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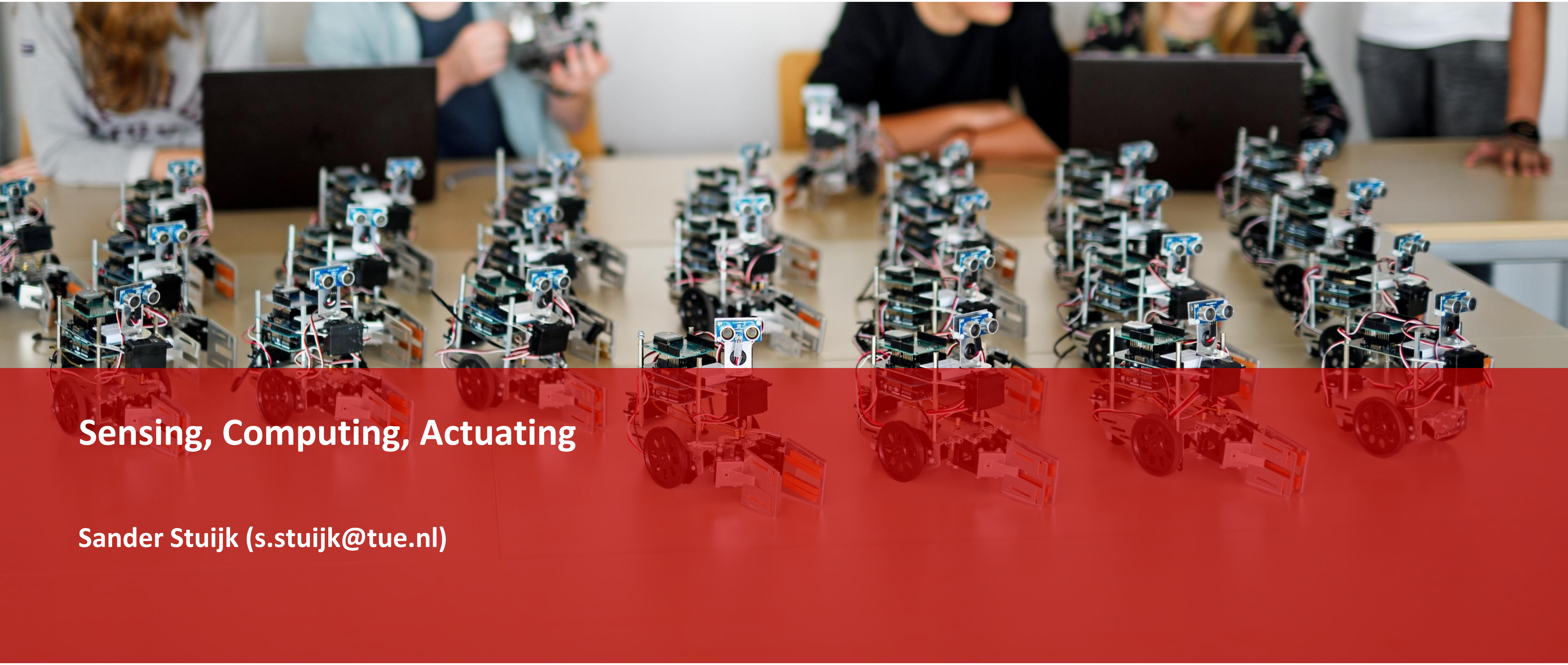


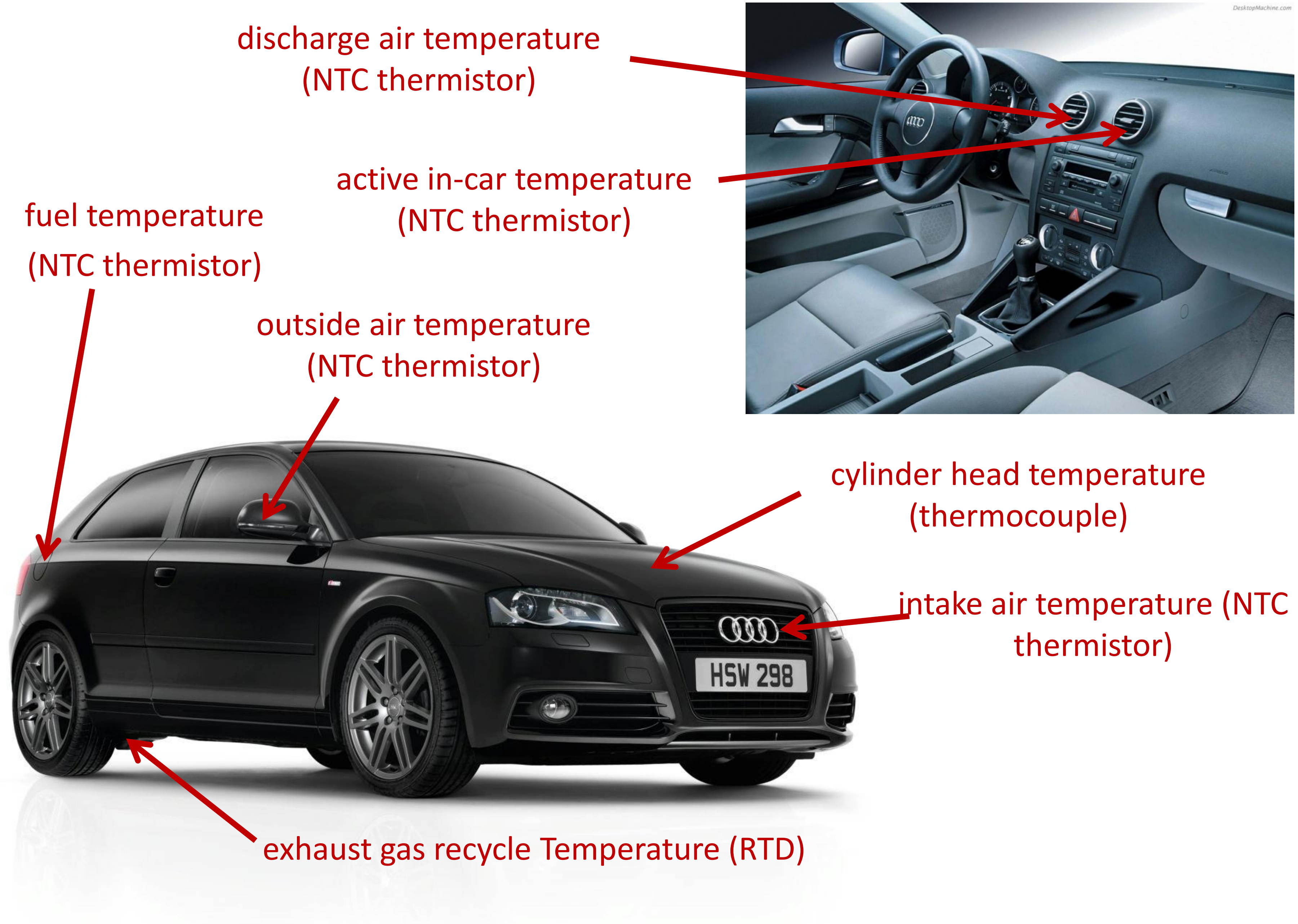
Sensing, Computing, Actuating

Sander Stuijk (s.stuijk@tue.nl)

THERMORESISTIVE SENSORS AND LINEARIZATION

(Chapter 2.9, 5.11)

Applications



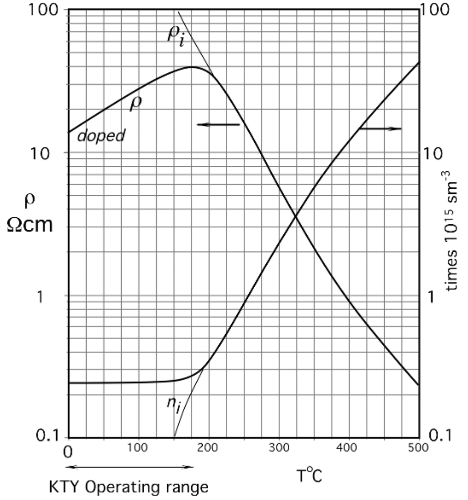
Temperature sensors

placement	excitation	physical effect	material	thermal sensor
contact	passive	thermal expansion	metal	bimetal
contact	**active**	**resistive effect**	**metal**	**RTD**
contact	**active**	**resistive effect**	**semiconductor**	**silicon resistive**
contact	**active**	**resistive effect**	**polymer or ceramic**	**thermistor**
contact	passive	thermoelectric effect	conductor	thermocouple
contact	active	PN junction	semiconductor	
non-contact	passive	pyroelectric effect	pyroelectric	pyroelectric
non-contact	active	ultrasound	-	acoustic

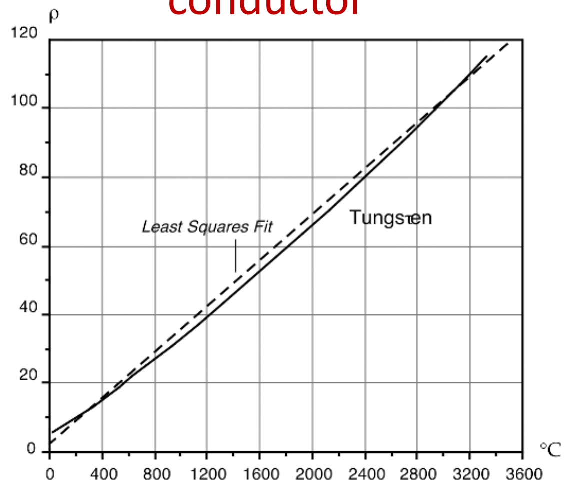
Thermoresistive effect

- resistivity of (semi)conductors depends on temperature
- temperature can be found by measuring resistance
- specific resistivity of a material $\rho = \frac{m}{ne^2\tau}$
  - temperature dependency through
    - number of free electrons (n) in semiconductors
      - NTC behavior: ↑T → ↑n → ρ↓
    - mean time between collisions (τ) in conductors
      - PTC behavior: ↑T → ↓τ → ρ↑

semiconductor



conductor



Resistance temperature detectors (RTDs)

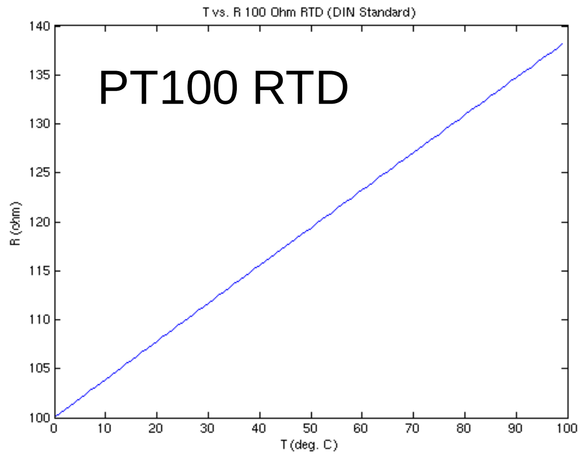
- specific resistivity of a material $\rho = \frac{m}{ne^2\tau}$
  - mean time between collisions (τ) in conductors
  - number of free electrons (n) in semiconductors

- RTD metal temperature sensor
  - positive temperature coefficient (PTC)
- relation between temperature and resistance

$$R = R_0[1 + \alpha_1(T - T_0) + \alpha_2(T - T_0)^2 + \ldots + \alpha_n(T - T_0)^n]$$

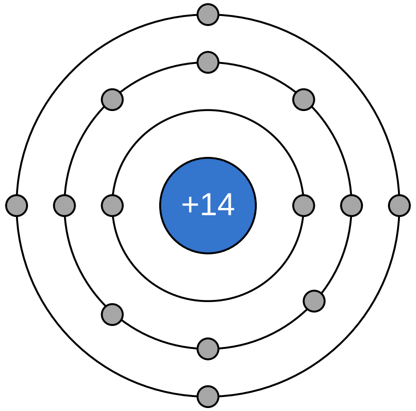
  - T_0 – reference temperature
  - R_0 – resistance at T_0

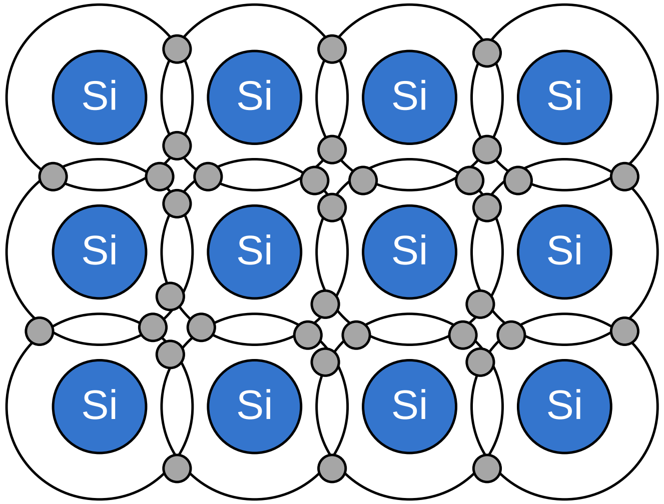
- example – PT100 RTD
  - $\alpha_1 \approx 3.89 \cdot 10^{-3}$/K, $\alpha_2 \approx -5.83 \cdot 10^{-7}/K^2$, $\alpha_3 \approx 1.92 \cdot 10^{-7}/K^3$
  - almost linear relation between temperature and resistance



Silicon resistive sensors

- specific resistivity of a material $\rho = \dfrac{m}{ne^2\tau}$
  - mean time between collisions (τ) in conductors
  - number of free electrons (n) in semiconductors
- pure silicon
  - four electrons in outer ring
  - electrons in outer ring form covalent bonds with neighboring atoms
  - these electrons are responsible for current





Silicon resistive sensors

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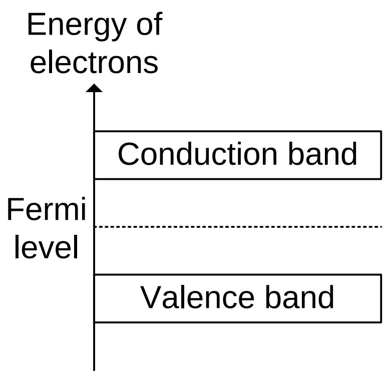
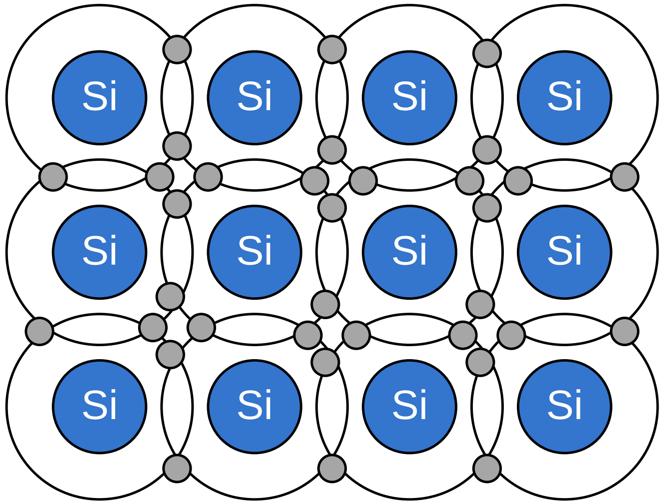
- specific resistivity of a material $\rho = \dfrac{m}{ne^2\tau}$
  - mean time between collisions (τ) in conductors
  - number of free electrons (n) in semiconductors

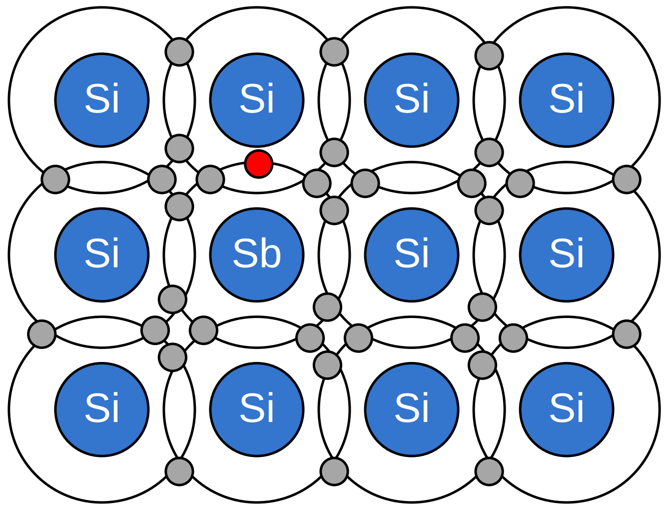
- pure silicon
  - four electrons in outer ring
  - electrons in outer ring form covalent bonds with neighboring atoms
  - these electrons are responsible for current
  - these electrons are at T = 0K in valence band
  - increasing energy moves electrons to conduction band





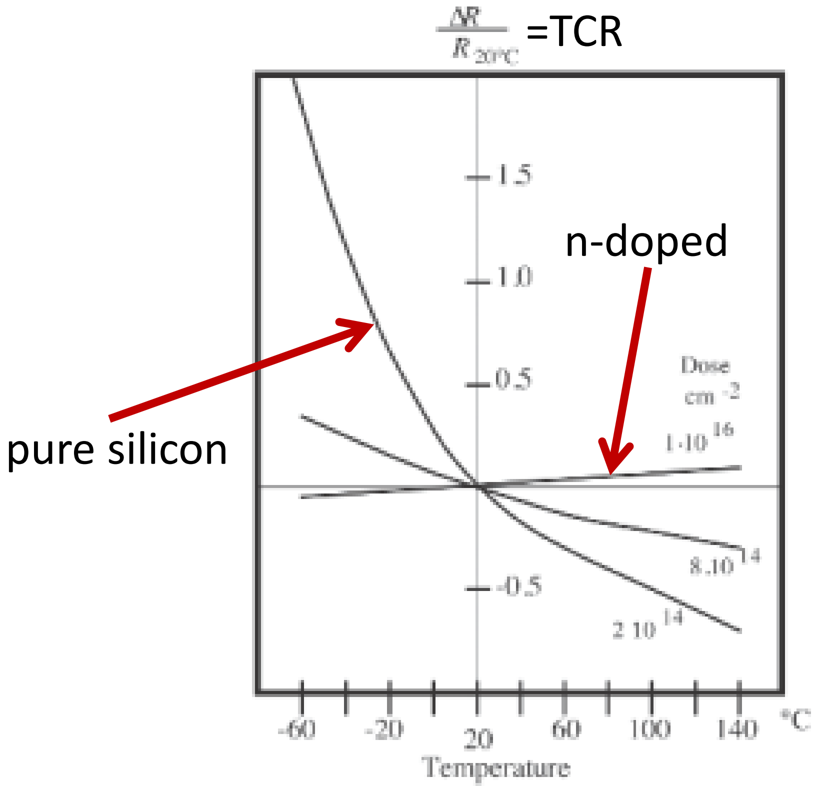
Silicon resistive sensors

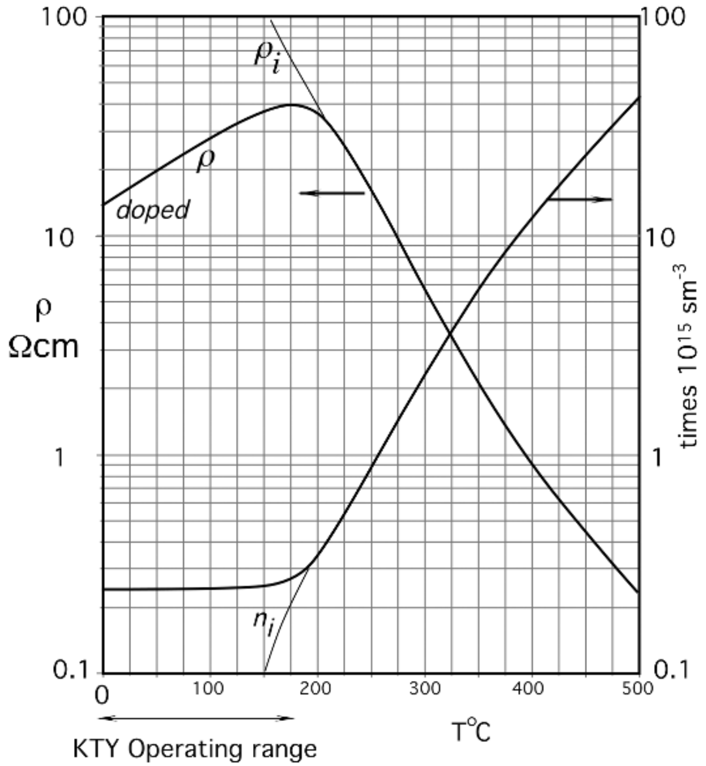
- specific resistivity of a material $\rho = \frac{m}{ne^2\tau}$
  - mean time between collisions (τ) in conductors
  - number of free electrons (n) in semiconductors
- pure silicon
  - temperature increase → lower resistance (NTC behavior)
- n-doped silicon (Sb - Stibium)
  - additional free electrons at given temperature which contribute to current
  - mean time between collisions dominates resistivity
  - device has PTC behavior



Silicon resistive sensor

- pure silicon has a negative temperature coefficient (NTC)
  - specific sensitivity ρ decreases when temperature increases
- doping with n-type impurity can change behavior to PTC
  - PTC behavior only in limited range (up-to 200°C)
  - above this temperature intrinsic behavior dominates



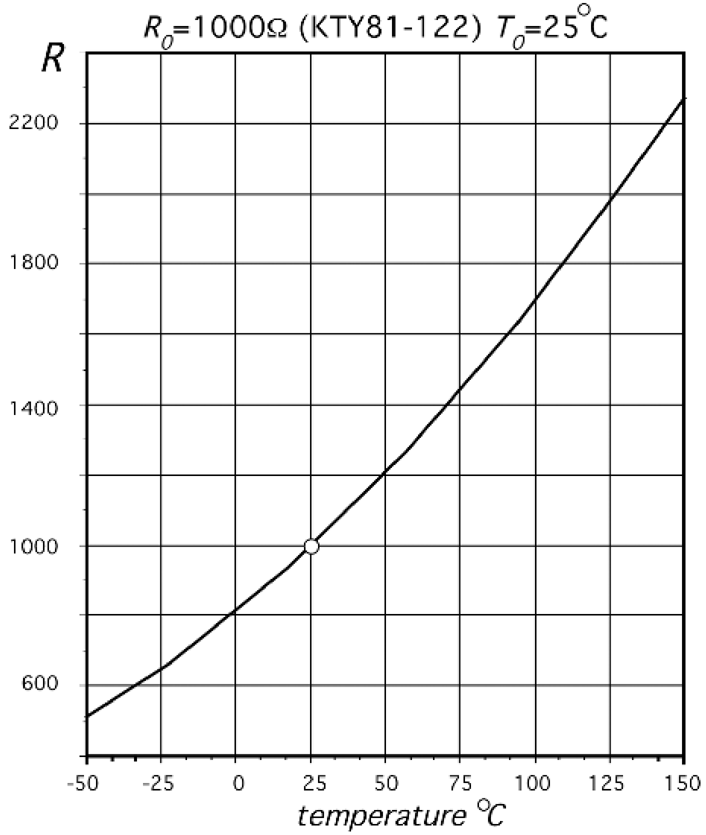


Silicon resistive sensor

- silicon resistive sensor has typically a TCR of 0.7%/K
- non-linearity of the sensor is limited
- transfer function can be approximated with

$$R = R_0[1 + \alpha_1(T - T_0) + \alpha_2(T - T_0)^2]$$

- example – KTY81-122 sensor
  - silicon resistive sensor with n-doping
  - range: -55°C to +150°C
  - high long-term stability: ±0.05K/year
  - almost linear transfer function
    - α_1 = 0.007847 (Ω/Ω)/K
    - α_2 = 1.874·10^{-5} (Ω/Ω)/K^2
  - resistance: 1kΩ at 25°C
  - sensitivity (at 25°C) ~ 7.9Ω/K



Thermistor

- thermistor comes from “thermally sensitive resistor”

- thermistors are made by mixing doped oxides of metals
  - leads to larger sensitivity then RTDs which use only metals
  - effect similar as seen in silicon resistive sensors

- temperature coefficient of thermistors
  - PTC when doping is heavy
  - NTC when doping is small

- only NTC thermistors are useful for precision temperature measurement

NTC thermistor

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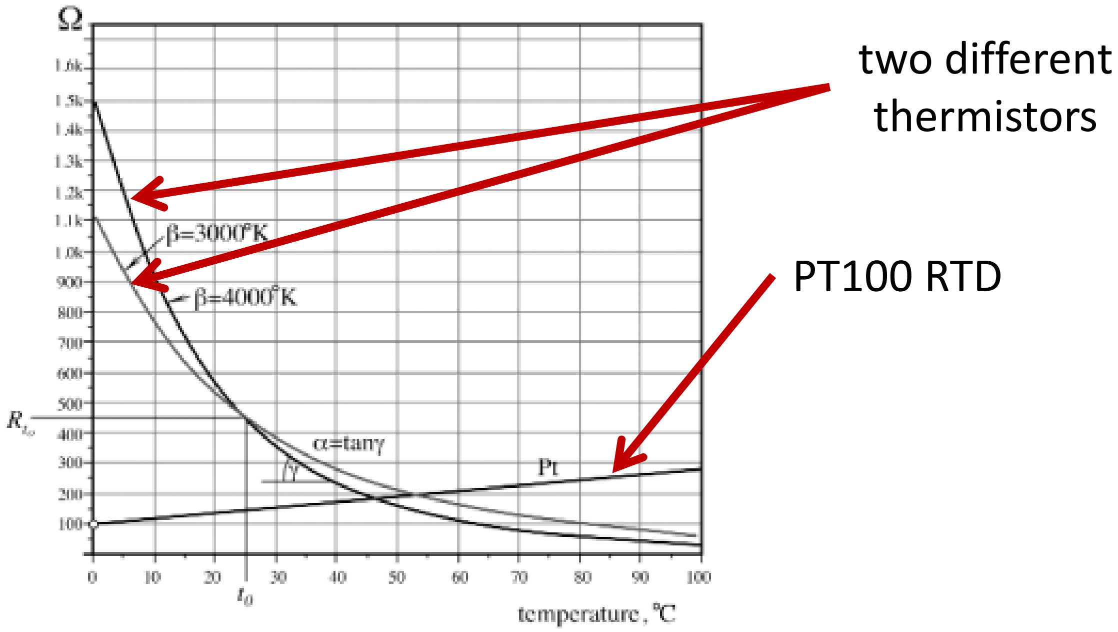
- relation between temperature and resistance is highly non-linear



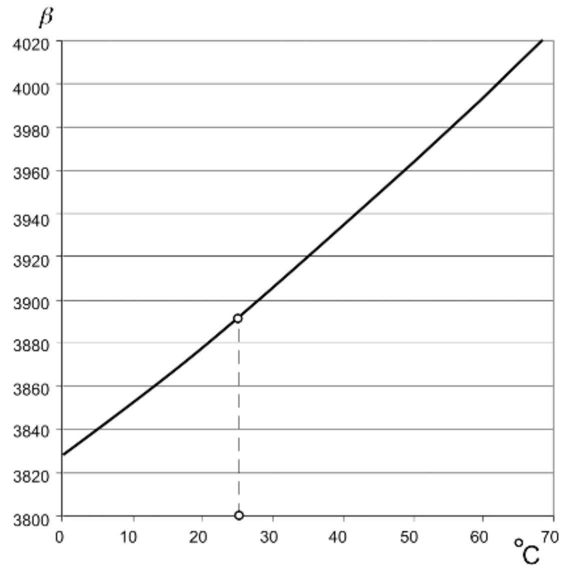
- transfer function $R_T = R_0 e^{B(1/T - 1/T_0)}$
  - T_0 – reference temperature
  - R_0 – resistance at T_0 (typically at 25°C)
  - B (or β) – characteristic temperature of the material

NTC thermistor

- characteristic temperature is material and temperature dependent
  - relation B and T can be approximated as linear (small error)
  - two-point calibration can be used to find B in certain range

$$B_{T_0/T_1} = \frac{\ln(R_1/R_0)}{(1/T_1 - 1/T_0)}$$

    - R_0 – resistance at T_0
    - R_1 – resistance at T_1



- example
  - calibration data
    - R = 5000Ω at 25°C, R = 1244Ω at 60°C
  - **what is the characteristic temperature from 25°C to 60°C?**

$$B_{25/60} = \frac{\ln\left(\frac{1244\Omega}{5000\Omega}\right)}{\frac{1}{(273+60)K} - \frac{1}{(273+25)K}} = 3944K$$

NTC thermistor

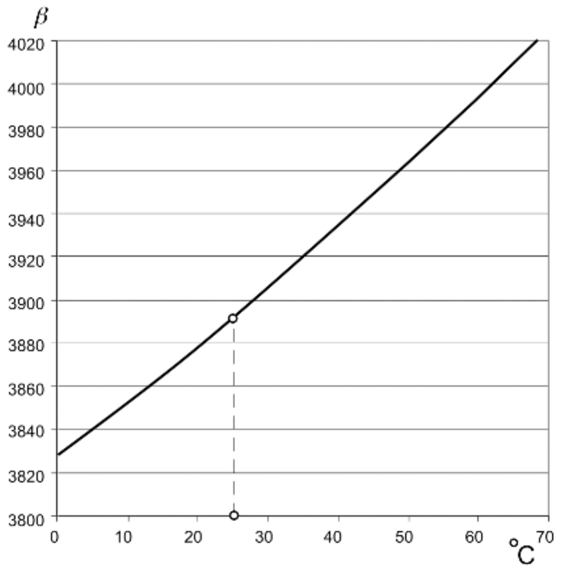
- characteristic temperature is material and temperature dependent
  - relation B and T can be approximated as linear (small error)
  - two-point calibration can be used to find B in certain range

$$B_{T_0/T_1} = \frac{\ln(R_1/R_0)}{(1/T_1 - 1/T_0)}$$

    - R_0 – resistance at T_0
    - R_1 – resistance at T_1
- temperature coefficient of resistivity (TCR)
    - TCR is often called relative sensitivity

$$\alpha = \frac{dR_T/dT}{R_T} = \frac{1}{R_T}\frac{d\left(R_0 e^{B(1/T-1/T_0)}\right)}{dT} = -\frac{B}{T^2}$$

  - non-linear R-T dependency
  - sensitive for low temperatures
  - sensitivity decreases quickly when temperature increases
    - typical: α = -8%/°C (at cold side), α = -2%/°C (at hot side)



NTC thermistor

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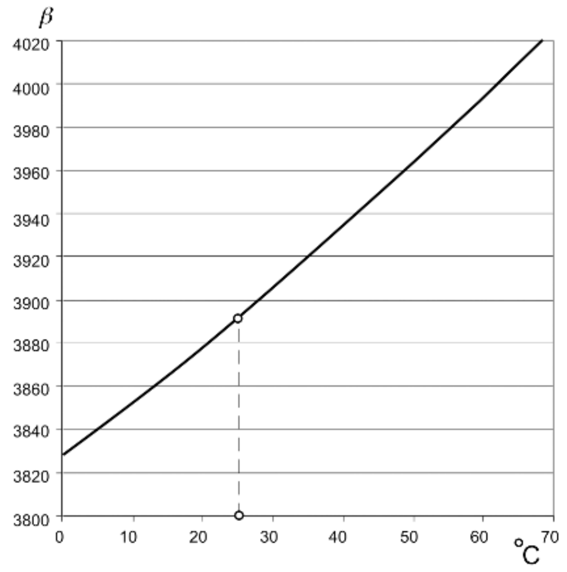
$$B_{T_0/T_1} = \frac{\ln(R_1/R_0)}{(1/T_1 - 1/T_0)}$$

    - R_0 – resistance at T_0
    - R_1 – resistance at T_1
- temperature coefficient of resistivity (TCR)

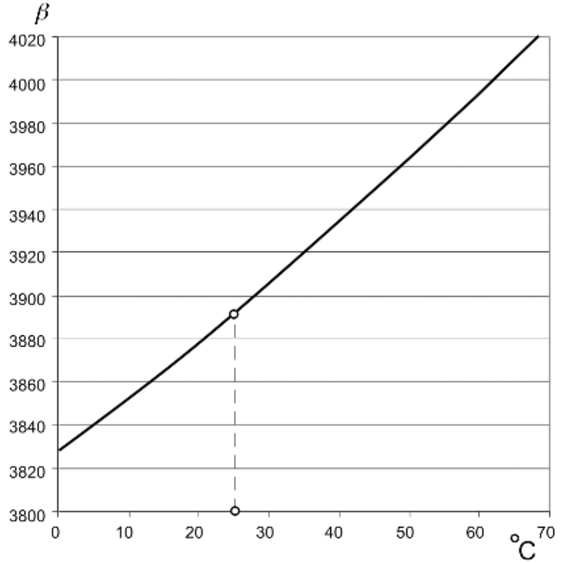
$$\alpha = \frac{dR_T/dT}{R_T} = \frac{1}{R_T}\frac{d\left(R_0 e^{B(1/T-1/T_0)}\right)}{dT} = -\frac{B}{T^2}$$

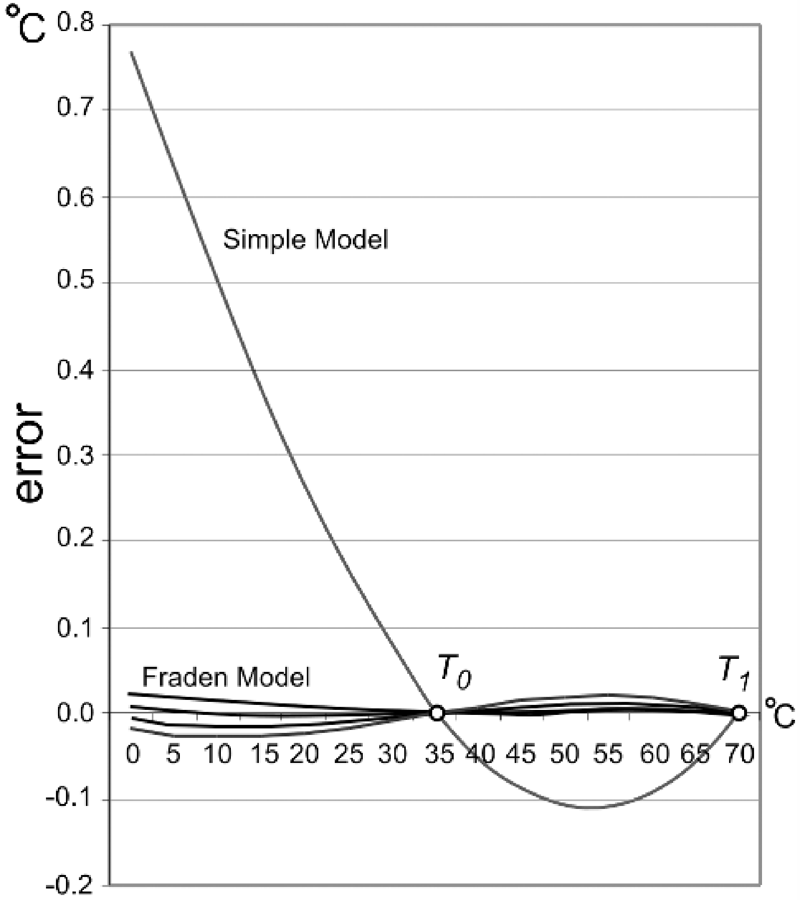
- example – Siemens thermistor
  - B = 4000K
  - TCR (at 25°C): α = -4.5%/K
  - thermistor has a 12.5x larger TCR as a PT100 RTD
  - **why do we prefer a large TCR?**
    - larger TCR means larger sensitivity



NTC thermistor – models

- two-parameter model $R_T = R_0 e^{B(1/T - 1/T_0)}$
  - model also known as simple model
  - provides accuracy of ±0.7°C for 70°C range
  - error due to non-linear relation B and T
- three or four parameter models exist
  - provide higher accuracy
  - more calibration points needed





NTC thermistor – models

example - alternative thermistor model

$$R_T = Ae^{B/T}$$

- **what value does A have when B = 4200K and R_T = 100kΩ at 25°C?**

$$R_{25} = Ae^{B/T_{25}} \Rightarrow A = \frac{R_{25}}{e^{B/T_{25}}} = \frac{100k\Omega}{e^{4200K/(273K+25K)}} = 0.0757\Omega$$

- **what is the TCR of this device at 0°C and 100°C?**
- at (0°C = 273K) it holds that

$$\alpha_0 = -\frac{B}{T^2} = \frac{-4200K}{(273K)^2} = -0.0564/K$$

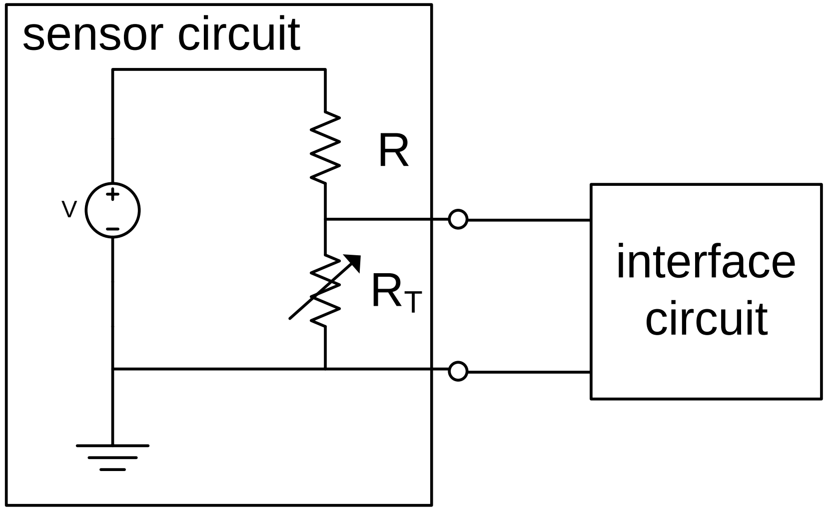
- at (100°C = 373K) it holds that

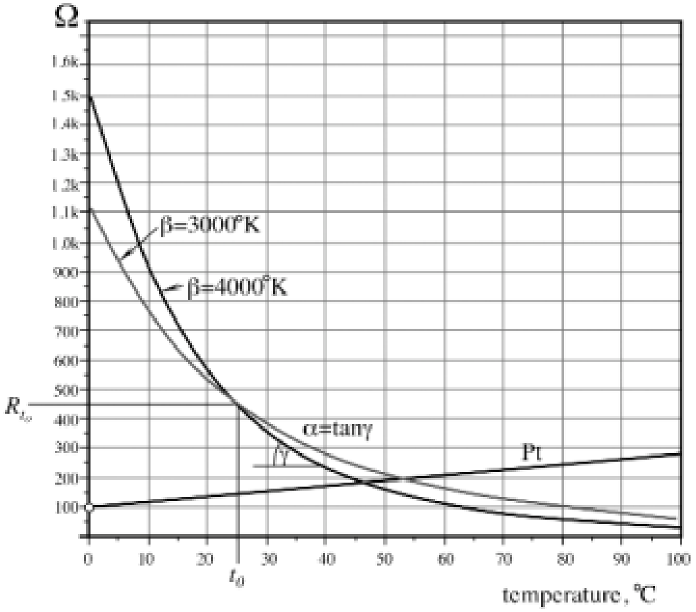
$$\alpha_{100} = \frac{-4200K}{(373K)^2} = -0.0302/K$$

- (relative) sensitivity decreases with increasing temperature

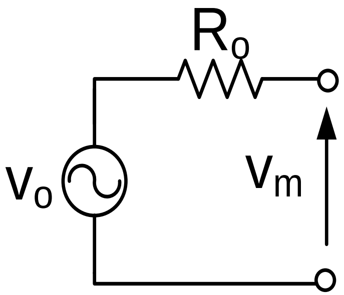
NTC thermistor – linearization

- relation between temperature and resistance is highly non-linear
- sensor will be connected to interface circuit





- Thévenin equivalent resistance R_o as seen by interface circuit

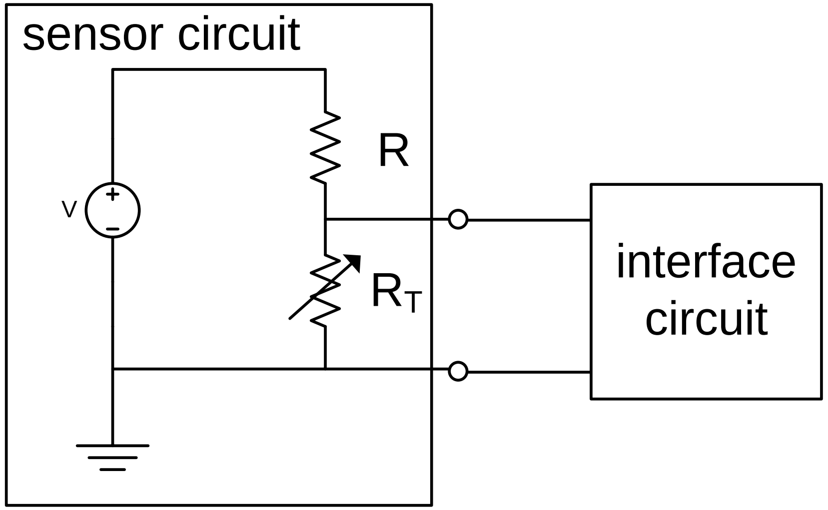


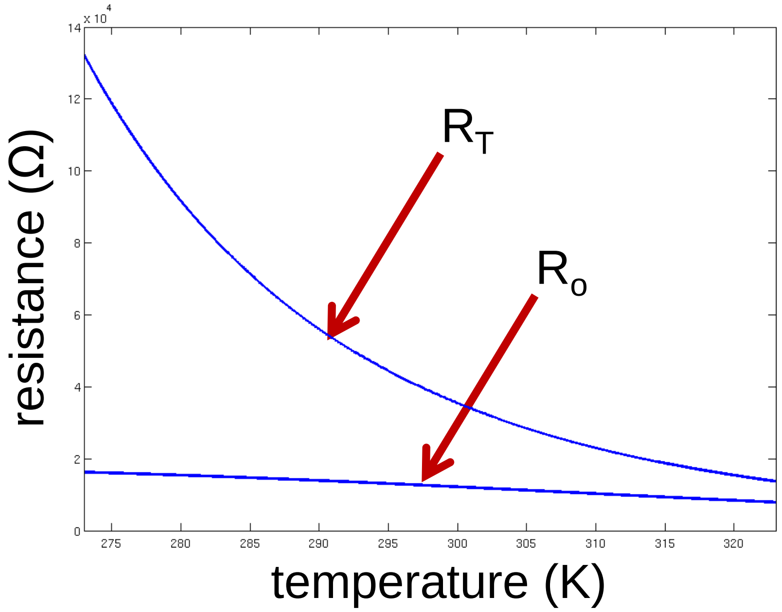
$$v_o = \frac{R_T}{R + R_T} v \quad R_o = \frac{R R_T}{R + R_T}, \quad R_T = R_0 e^{B\left(1/T - 1/T_0\right)} \Rightarrow R_o = \frac{R R_0}{R e^{-B\left(1/T - 1/T_0\right)} + R_0} = \frac{R R_0}{R e^{B\left(1/T_0 - 1/T\right)} + R_0}$$

NTC thermistor – linearization

example - resistance-temperature characteristic of sensor circuit

- R_0 = 25kΩ, B = 4000K, R=18500Ω





- sensor circuit has smaller non-linearity error compared to thermistor
- reduction in non-linearity error is not for free... (**why?**)
  - sensor circuit has smaller sensitivity compared to thermistor
  - linearity and sensitivity form a trade-off

NTC thermistor – linearization

- sensitivity for temperature change of sensor circuit: $\frac{dR_o}{dT}$

$$\left.\begin{aligned} R_o &= \frac{RR_T}{R+R_T} \\ R_T &= R_0 e^{B(1/T-1/T_0)} \end{aligned}\right\} \Rightarrow \frac{dR_o}{dT} = \frac{R^2}{(R+R_T)^2}\frac{dR_T}{dT}$$

  - sensitivity of R_o is non-linear

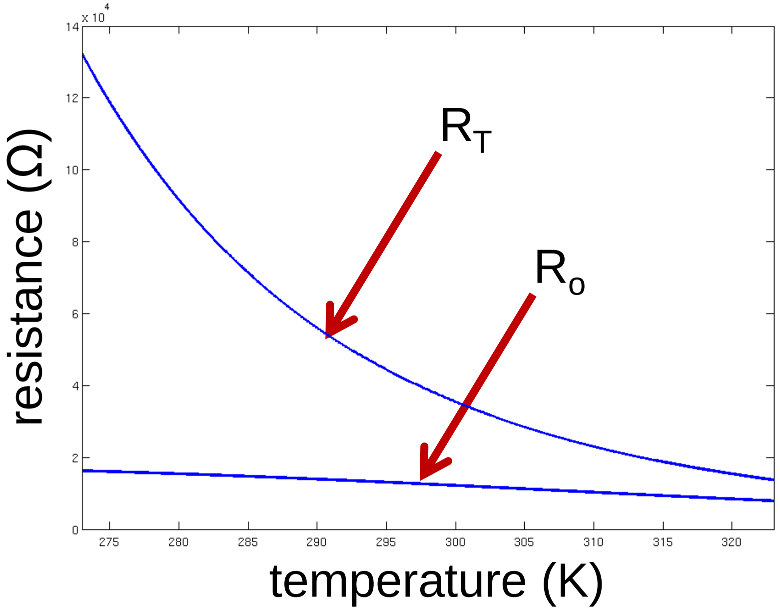
- sensitivity for temperature change of thermistor: $\frac{dR_T}{dT}$

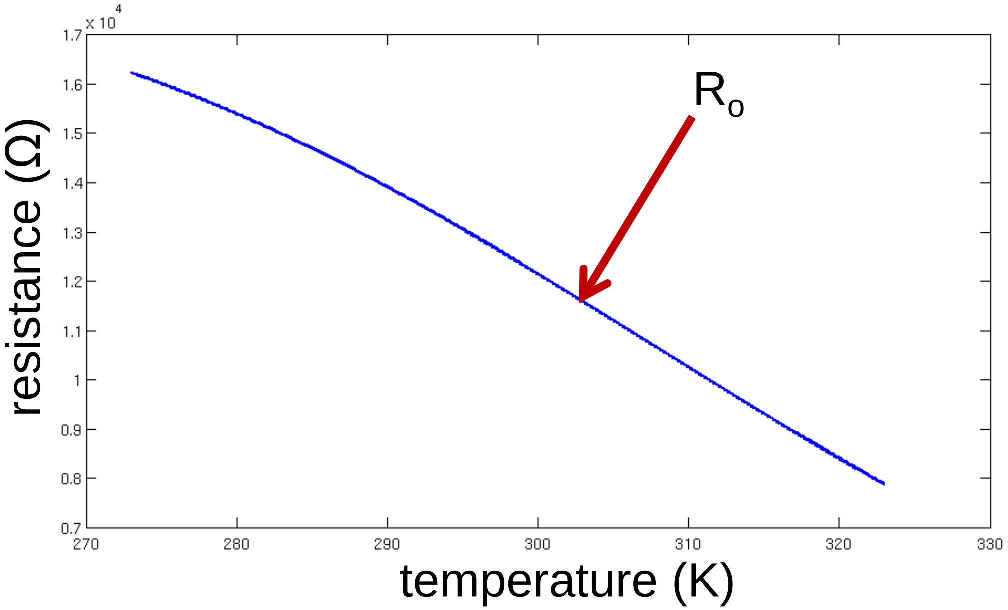
- comparing sensitivity of sensor circuit and thermistor

$$\frac{R^2}{(R+R_T)^2} < 1 \qquad \Rightarrow \frac{dR_o}{dT} < \frac{dR_T}{dT}$$

  - sensor circuit has lower sensitivity compared to thermistor alone
  - non-linearity error will also be smaller

NTC thermistor – linearization





- comparing sensitivity of sensor circuit and thermistor

$$\frac{R^2}{(R+R_T)^2} < 1 \qquad \Rightarrow \frac{dR_o}{dT} < \frac{dR_T}{dT}$$

  - sensor circuit has lower sensitivity compared to thermistor alone
  - non-linearity error will also be smaller

NTC thermistor – linearization

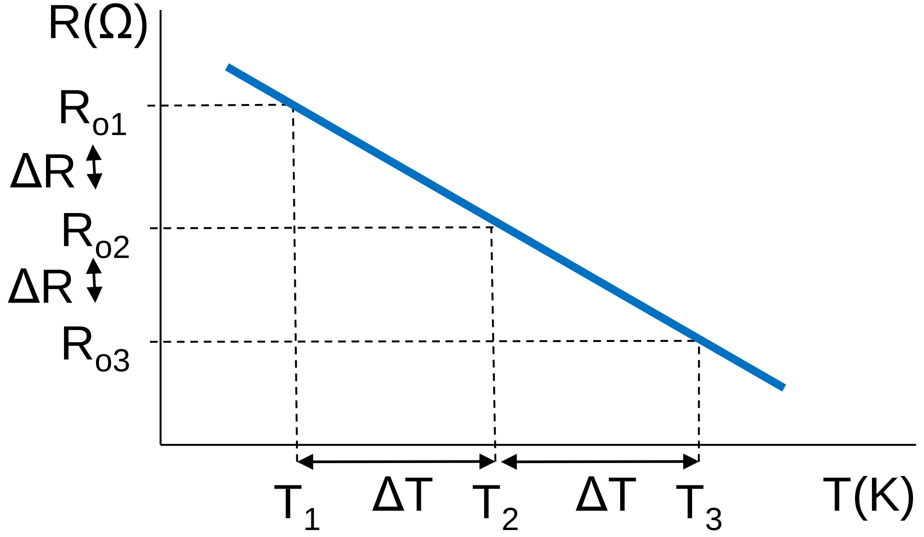
- linearity in range [T_1, T_3] can be improved in measurement range by choosing R
  - constraints

$$T_1 - T_2 = T_2 - T_3$$

$$R_{o1} - R_{o2} = R_{o2} - R_{o3}$$

  - it holds that

$$R_o = \frac{RR_T}{R + R_T}$$



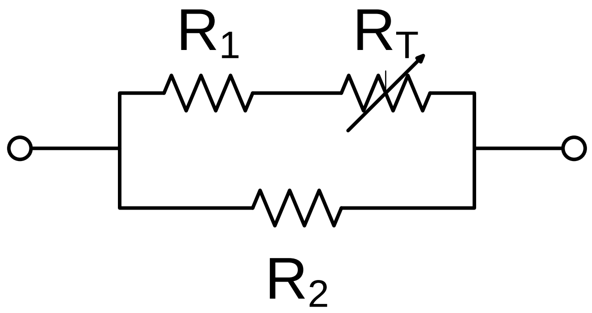
  - solving R_o for R gives

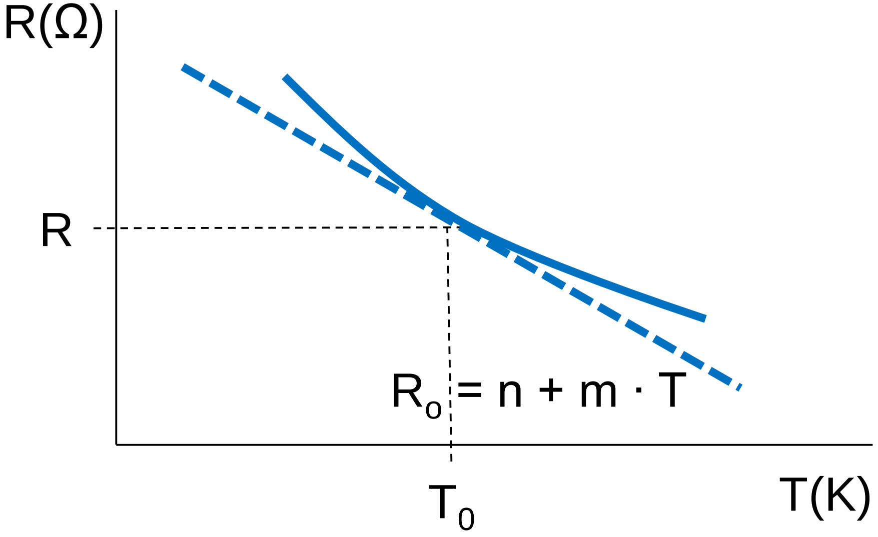
$$\frac{RR_{T1}}{R + R_{T1}} - \frac{RR_{T2}}{R + R_{T2}} = \frac{RR_{T2}}{R + R_{T2}} - \frac{RR_{T3}}{R + R_{T3}} \Rightarrow R = \frac{R_{T2}(R_{T1} + R_{T3}) - 2R_{T1}R_{T3}}{R_{T1} + R_{T3} - 2R_{T2}}$$

  - the expression is independent from the model of R_T
  - this technique can be used for any nonlinear resistive sensor
  - method minimizes non-linearity near the adjusting points

NTC thermistor – linearization

- minimize non-linearity near specific point in measurement range
- use circuit



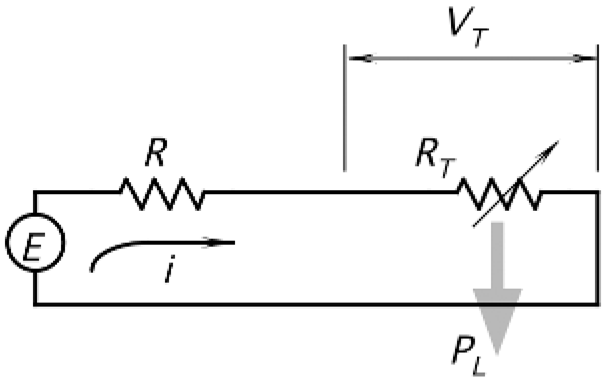


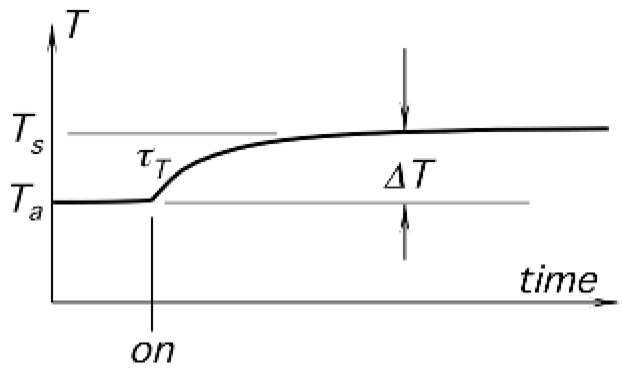
- objective
  - equivalent resistance of circuit shows inflection point around temperature T_0 and slope m

- **what value should R1 and R2 have to fulfill the objective?**
  - homework exercise with this lecture

NTC thermistor – self-heating

- current through NTC thermistor causes self-heating similar to RTDs





- PL is the thermal loss to the environment $P_L = \delta(T_S - T_a) = \delta \cdot \Delta T$
  - T_s, T_a – thermistor, ambient temperature
  - δ – heat dissipation factor
- power consumption of the thermistor

$$P_D = V_T \cdot i = i^2 \cdot R_T = \delta \cdot \Delta T + C\frac{dT_S}{dt}$$

heat accumulation rate

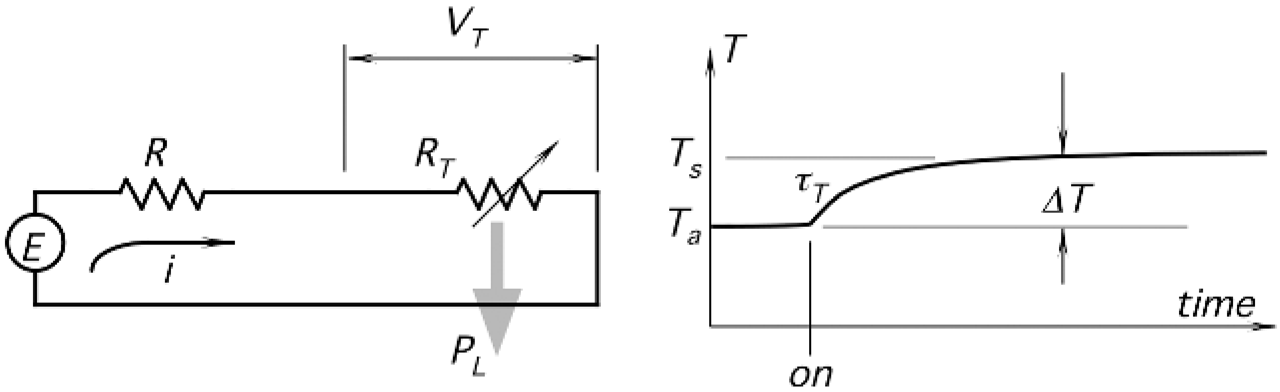
heat loss rate

  - C – thermal capacity (mass x specific heat)
- steady-state condition

$$\frac{dT_S}{dt} = 0 \;\Rightarrow V_T \cdot i = \delta \cdot \Delta T \;\Rightarrow \Delta T = \frac{V_T \cdot i}{\delta} = \frac{i^2 \cdot R_T}{\delta} = \frac{P_D}{\delta}$$

NTC thermistor – self-heating

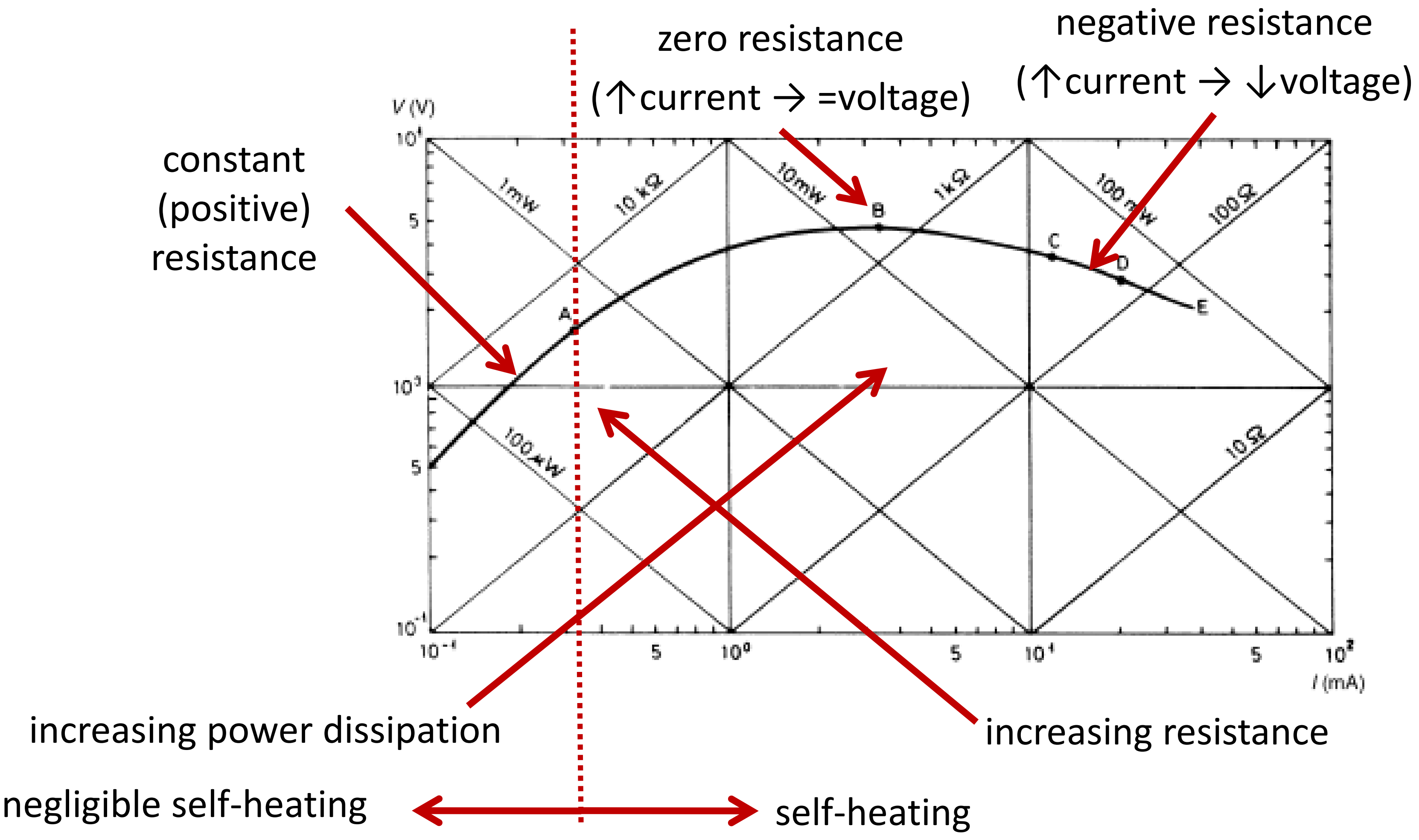
- current through NTC thermistor causes self-heating similar to RTDs



- self-heating can be minimized by minimizing current through R_T
  - use small supply voltage
  - use large series resistor in front of R_T
  - use sensor with small duty-cycle voltage source

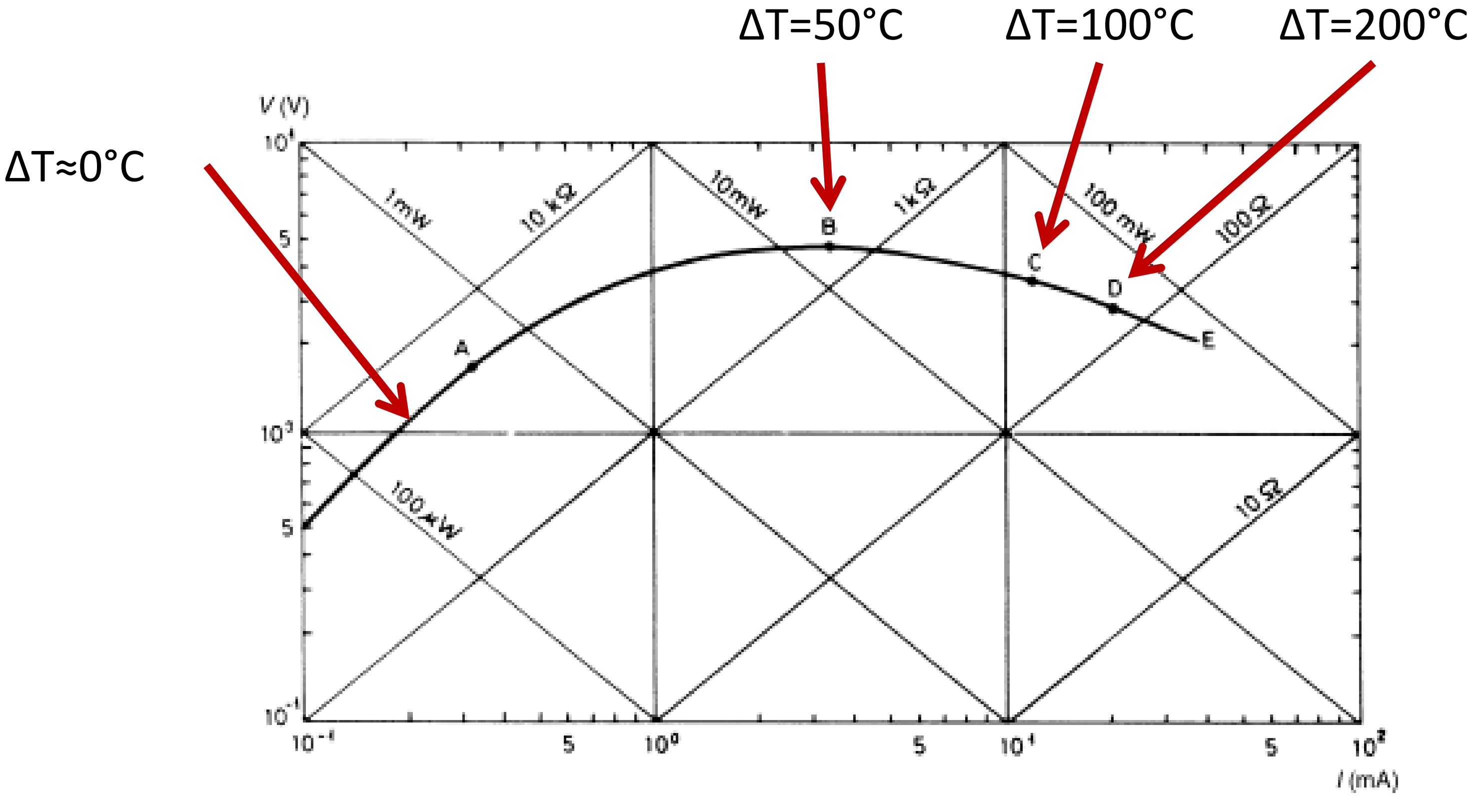
NTC thermistor – self-heating

- voltage-current characteristic for a thermistor in still air at 25°C



NTC thermistor – self-heating

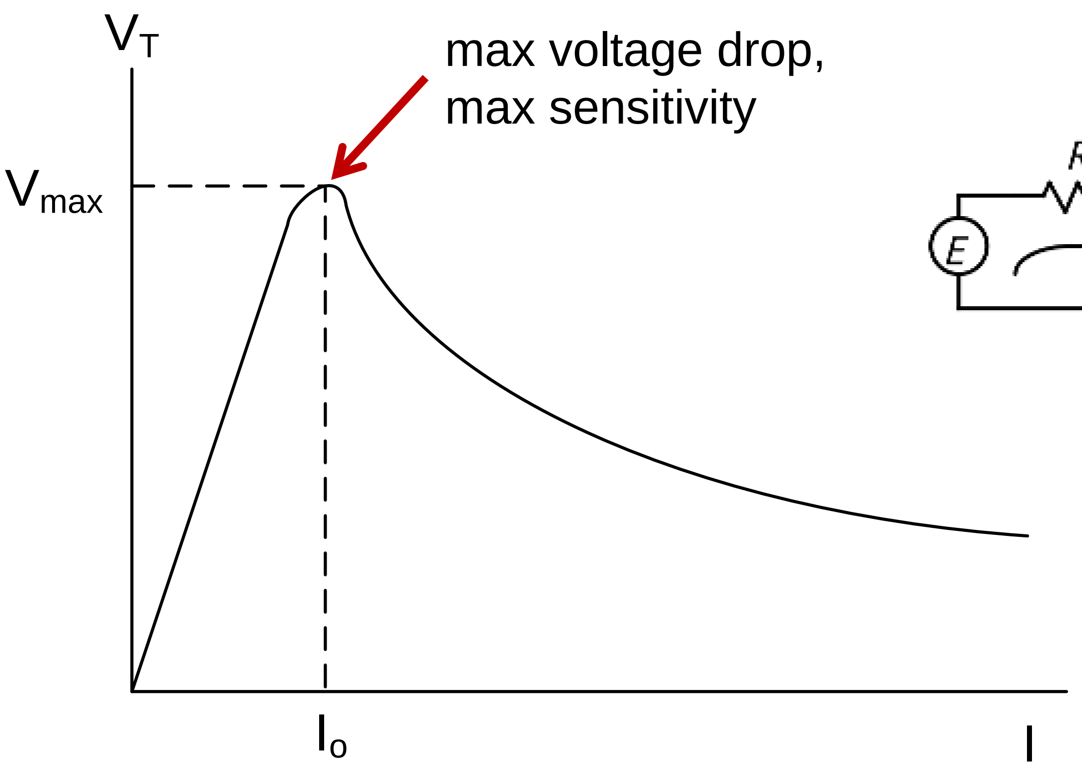
- voltage-current characteristic for a thermistor in still air at 25°C

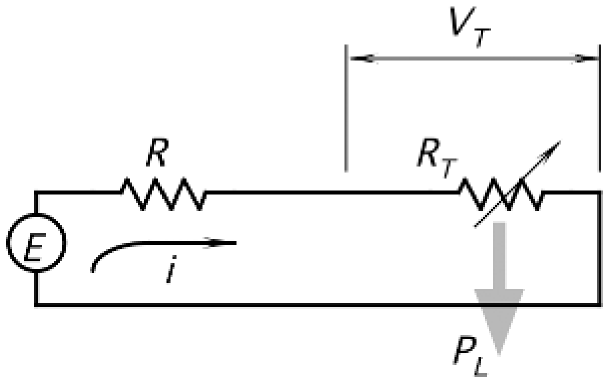


- self-heating can lead to thermal run-away and destruction of sensor
- some applications use self-heating to get maximal voltage drop across sensor (e.g. flow rate sensor)

NTC thermistor – self-heating

- voltage-current characteristic for a thermistor in still air at 25°C





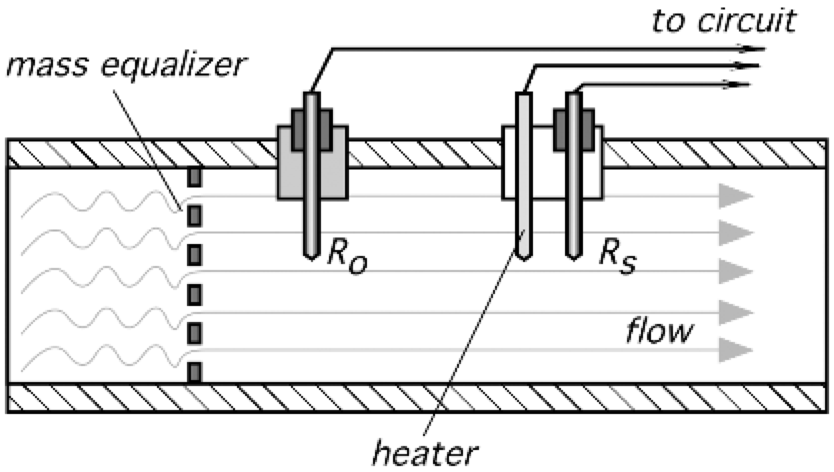
- small current (negligible heating) – use Ohm's law to find temperature
- increasing current: I↑ → P_D↑ → ΔT↑ → R_T↓ → V_T↓

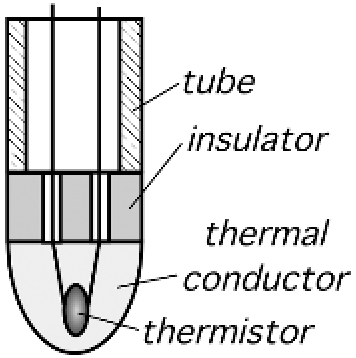
Thermal flow sensors

- example - wind chill factor
  - when cycling on a cold day, your hands are colder then when standing still
  - air flow causes the hands of the cyclist to cool down
  - by measuring the temperature of his hands, the cyclist can get a good measure of the air flow

Thermal flow sensors

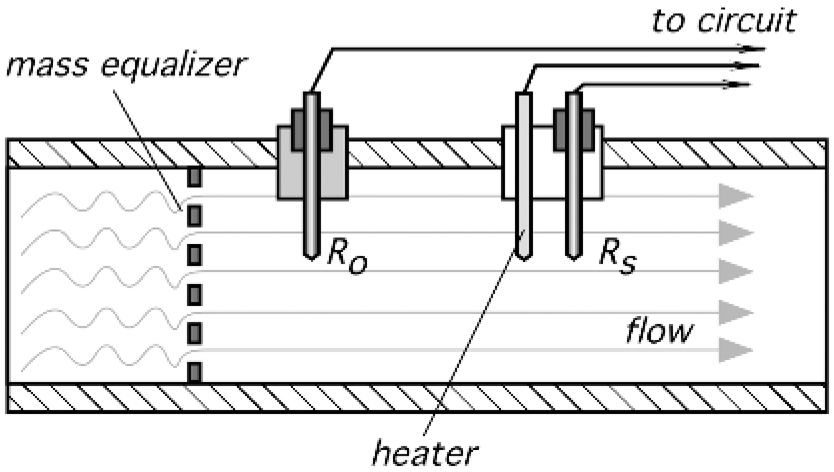
- construction
  - three tubes immersed into a moving medium
  - two tubes contain temperature detectors (R_0 and R_S)
  - detectors thermally coupled to medium
  - detectors thermally isolated from structural elements
  - mass equalizer ensures that medium moves through the detectors without turbulence

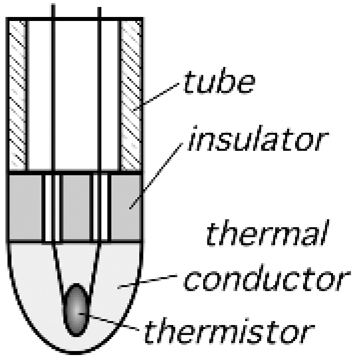




Thermal flow sensors

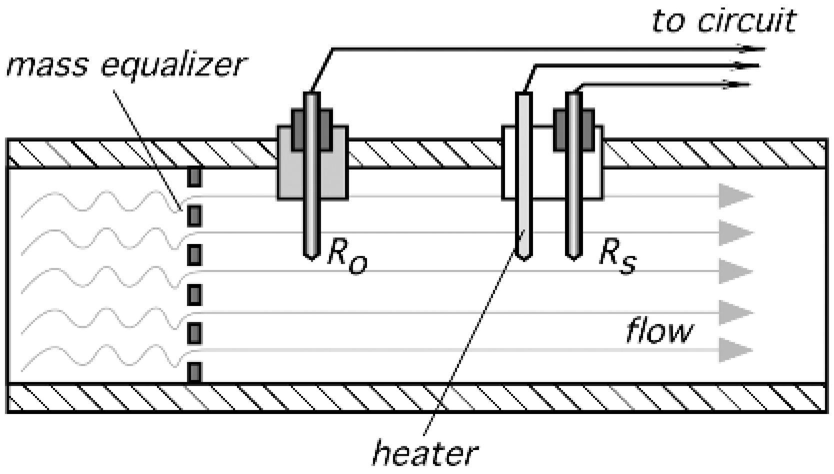
- operation
  - detector R_0 measures the temperature of the flowing medium
  - medium heated with heater
  - detector R_S measures the elevated temperature

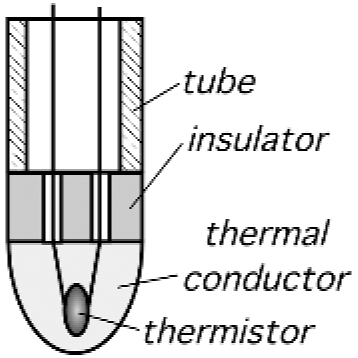




Thermal flow sensors

- still medium
  - heat dissipated from heater to both detectors
  - heat moves out of the heater by thermal conduction and gravitational convection
  - heater is closer to R_S then to R_0
    - R_S will register a higher temperature then R_0

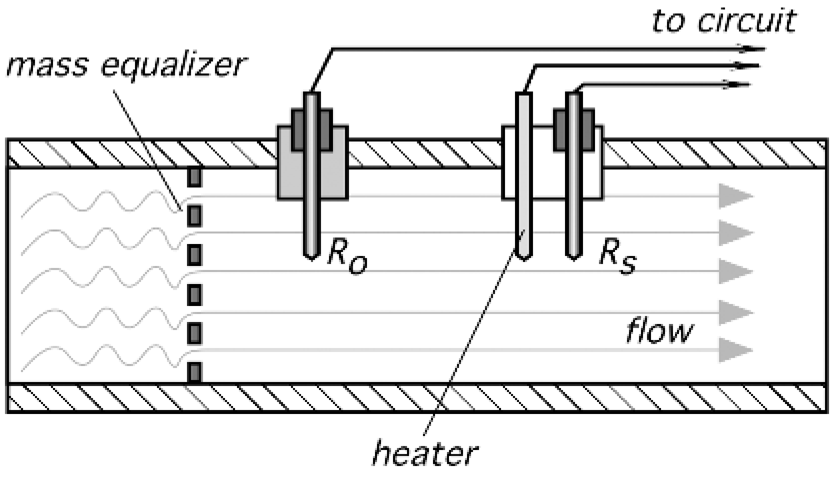


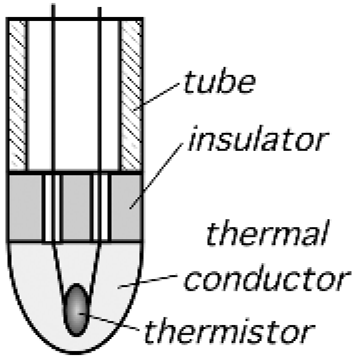


Thermal flow sensors

- moving medium
  - heat dissipation increased due to forced convection
  - increasing flow rate, leads to the higher heat dissipation
  - this leads to a lower temperature registered by R_S

- thermal flow sensor measures the heat loss and converts it into the flow rate of the medium



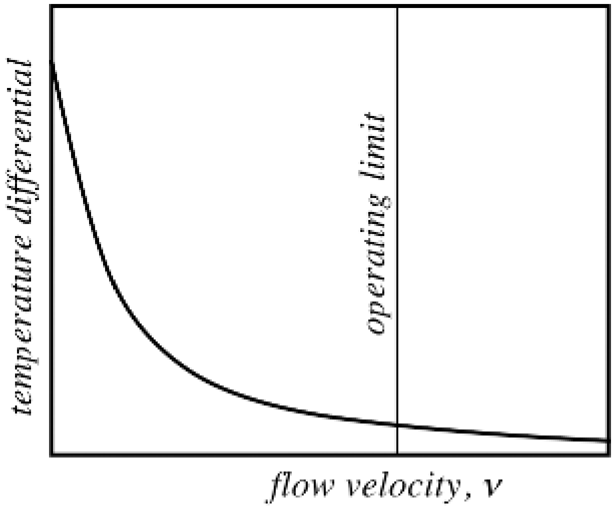


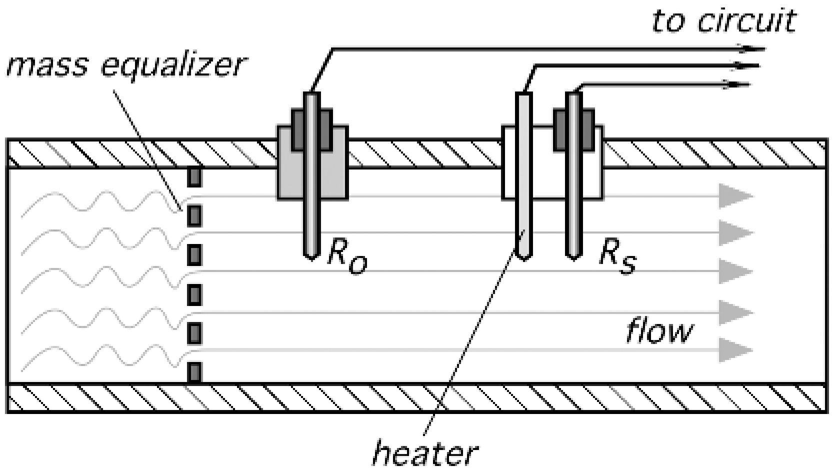
Thermal flow sensors

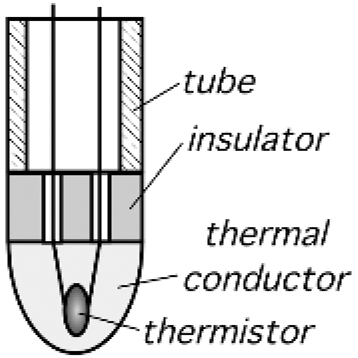
- non-linear relation between flow velocity v and temperature difference
- relation between flow velocity v and temperature difference

$$v = \frac{K}{\rho}\left(\frac{dQ}{dt}\frac{1}{T_s - T_0}\right)^{1.87}$$

- K – calibration constant, ρ – density of the medium, Q – heat transfer
- Q depends on dissipation factor (velocity)





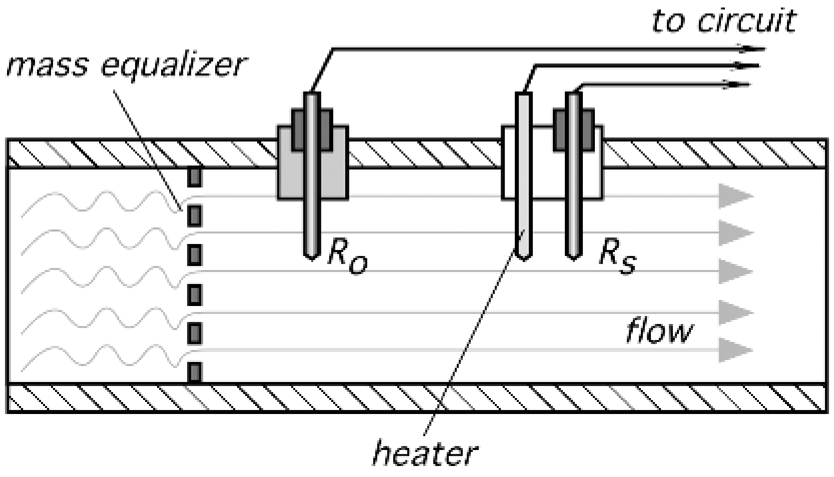


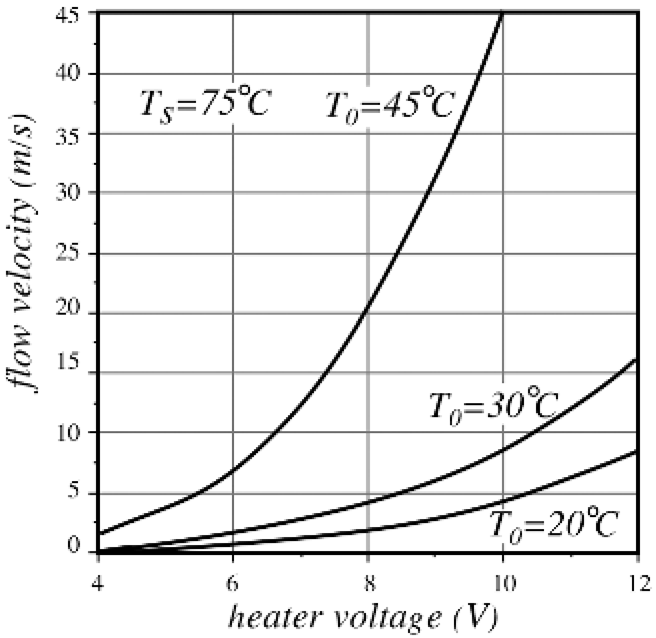
Thermal flow sensors

- heater can be integrated into R_S by using it in self-heating mode
- relation between voltage across self-heating detector (e) and velocity of the flow (v) given by

$$v = \frac{K}{\rho}\left(\frac{e^2}{R_S}\frac{1}{T_s - T_0}\right)^{1.87}$$

  - K – calibration constant
  - ρ – density of the medium





Thermal flow sensors

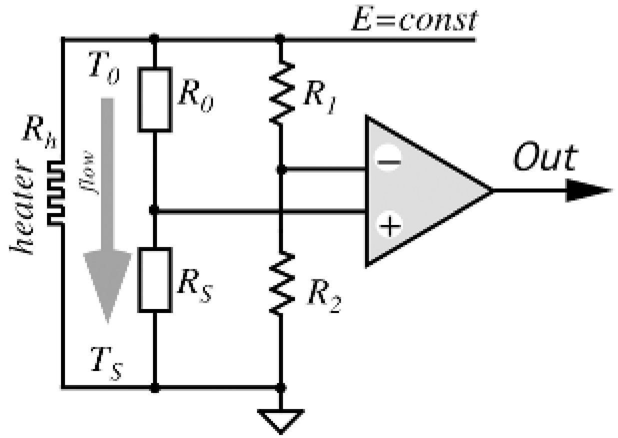
- relation between voltage across self-heating detector (*e*) and velocity of the flow (*v*) suggests two method to measure flow

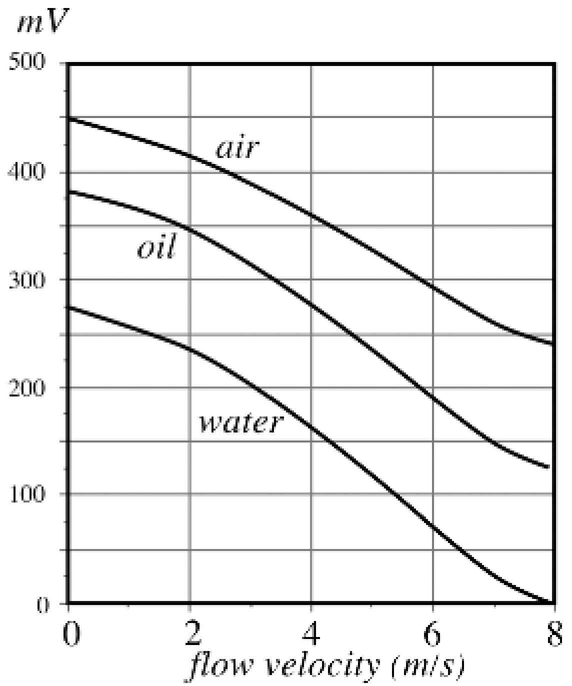
$$v = \frac{K}{\rho}\left(\frac{e^2}{R_S}\frac{1}{T_s - T_0}\right)^{1.87}$$

- method 1
  - voltage (e) and resistance of detector R_S are kept constant and temperature difference is outputted
- method 2
  - temperature difference is kept constant by a control circuit which regulates voltage (e) of the heater
  - voltage (e) is output of the sensor
- method 2 is easier to realize in miniature sensors

Thermal flow sensors – method 1

- thermal flow sensor can be placed in a bridge circuit
- bridge is imbalanced at low flow rate
  - high output voltage
- increasing flow heats detector R_S
  - its temperature comes closer to temperature of R_0
  - balances the bridge and decreases the output voltage
- voltage outputted by the sensor depends
  - on the flow velocity
  - on the medium



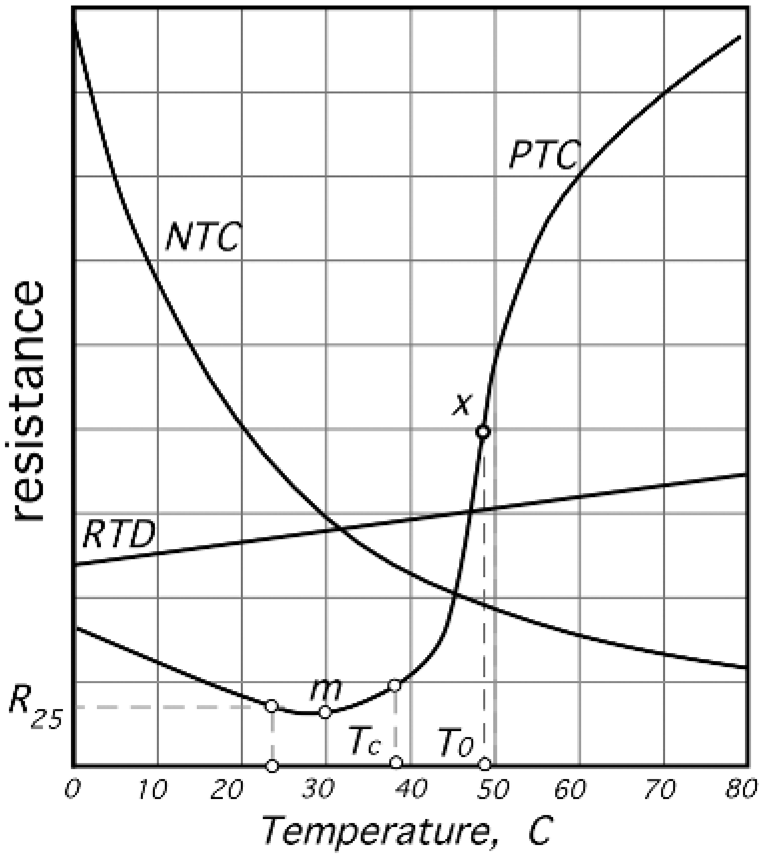


PTC thermistors

- PTC thermistor NTC behavior till some temperature (point m)

- PTC thermistors show abrupt change in resistance
  - switch occurs around Curie temperature TC
  - temperature coefficient of resistivity (TCR) can be 200%/°C

- transfer function is not suitable for precision temperature measurement



PTC thermistors

example – PTC thermistor

- PTC thermistor modeled with

$$R_T = R_{25} \cdot \left(\frac{273.15K + T}{298.15K}\right)^{2.3}$$

- **what is the TCR for this PTC thermistor at 25°C?**
- TCR is defined as

$$\frac{dR/dT}{R}$$

- derivative of the given equation yields

$$\frac{dR_T}{dT} = 2.3 \cdot R_{25} \cdot \left(\frac{273.15K + T}{298.15K}\right)^{1.3} \cdot \frac{1}{298.15K}$$

- at 25°C it holds that

$$\left.\frac{dR_T}{dT}\right|_{T=25^\circ C} = 2.3 \cdot R_{25} \cdot \left(\frac{273.15K + 25}{298.15K}\right)^{1.3} \cdot \frac{1}{298.15K} = \frac{2.3}{298.15K} \cdot R_{25} = (0.0077 \cdot R_{25})/K$$

PTC thermistors

example – PTC thermistor

- PTC thermistor modeled with

$$R_T = R_{25} \cdot \left(\frac{273.15K + T}{298.15K} \right)^{2.3}$$

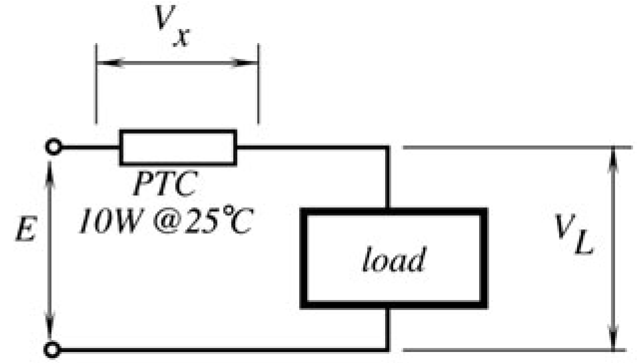
- **what is the TCR for this PTC thermistor at 25°C?**
- therefore

$$TCR(25^\circ C) = \left. \frac{dR/dT}{R} \right|_{T=25^\circ C} = \frac{dR/dT}{R_{25}} \cdot 100\% = 0.77\%/K$$

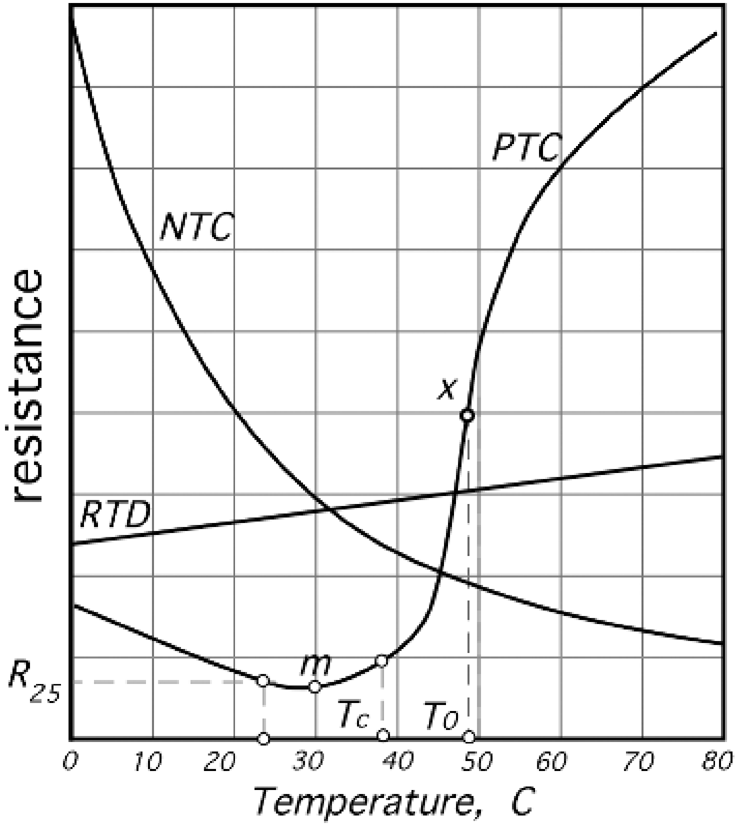
- % is used to indicate the fractional change in resistance when a unit change in the temperature occurs

PTC thermistors

- application: circuit protection



- short circuit of load causes PTC to get very large resistance
- current in circuit goes to zero
- circuit "resets" itself when short circuit is removed



- PTC characteristic provides build-in protection against overheating
  - overheating will increase resistance
  - increasing resistance will decrease current
  - decreasing current will decrease self-heating

Comparing NTC, PTC thermistors and RTDs

NTC thermistors versus RTDs

(+) higher resistivity (small lead-wire error)

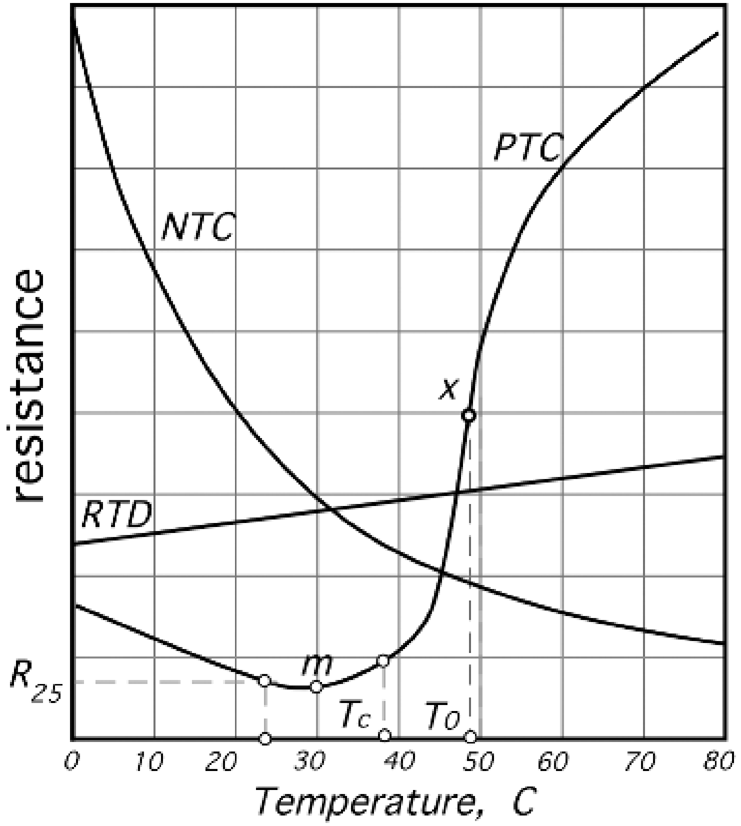
(+) higher sensitivity (up-to 1000x)

(-) larger self-heating effect

(-) smaller operating range

(-) higher non-linearity

(-) lower accuracy (±0.7°C)



PTC versus NTC

(+) build-in self-protection against overheating

(-) complex temperature-resistance relation (partially NTC behavior)

(-) smaller operating range (some types only usable as switch)

(-) not suitable for precision temperature measurement