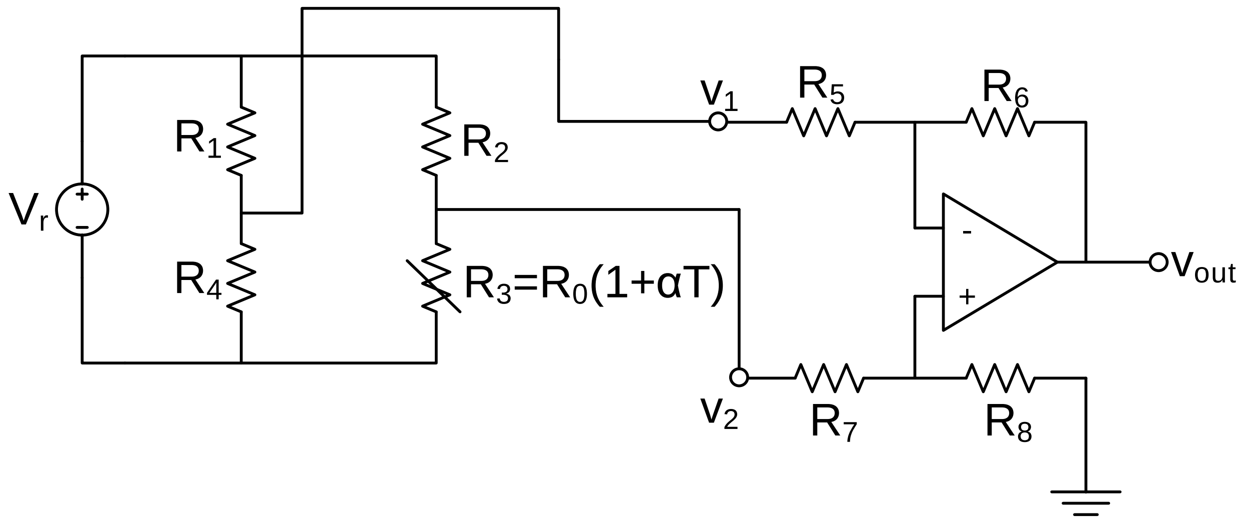


differential mode



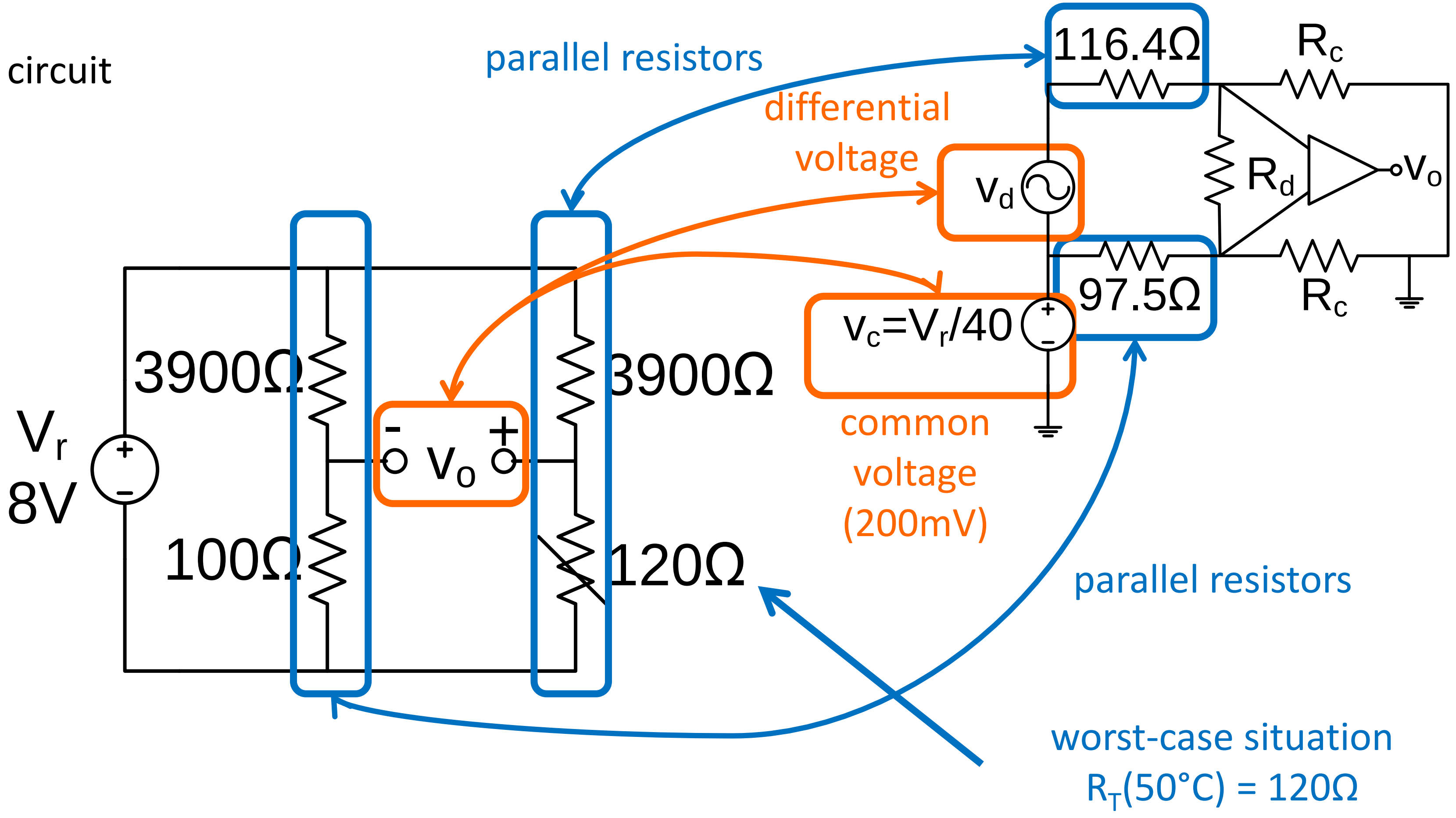
Differential amplifier

- example – PT100 temperature sensor
- PT100 (R_0=100Ω and α=0.004Ω/Ω/K at 0°C)
- measure temp from -10°C to +50°C in environment with δ=5mW/K
- output must range from -1V to +5V with error < 0.5% of the reading (due to non-linearity) plus 0.2% FSO (due to self-heating)
- we computed earlier: R_4=100Ω, R_1=R_2=3900Ω, V_r=8V, G=128.2
- **what are the minimal input differential and common mode resistances and the CMRR of the differential amplifier to have a negligible error (at most 10% of errors considered so far)?**



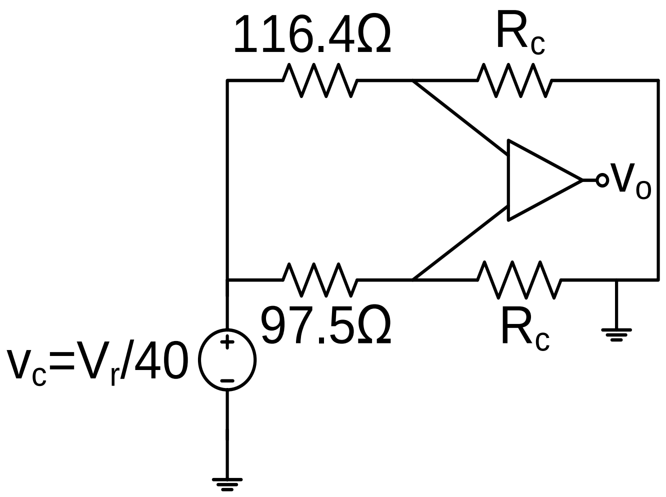
Differential amplifier

- example – PT100 temperature sensor
- **what are the minimal input differential and common mode resistances and the CMRR of the differential amplifier to have negligible error?**
- equivalent circuit



Differential amplifier

- example – PT100 temperature sensor
- **what are the minimal input differential and common mode resistances and the CMRR of the differential amplifier to have negligible error?**
- common mode voltage produces
- differential voltage across bridge
- due to unequal resistors in both arm
- error amplified by differential gain
- choose error to be 10% of constant error



common mode voltage ($V_r/40$) — bridge expression — differential gain

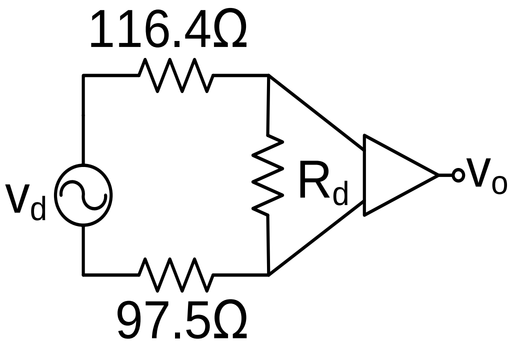
$$(200mV)\left(\frac{R_c}{R_c + 116.4\Omega} - \frac{R_c}{R_c + 97.5\Omega}\right) \cdot 128.2 < 0.1 \cdot 0.002 \cdot 5V$$

- this gives R_c > 484kΩ

Differential amplifier

- example – PT100 temperature sensor
- **what are the minimal input differential and common mode resistances and the CMRR of the differential amplifier to have negligible error?**
- output voltage v_d sensed by differential
- amplifier (voltage across) R_d lower
- then v_o due to resistors

$$v_o = \frac{R_d}{R_d + 116.4\Omega + 97.5\Omega} v_d = \frac{R_d}{R_d + 213.9\Omega} v_d$$



- leads to relative error

$$\varepsilon = \left|\frac{v_o - v_d}{v_d}\right| = \left|\left(\frac{R_d}{R_d + 213.9\Omega} v_o - v_o\right) / v_o\right| = \frac{213.9\Omega}{R_d + 213.9\Omega}$$

- choose error to be 10x smaller than allowed relative error (0.5%)

$$\frac{213.9\Omega}{R_d + 213.9\Omega} < 0.1 \cdot 0.5\%$$

- this gives R_d > 428kΩ

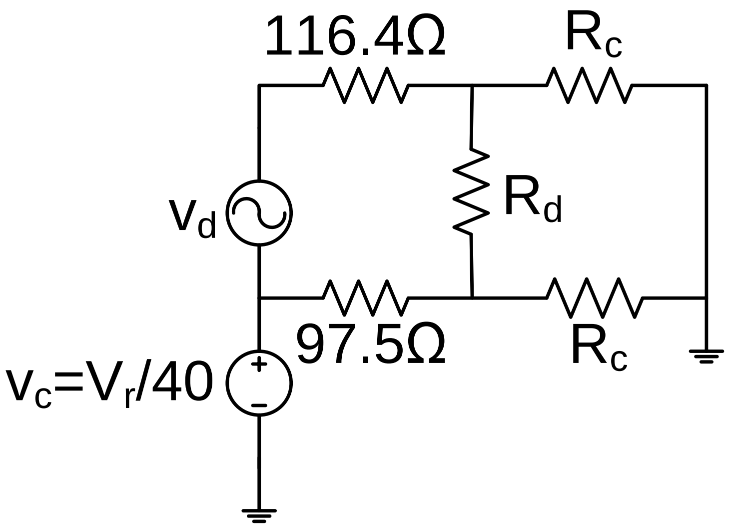
Differential amplifier

- example – PT100 temperature sensor
- **what are the minimal input differential and common mode resistances and the CMRR of the differential amplifier to have negligible error?**
- choose negligible error to be an error
- which is at most 10% of constant allowed
- error (0.2% of 5V)
- maximal common mode gain

$$G_c = \frac{0.1 \cdot (0.002 \cdot 5V)}{200mV} = 0.005$$

- differential gain is 128.2
- required common mode rejection ratio

$$CMRR > \frac{128.2}{0.005} = 25640 = 88dB$$



Differential amplifier

- example – PT100 temperature sensor
- **what are the minimal input differential and common mode resistances and the CMRR of the differential amplifier to have negligible error?**

- summary
  - CMRR = 88dB
  - R_c > 484kΩ
  - R_d > 428kΩ

- observations
  - differential impedance high to prevent signal loading
  - common mode impedance high to prevent common mode signal from producing differential voltage
  - result of high impedances is a low CMRR

Instrumentation amplifier

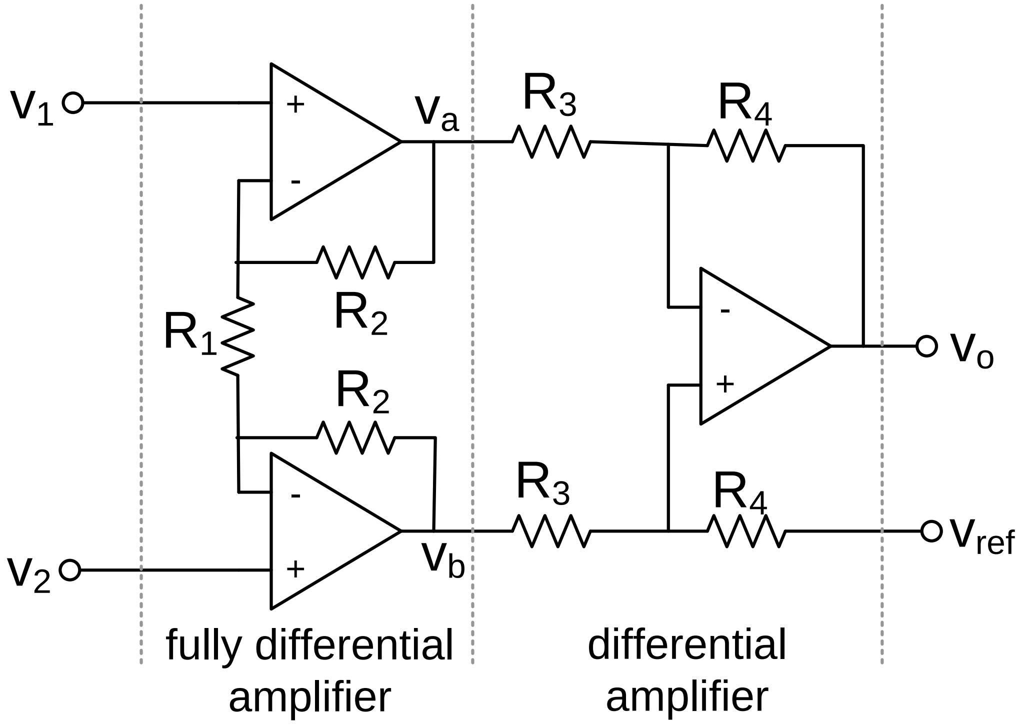
- instrumentation amplifier is a differential amplifier with
  - high input and low output impedance
  - high CMRR
  - high gain (adjustable with resistor R_1)

- output of first stage

$$v_a = \left(1 + \frac{R_2}{R_1}\right) v_1 - \frac{R_2}{R_1} v_2$$

$$v_b = \left(1 + \frac{R_2}{R_1}\right) v_2 - \frac{R_2}{R_1} v_1$$

  - output is combination of
    - non-inverting amplifier
    - inverting amplifier



Instrumentation amplifier

- instrumentation amplifier is a differential amplifier with
  - high input and low output impedance
  - high CMRR
  - high gain (adjustable with resistor R_1)

- output of second stage

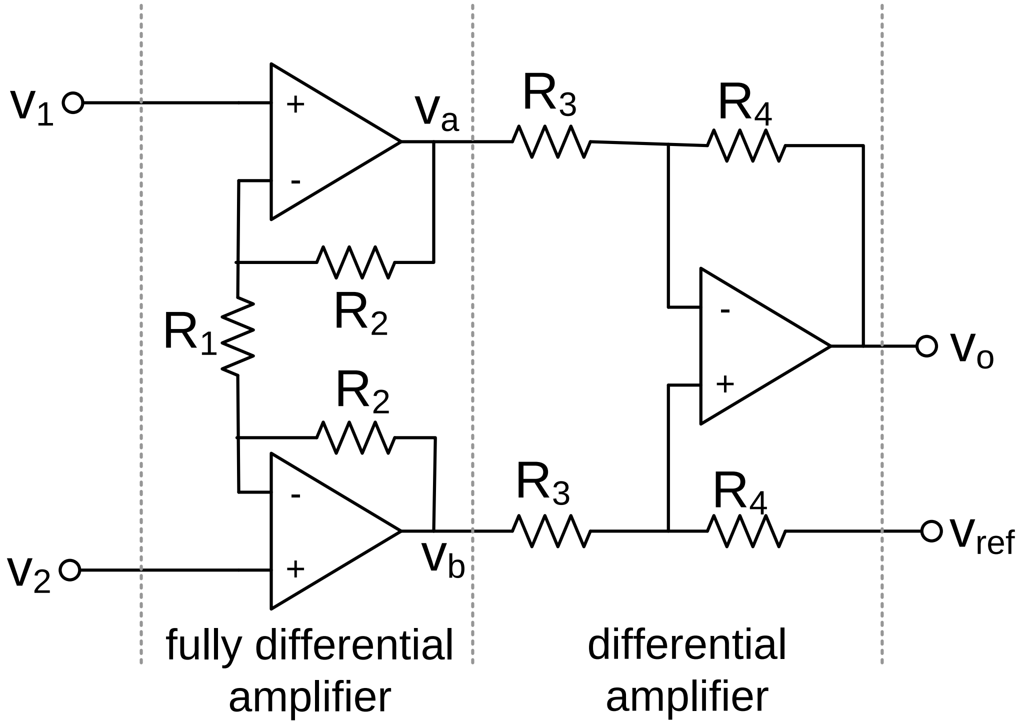
$$v_a = \left(1 + \frac{R_2}{R_1}\right) v_1 - \frac{R_2}{R_1} v_2$$

$$v_b = \left(1 + \frac{R_2}{R_1}\right) v_2 - \frac{R_2}{R_1} v_1$$

$$v_o - v_{ref} = (v_b - v_a)\frac{R_4}{R_3}$$

$$= \left(1 + \frac{2R_2}{R_1}\right)\frac{R_4}{R_3}(v_2 - v_1)$$

$$= (1 + G)k(v_2 - v_1) \qquad \text{with} \quad G = \frac{2R_2}{R_1}$$



Instrumentation amplifier

- instrumentation amplifier is a differential amplifier with
  - high input and low output impedance
  - high CMRR
  - high gain (adjustable with single resistor)

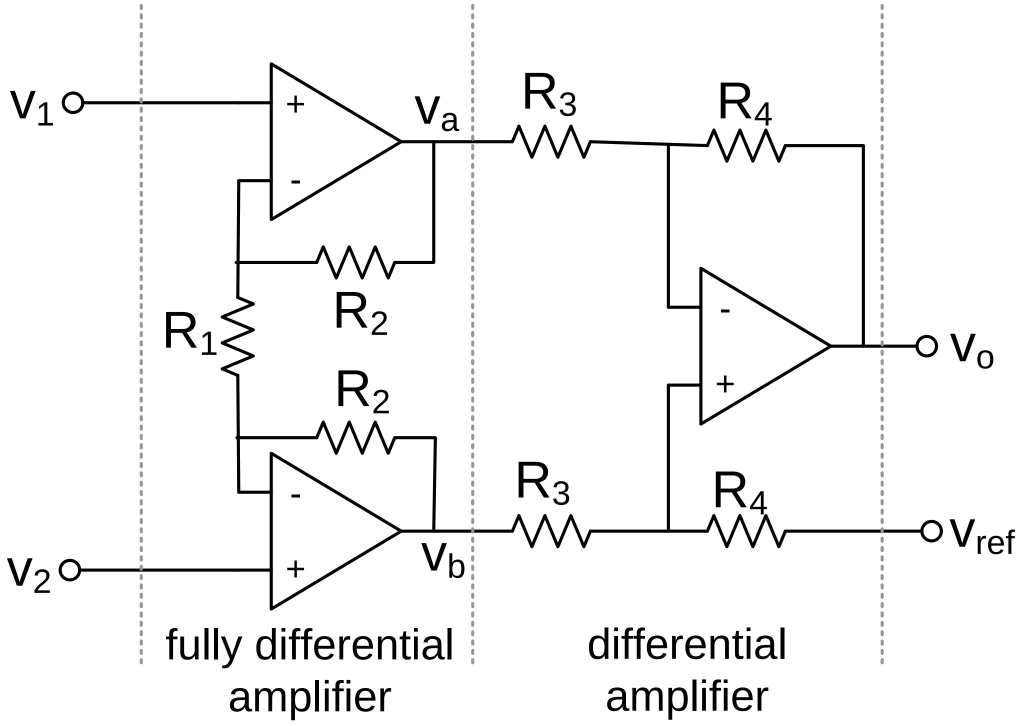
- output of second stage

$$v_o - v_{ref} = (1 + G)k(v_2 - v_1)$$

$$G = \frac{2R_2}{R_1} \qquad k = \frac{R_4}{R_3}$$

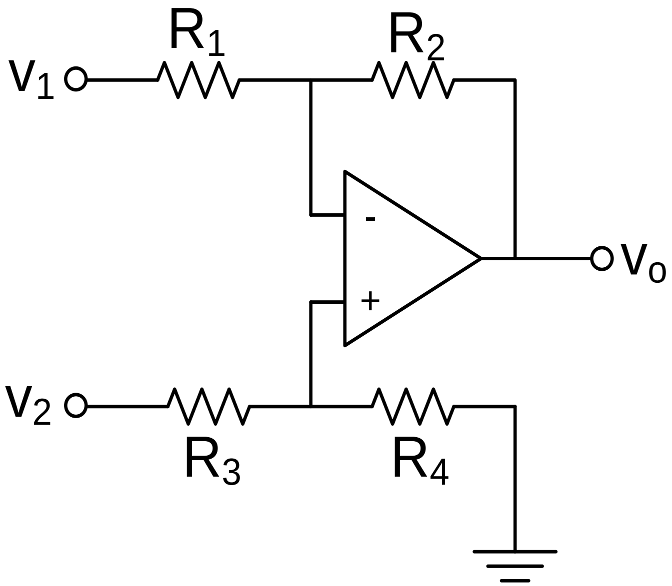
- R_1 plays no role in matching of differential amplifier
- differential mode gain independent of CMRR

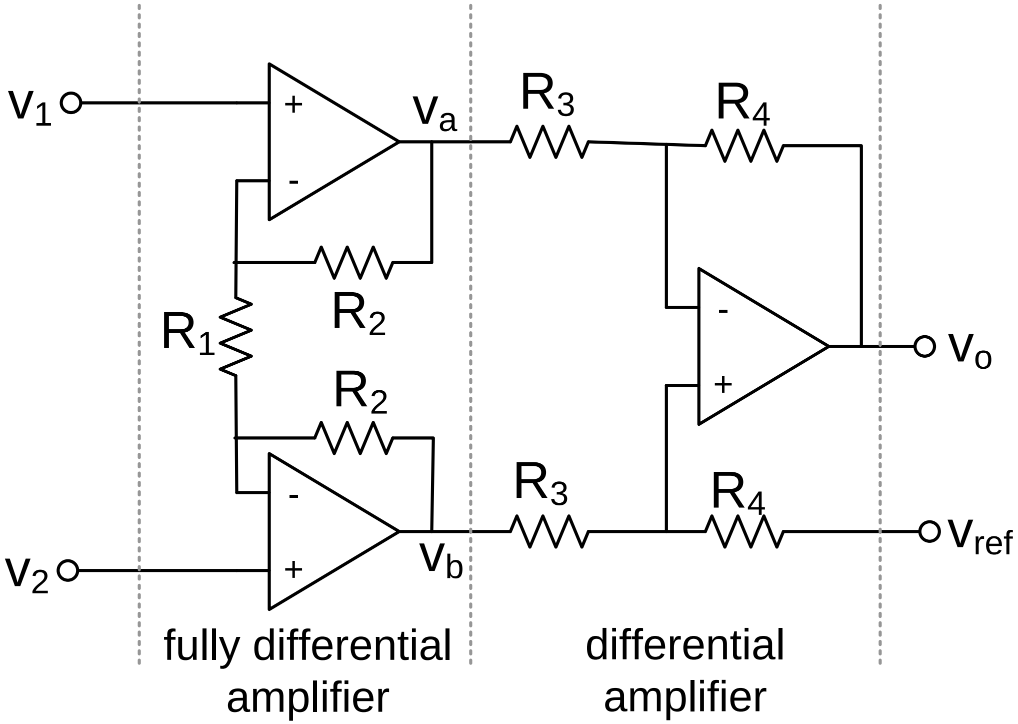
$$CMRR_{IA} \approx (1 + G) \cdot CMRR_{DA}$$



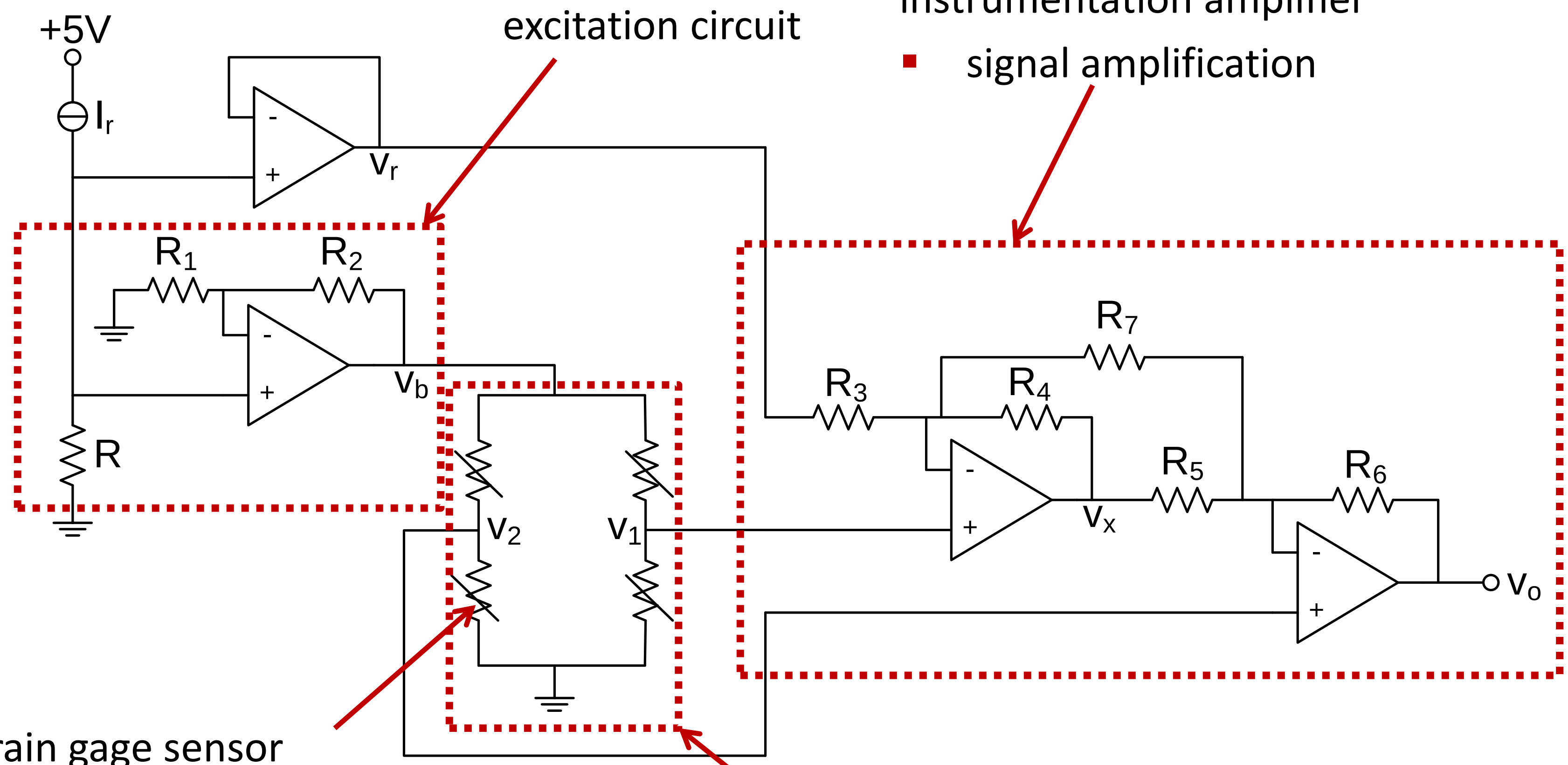
Summary

- operational amplifiers can increase output signal
- errors are increased with same magnitude
- two types of amplifiers studied
  - differential amplifiers
  - instrumentation amplifiers





Example – pressure sensor



excitation circuit

instrumentation amplifier

- signal amplification

strain gage sensor

- active sensor
- absolute sensor
- contact sensor

signal processing circuit

- bridge circuit
- remove error sources (e.g. temperature)
- remove offset voltage
- improve linearity
- improve sensitivity

Signal processing

- increase sensitivity by adding sensor on other side of opposing arm
- bridge output voltage

$$\left.\begin{array}{l} v_o = \left(\dfrac{R_3}{R_3 + R_4} - \dfrac{R_1}{R_1 + R_2}\right) V_r \\ R_2 = R_3 = R_0(1 + x)\,,\; k = \dfrac{R_0}{R_4} = \dfrac{R_1}{R_0} \end{array}\right\} \Rightarrow v_o = \frac{x}{1 + x + k} V_r$$

$$\left.\begin{array}{l} v_o = \dfrac{x}{1 + x + k} V_r \\ R_1 = R_4 \Rightarrow k = 1 \end{array}\right\} \Rightarrow v_o = \frac{x}{2 + x} V_r$$

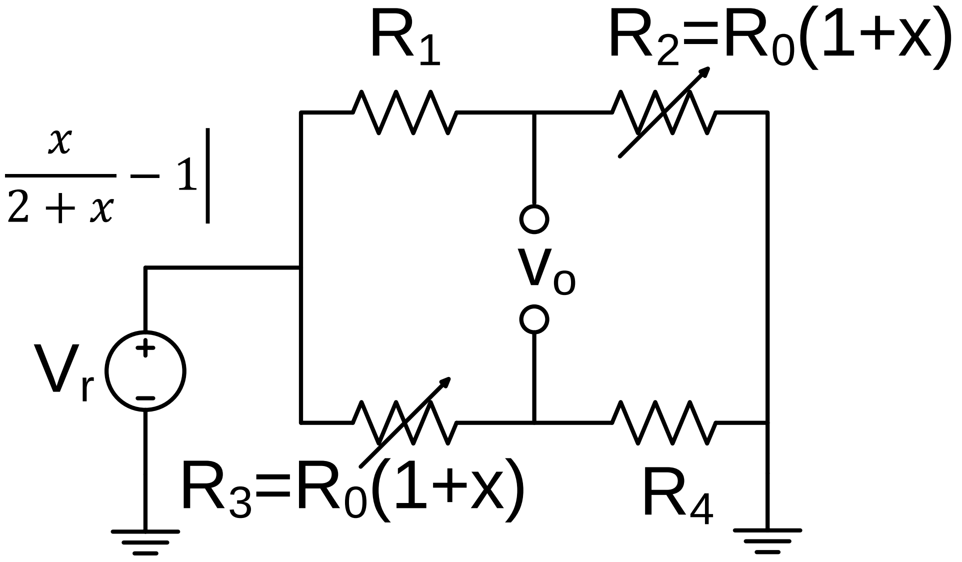
- relative non-linearity error due to bridge circuit
  - ideal output voltage $v_i = \frac{x}{2} V_r$

absolute error

$$\varepsilon = \left|\frac{v_o - v_i}{v_i}\right| = \left|\frac{\frac{x}{2 + x} V_r - \frac{x}{2} V_r}{\frac{x}{2} V_r}\right| = \left|\frac{2}{x}\frac{x}{2 + x} - 1\right|$$

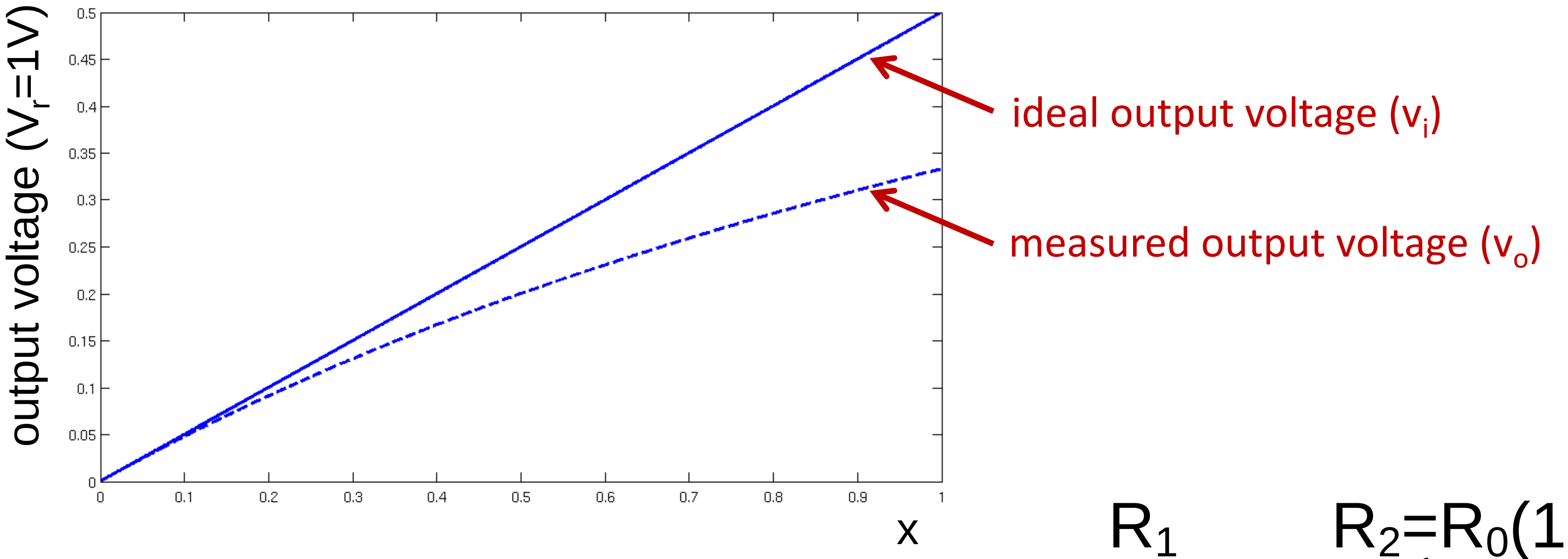
relative error

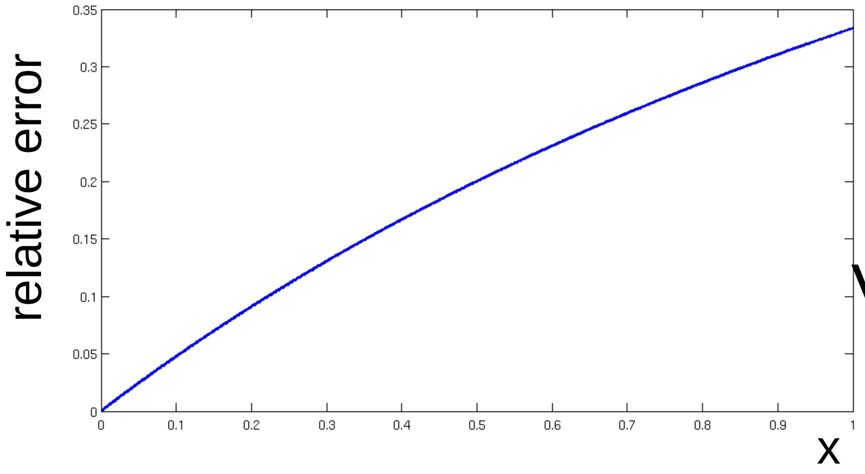
$$\Leftrightarrow \varepsilon = \left|\frac{2}{2 + x} - 1\right| = \left|\frac{-x}{2 + x}\right| = \left|\frac{x}{2 + x}\right|$$

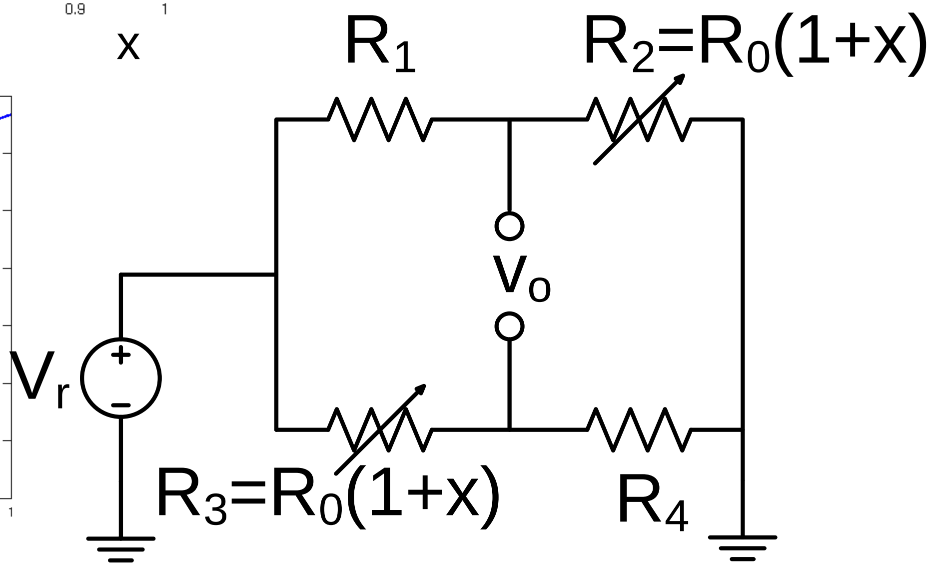


Signal processing

- increase sensitivity by adding sensor on other side of opposing arm







Signal processing

example – pseudo-bridge with two resistive sensors

- two equal linear resistance sensors: $R_3=R_2=R_0(1+x)$
- two equal fixed resistors: $R_1=R_4$

what value should R_1 and R_4 have to get an output voltage (V_o) which is directly proportional to the measured quantity x?

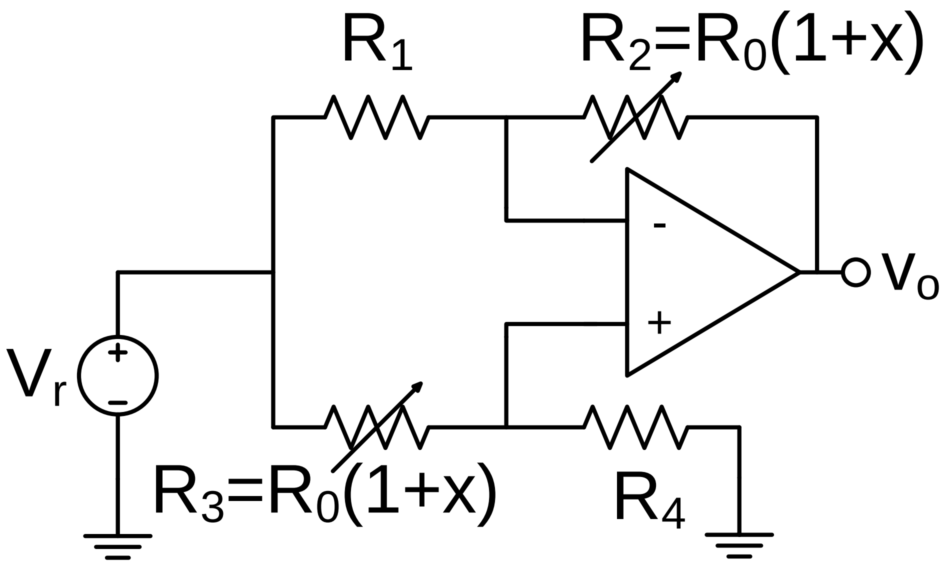
$$\frac{V_r - v_p}{R_3} = \frac{v_p}{R_4} \Leftrightarrow v_p = \frac{R_4}{R_3 + R_4} V_r \qquad \text{(=voltage divider)}$$

$$\frac{V_r - v_n}{R_1} = \frac{v_n - v_o}{R_2} \Leftrightarrow \frac{R_2}{R_1} V_r - \frac{R_2}{R_1} v_n - v_n = -v_o$$

$$\Leftrightarrow v_o = -\frac{R_2}{R_1} V_r + \left(\frac{R_2}{R_1} + 1\right) v_n$$

using $v_p=v_n$ we find

$$v_o = -\frac{R_2}{R_1} V_r + \left(\frac{R_2 + R_1}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_r$$



Signal processing

example – pseudo-bridge with two resistive sensors

- two equal linear resistance sensors: $R_3=R_2=R_0(1+x)$
- two equal fixed resistors: $R_1=R_4$

what value should R_1 and R_4 have to get an output voltage (V_o) which is directly proportional to the measured quantity x?

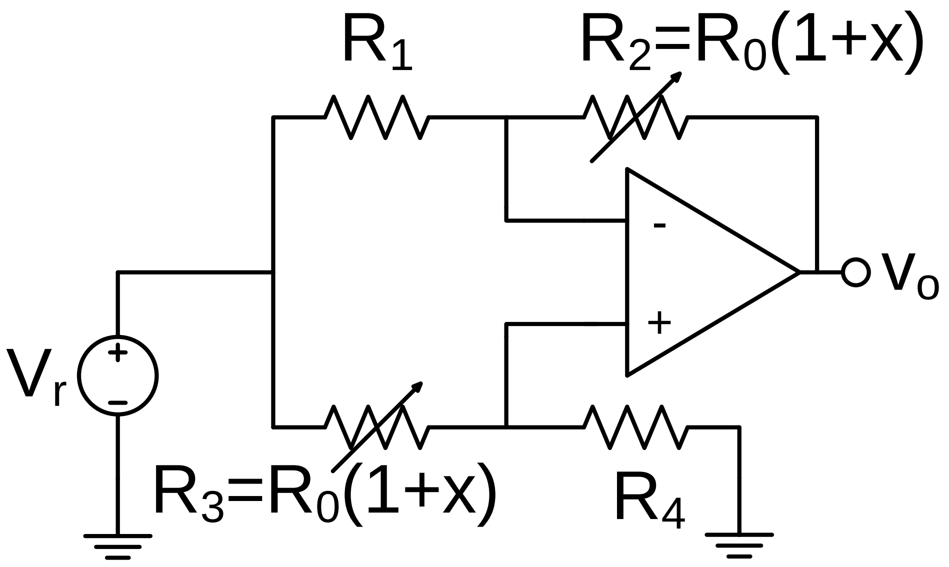
$$v_o = -\frac{R_2}{R_1}V_r + \left(\frac{R_2+R_1}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)V_r$$

using $R_1=R_4$ and $R_2=R_3$ we find

$$v_o = -\frac{R_2}{R_1}V_r + \left(\frac{R_2+R_1}{R_2+R_1}\right)V_r$$

using $R_2=R_0(1+x)$ we find

$$v_o = -\frac{R_0(1+x)}{R_1}V_r + V_r = \left[1-\frac{R_0}{R_1}-\frac{R_0}{R_1}x\right]V_r$$



Signal processing

example – pseudo-bridge with two resistive sensors

- two equal linear resistance sensors: $R_3=R_2=R_0(1+x)$
- two equal fixed resistors: $R_1=R_4$

what value should R_1 and R_4 have to get an output voltage (V_o) which is directly proportional to the measured quantity x?

$$v_o = -\frac{R_0(1+x)}{R_1}V_r + V_r = \left[1 - \frac{R_0}{R_1} - \frac{R_0}{R_1}x\right]V_r$$

v_o directly proportional to x when

$$1 - \frac{R_0}{R_1} = 0 \Rightarrow R_0 = R_1$$

output voltage then equal to

$$v_o = -xV_r$$

non-linearity is removed by circuit