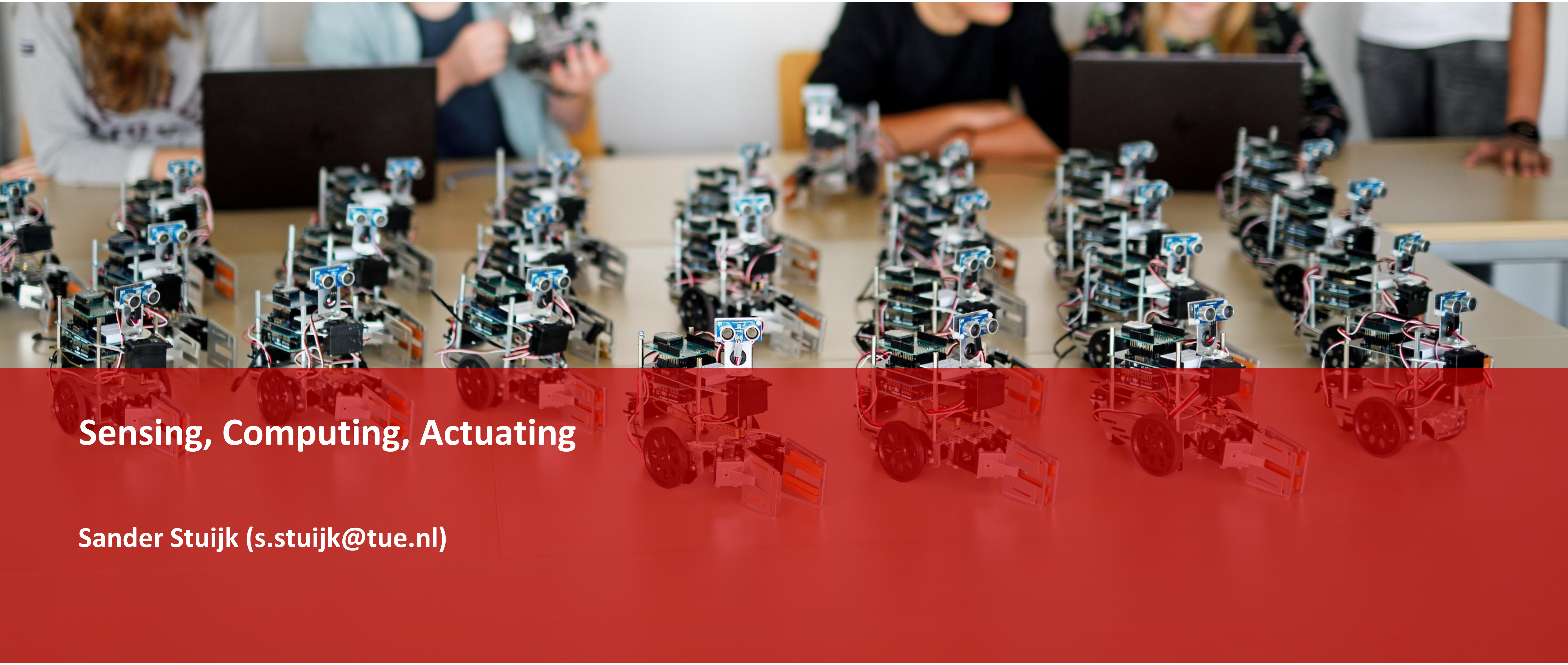


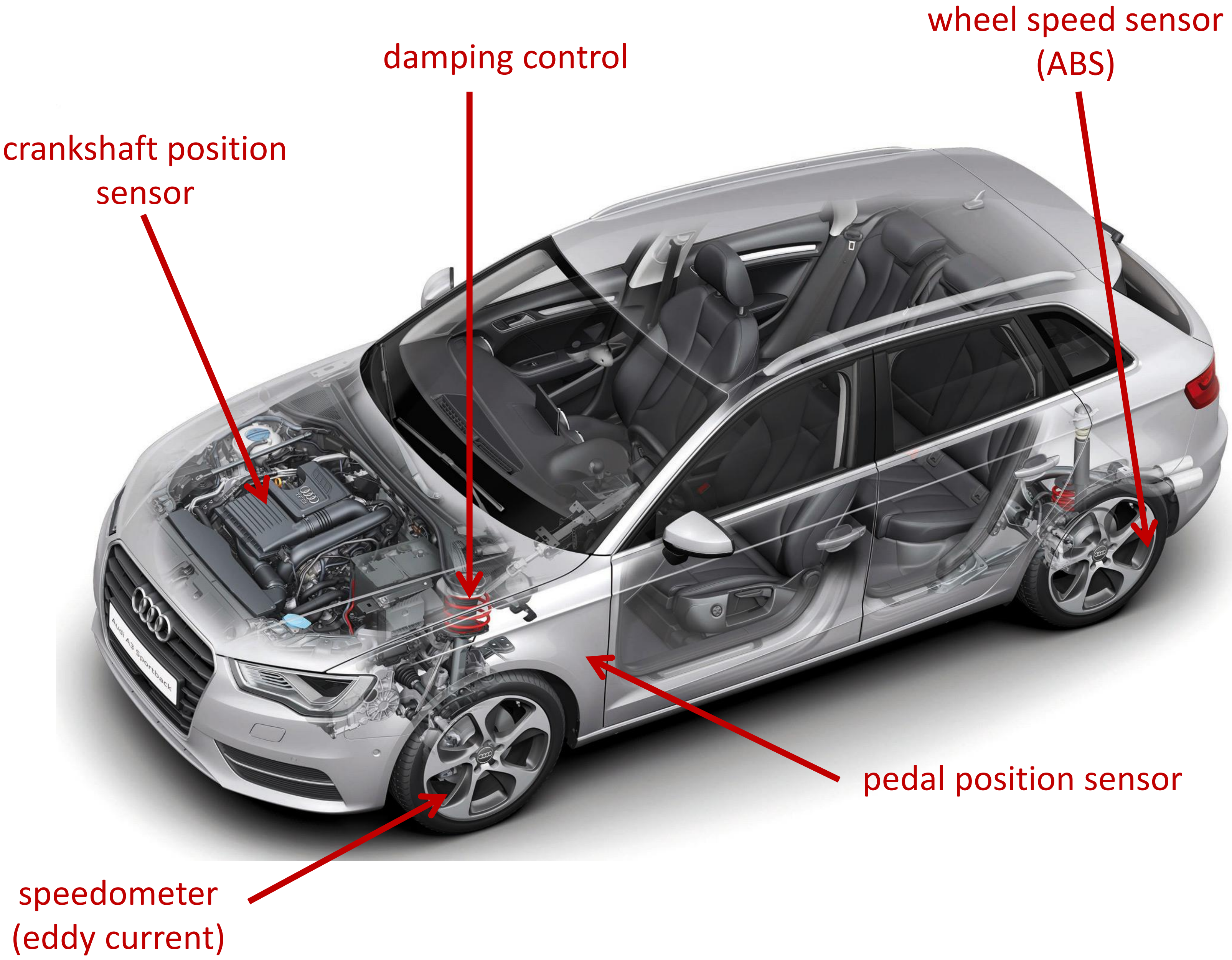
Sensing, Computing, Actuating

Sander Stuijk (s.stuijk@tue.nl)

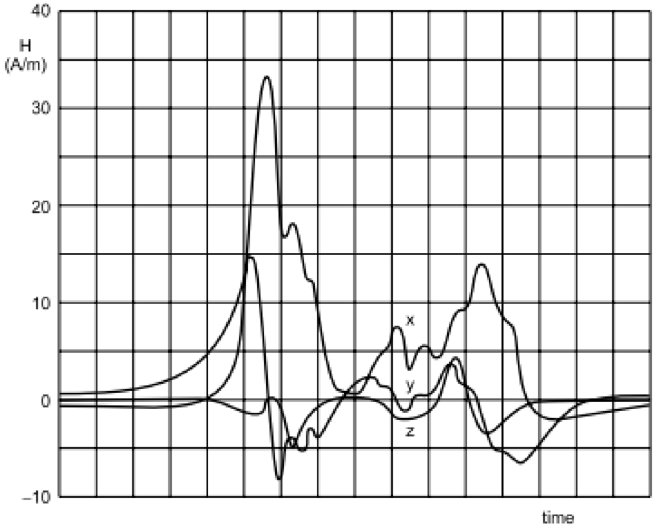
INDUCTIVE SENSORS

(Chapter 2.5, 2.6, 2.10, 5.4)

Inductive sensors



Inductive sensors



Sensor classification – type / quantity measured

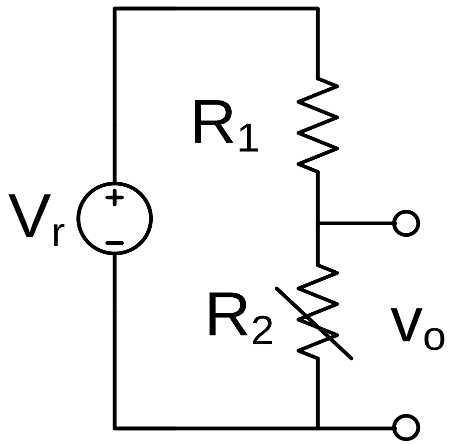
Sensor type		Quantity			
		Position, distance, displacement	**Flow rate / Point velocity**	**Force**	**Temperature**
	Resistive	Magnetoresistor	Thermistor	Strain gage	RTD
		Potentiometer			Thermistor
	Capacitive	Differential capacitor		Capacitive strain gage	Capacitor
	Inductive and electro-magnetic	Eddy currents	LVDT	Load cell + LVDT	LVDT
		Hall effect		Magnetostriction	
		LVDT			
		Magnetostriction			
	Self-generating		Thermal transport + thermocouple	Piezoelectric sensor	Pyroelectric sensor
					Thermocouple
	PN junction	Photoelectric sensor			Diode
					Bipolar transistor

- reactance variation sensors (capacitive and inductive sensors)
  - typically require no physical contact
  - exert minimal mechanical loading

Magnetic reluctance

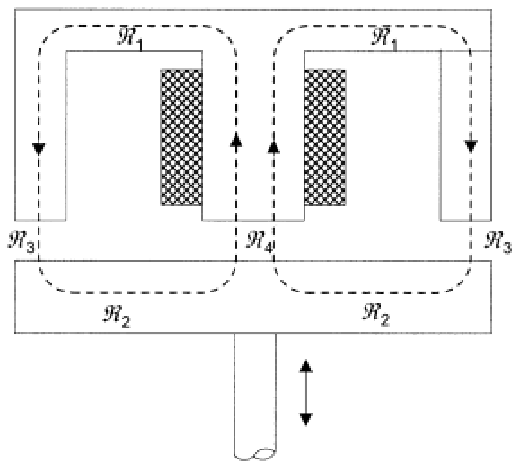
- electrical circuit may offer resistance to charge flow
  - resistor: R
  - resistor dissipates electrical energy
  - current follows path of least resistance
  - total resistance

$$R_{tot} = R_1 + R_2$$



- magnetic circuit may offer reluctance to magnetic flux
  - reluctance: $\Re$
  - reluctant circuit stores magnetic energy
  - magnetic flux follows path of least reluctance
  - total reluctance computed in similar way as resistance in electrical circuit

$$\Re_{tot} = \frac{\Re_1}{2} + \frac{\Re_2}{2} + \frac{\Re_3}{2} + \Re_4$$



Magnetic reluctance

- reluctance depends on physical properties of the device

$$\Re = \frac{1}{\mu\mu_0}\frac{l}{A}$$

  - l – length of the device
  - A – cross-sectional area
  - μ_0 – permeability of free space (4x10-7 H/m)
  - μ – relative permeability of the material
    - “soft” ferromagnetic material (typically 1000 to 10000)
    - permeability of air (approx. 1)

- options to vary reluctance
  - modify length l (variable gap sensor)
  - modify magnetic permeability μ (moving core sensor)
  - modify cross-sectional area A (not frequently used)

Magnetic reluctance

- reluctance depends on physical properties of the device

$$\Re = \frac{1}{\mu\mu_0}\frac{l}{A}$$

- sensor requires conversion of magnetic signal to electric signal

- Faraday's law relates magnetic reluctance to electric current

$$v = \frac{N^2}{\Re}\frac{di}{dt} = L\frac{di}{dt}$$

  - change in reluctance changes output voltage
  - self-inductance L and reluctance are related: $L = \frac{N^2}{\Re}$

- device can also be used as sensor without changing reluctance
  - changing magnetic field causes electrons to move
  - induces additional (eddy) current (eddy current sensor)

Variable gap sensor

- **what is the output voltage (in terms of x) of a sensor with N windings?**

$$\Re_{core} = \frac{l_{core}}{\mu_{core}\mu_0 A}, \quad \Re_{object} = \frac{l_{object}}{\mu_{object}\mu_0 A}, \quad \Re_{air} = \frac{x}{\mu_{air}\mu_0 A}$$

$$\Re_{total} = \Re_{core} + \Re_{object} + 2 \cdot \Re_{air}$$

$$\Rightarrow \Re_{total} = \frac{l_{core}}{\mu_{core}\mu_0 A} + \frac{l_{object}}{\mu_{object}\mu_0 A} + \frac{2x}{\mu_{air}\mu_0 A}$$

  - reluctance of core and object are constant

$$\Re_0 = \frac{l_{core}}{\mu_{core}\mu_0 A} + \frac{l_{object}}{\mu_{object}\mu_0 A}$$

  - reluctance of the circuit

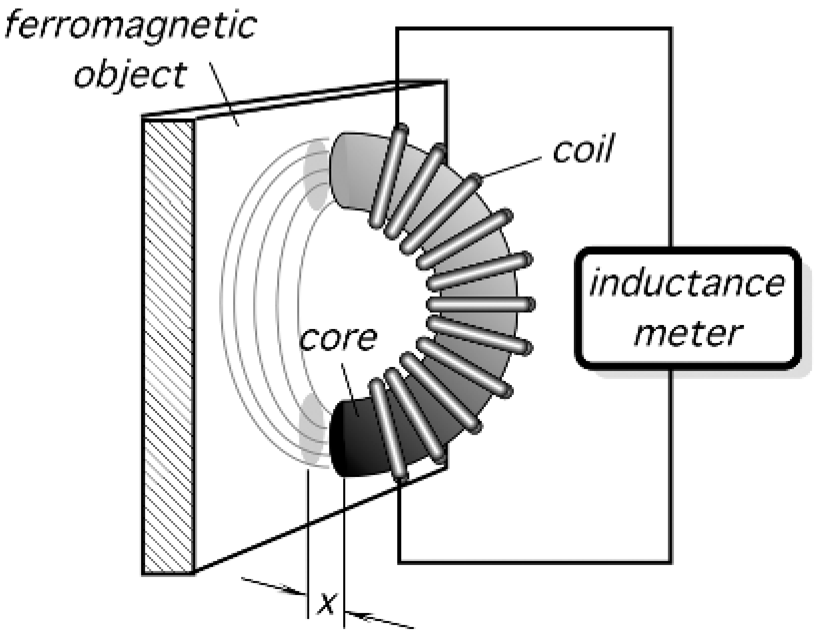
$$\Re_{total} = \Re_0 + \frac{2x}{\mu_{air}\mu_0 A} = \Re_0 + kx$$

  - self-inductance of the circuit

$$L = \frac{N^2}{\Re_{total}} = \frac{N^2}{\Re_0 + kx}$$

  - output voltage of the sensor

$$v = L\frac{di}{dt} = \frac{N^2}{\Re_0 + kx}\frac{di}{dt}$$



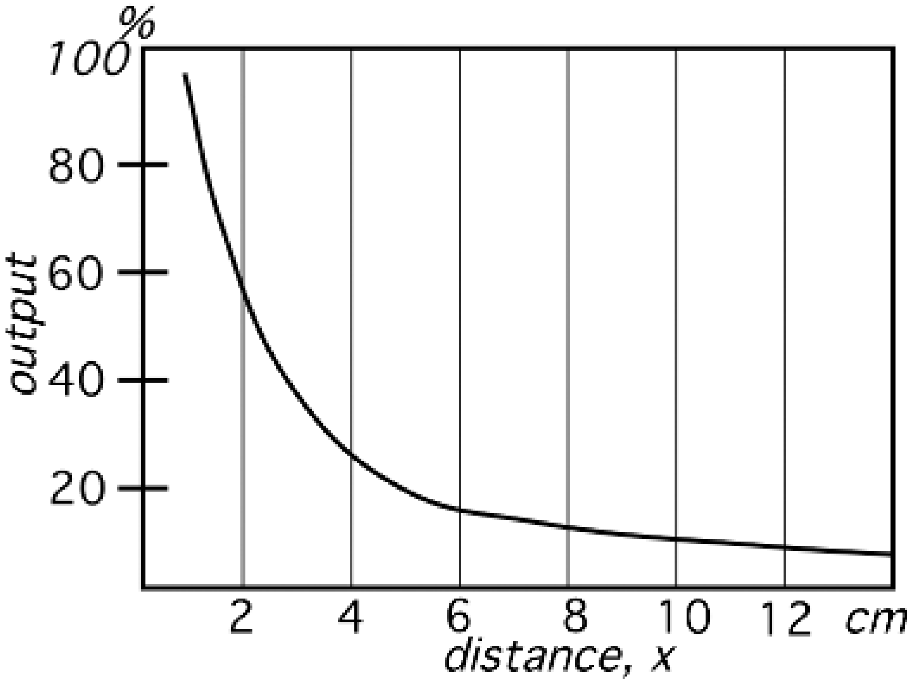
Variable gap sensor

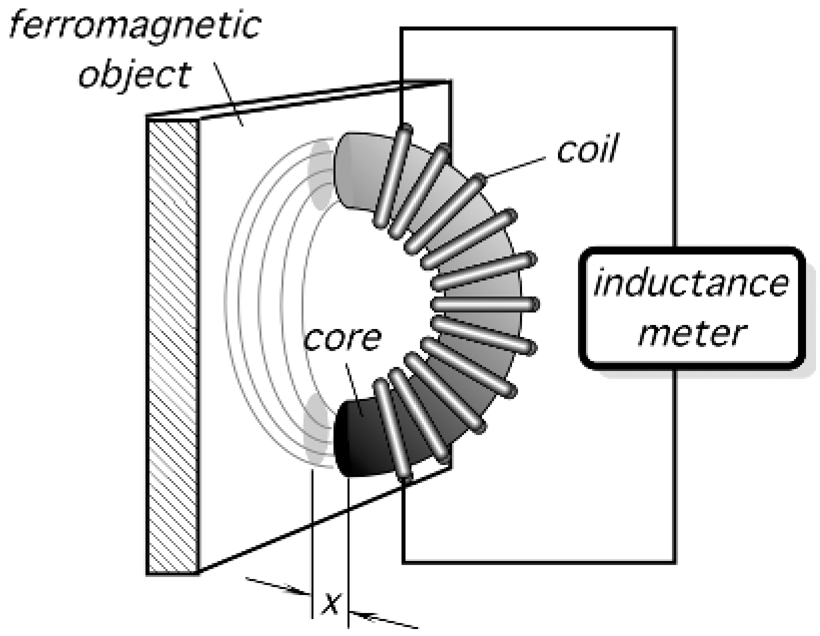
TU/e

- output voltage of the sensor

$$v = L\frac{di}{dt} = \frac{N^2}{\Re_0 + kx}\frac{di}{dt}$$

  - highly non-linear relation between output and displacement x
  - use of sensor limited to proximity sensor





Linear displacement transformer

- two coils in series, moving object
  - increases reluctance in one coil
  - decreases reluctance in other coil

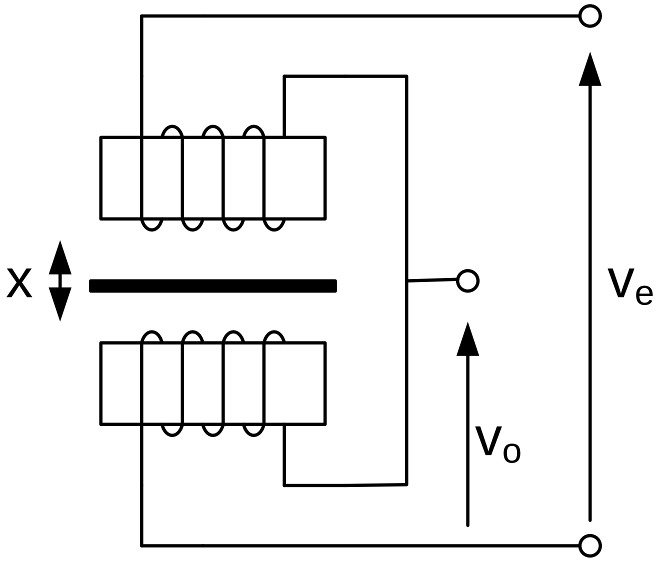
- circuit is differential voltage divider
- impedance of coil is equal to

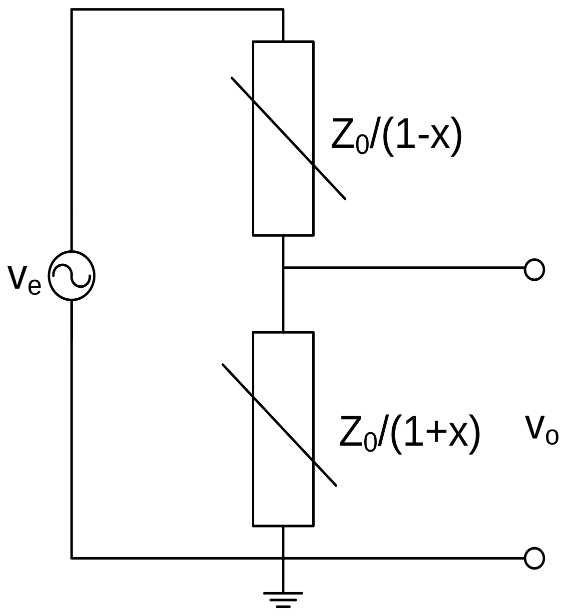
$$Z = j\omega L$$

$$\left.\begin{aligned} L &= \frac{N^2}{\Re} \\ \Re &= \frac{1}{\mu\mu_0}\frac{l}{A} \end{aligned}\right\} \Rightarrow Z = j\omega\frac{N^2}{\Re} = j\omega\frac{N^2\mu\mu_0 A}{l}$$

- changing l with a relative amount x

$$Z = j\omega\frac{N^2\mu\mu_0 A}{l(1+x)} = \frac{j\omega L_0}{(1+x)} = \frac{Z_0}{(1+x)}$$





Linear displacement transformer

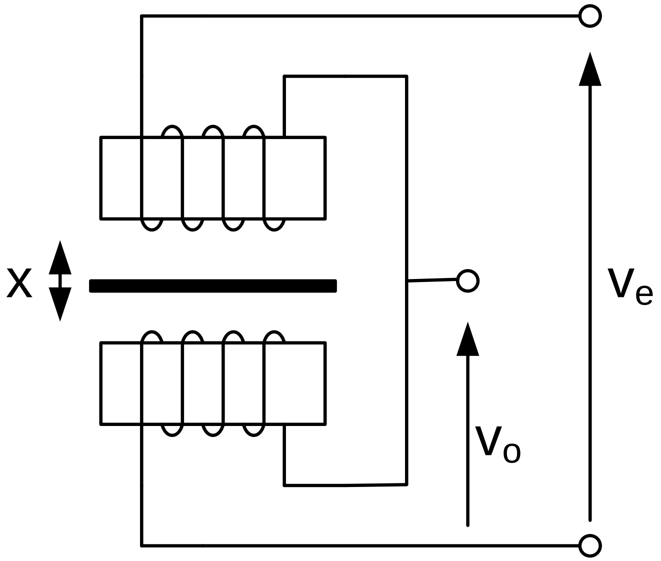
- two coils in series, moving object
  - increases reluctance in one coil
  - decreases reluctance in other coil

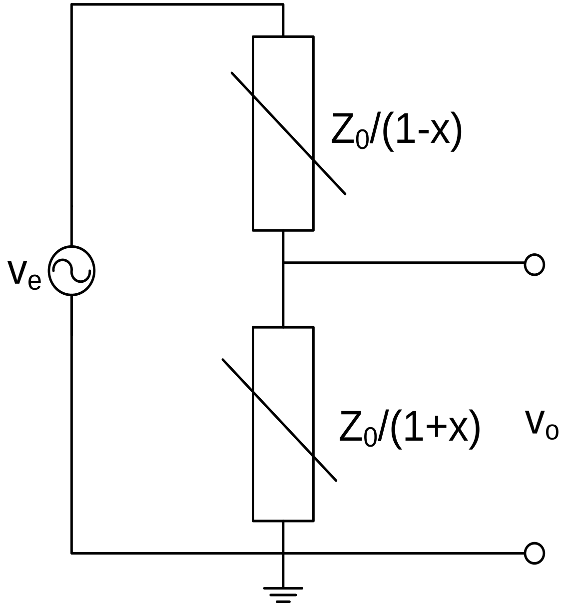
- circuit is differential voltage divider
- output of the voltage divider

$$v_o = \frac{Z_0/(1+x)}{Z_0/(1-x) + Z_0/(1+x)} v_e = \frac{1-x}{2} v_e$$

  - linear relation between output voltage and displacement
  - offset voltage present

- displacement (x) should be small
- sensor often not practical





Mutual inductance

- self-inductance
  - induced voltage due to change in own current

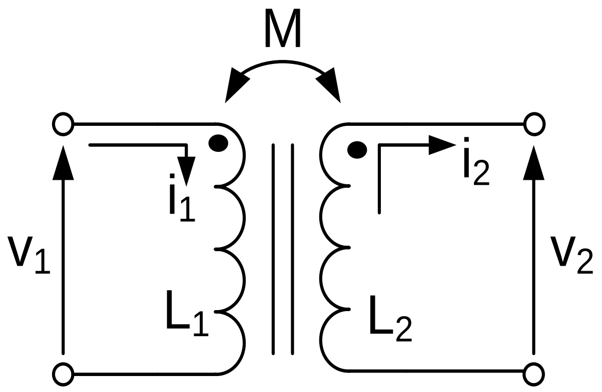
$$v = L\frac{di}{dt}$$

- mutual inductance
  - induced voltage due to change in current in neighboring circuit

$$v_2 = L_2\frac{di_2}{dt} - M\frac{di_1}{dt}$$

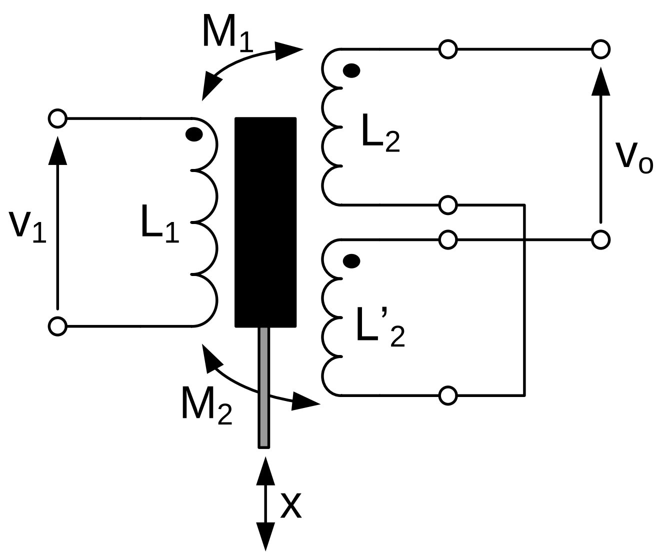
  - depends on reluctance of the space between the coils

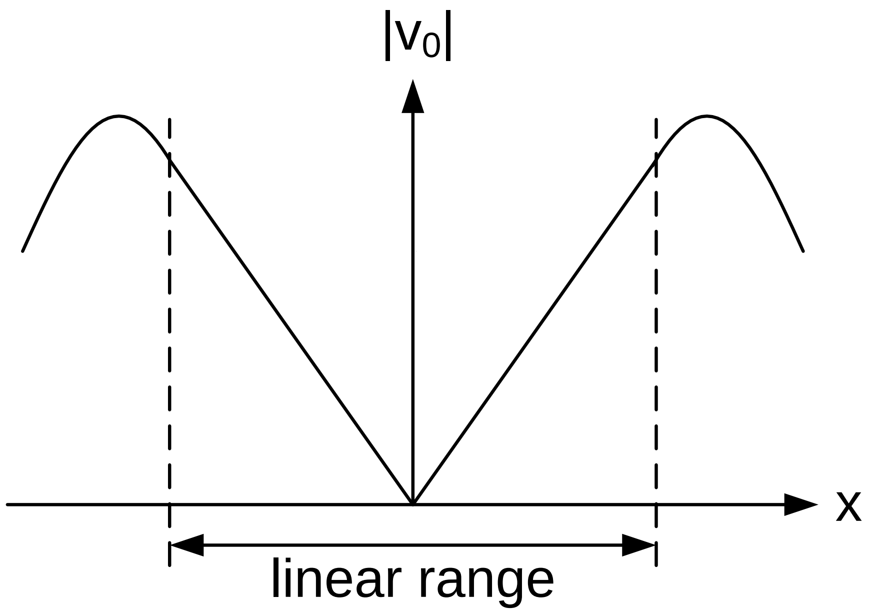
- changing reluctance between coils alters mutual inductance
  - device usable as sensor
  - two coil solution still not practical (large offset, small fluctuation)



Linear Variable Differential Transformer

- Linear Variable Differential Transformer (LVDT)
  - two secondary coils in series-opposition
  - linear relation between output voltage and core displacement
  - operation based on mutual inductance





Linear Variable Differential Transformer

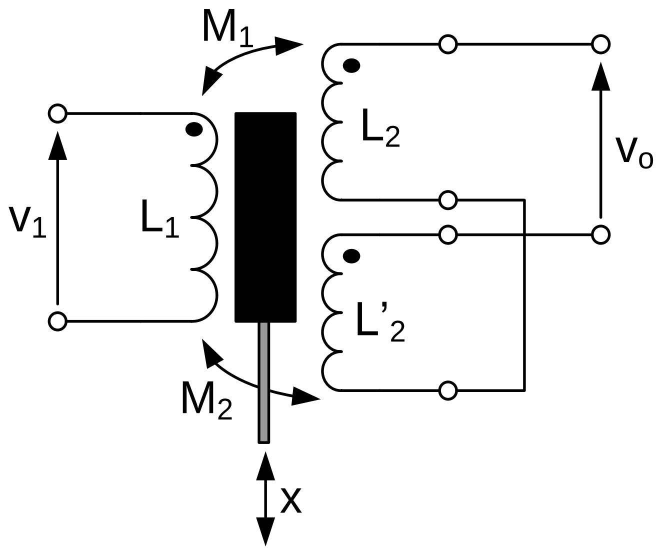
- assume sinusoidal excitation of primary circuit

  $$v_1(t) = V_1 \sin(\omega t)$$

- output voltage of secondary circuit

  $$v_o(t) = S_\omega \cdot x \cdot V_1 \sin(\omega t + \phi)$$

  - S_ω – sensitivity at frequency ω
  - x – displacement of the core from center
  - ϕ – phase shift (in voltage) from primary to secondary circuit

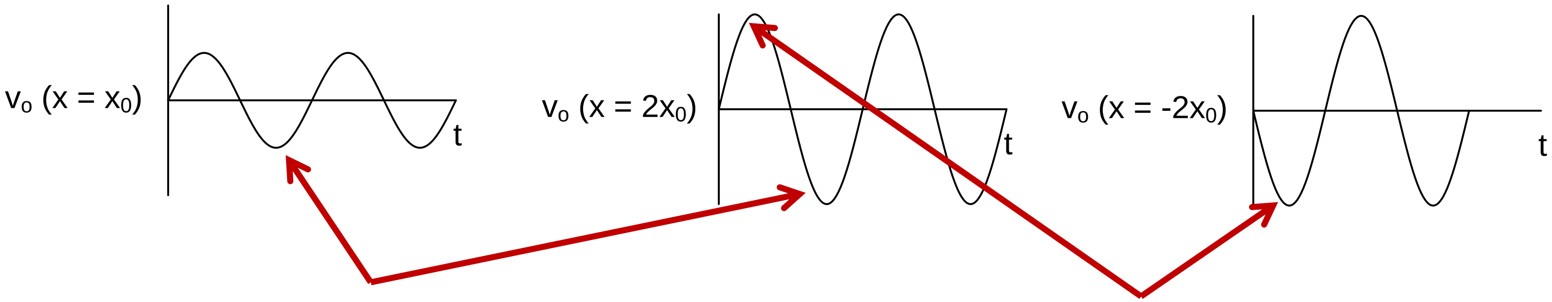


- S_ω and ϕ depend on
  - load R_L of measurement circuit
  - excitation frequency ω

- phase shift can be compensated

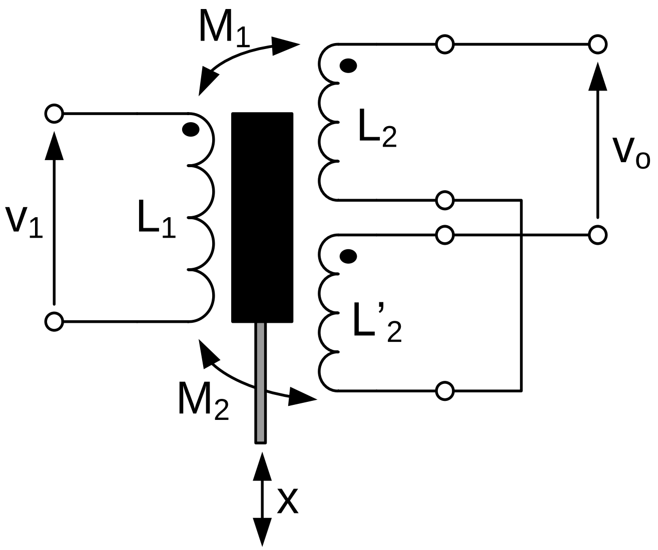
Signal conditioning for LVDT sensors

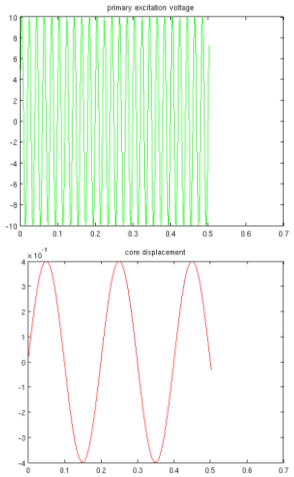
- output signal of LVDT is amplitude modulated ac signal

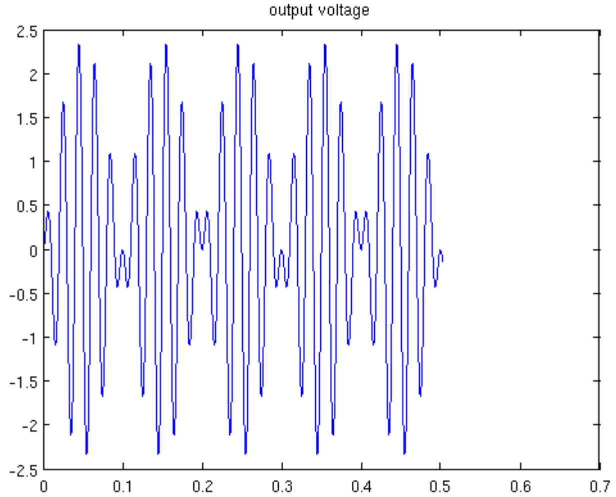


amplitude indicates magnitude of displacement

phase indicates direction of displacement







Linear Variable Differential Transformer

- output voltage (no load connected to secondary winding)
  - no current in secondary circuit ($I_2 = 0$)

$$V_1 = I_1(R_1 + j\omega L_1) \quad \Leftrightarrow I_1 = \frac{V_1}{j\omega L_1 + R_1}$$

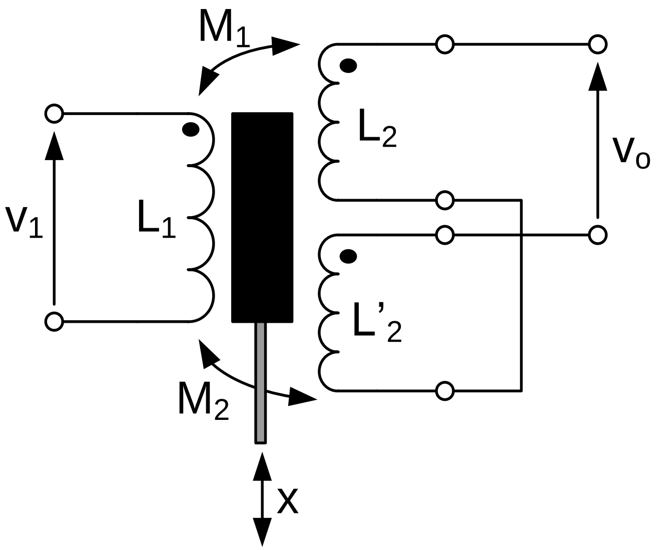
$$V_o = I_1(-j\omega M_1 + j\omega M_2) = j\omega(M_2 - M_1)I_1$$

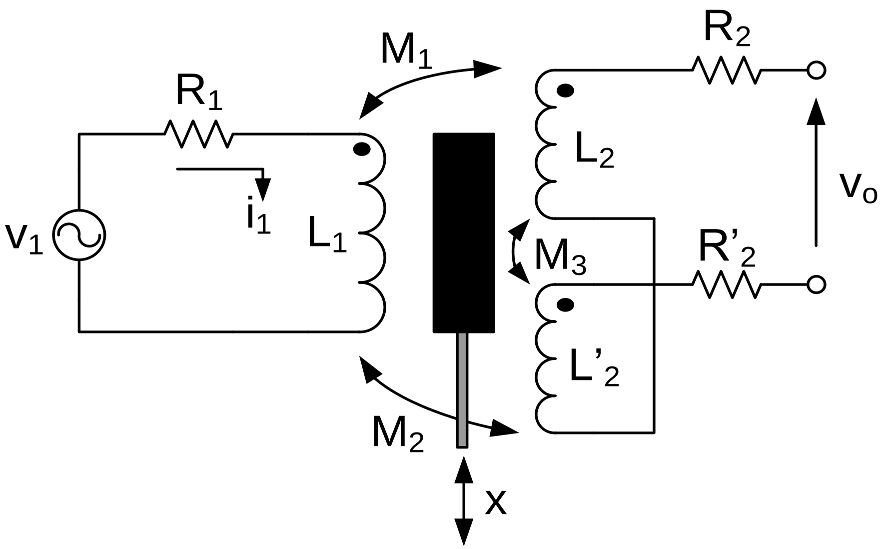
$$\Rightarrow V_o = \frac{j\omega(M_2 - M_1)V_1}{j\omega L_1 + R_1}$$

$$(M_2 - M_1) = k_x x$$

$$\Rightarrow V_o = j\omega k_x x I_1 = \frac{j\omega k_x x V_1}{j\omega L_1 + R_1}$$

  - primary current I_1 independent of core position
  - output voltage V_o proportional to core position





Linear Variable Differential Transformer

- output voltage (no load connected to secondary winding)

$$V_o = j\omega k_x x I_1 = \frac{j\omega k_x x V_1}{j\omega L_1 + R_1}$$

- sensitivity

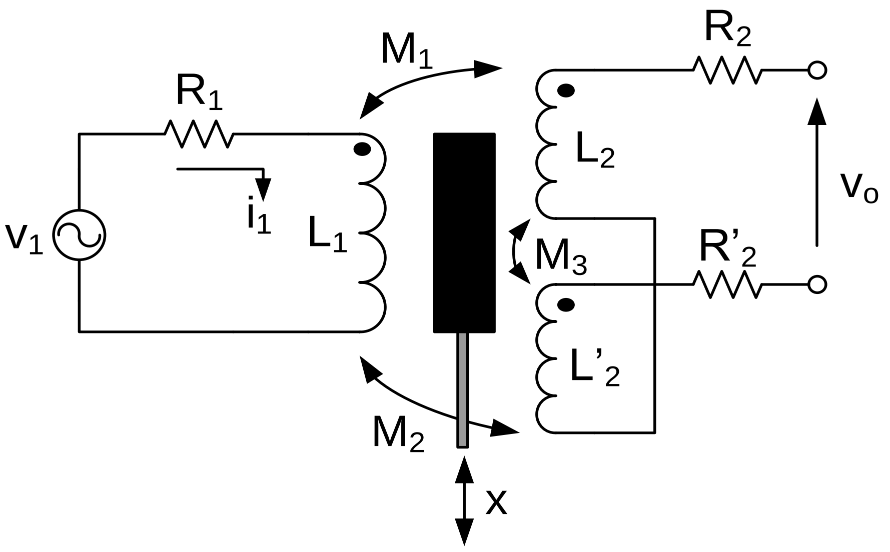
$$S = \frac{|V_o/V_1|}{x} = \left|\frac{j\omega k_x}{j\omega L_1 + R_1}\right| = \left|\frac{k_x}{L_1 + \frac{R_1}{j\omega}}\right| = \left|\frac{k_x}{L_1 - j\frac{R_1}{\omega}}\right| = \frac{k_x}{\sqrt{L_1^2 + \frac{R_1^2}{\omega^2}}}$$

  - sensitivity increases with increasing frequency

- phase shift
  - output voltage 90° out of phase
  - with primary current
  - phase shift between V_1 and V_0

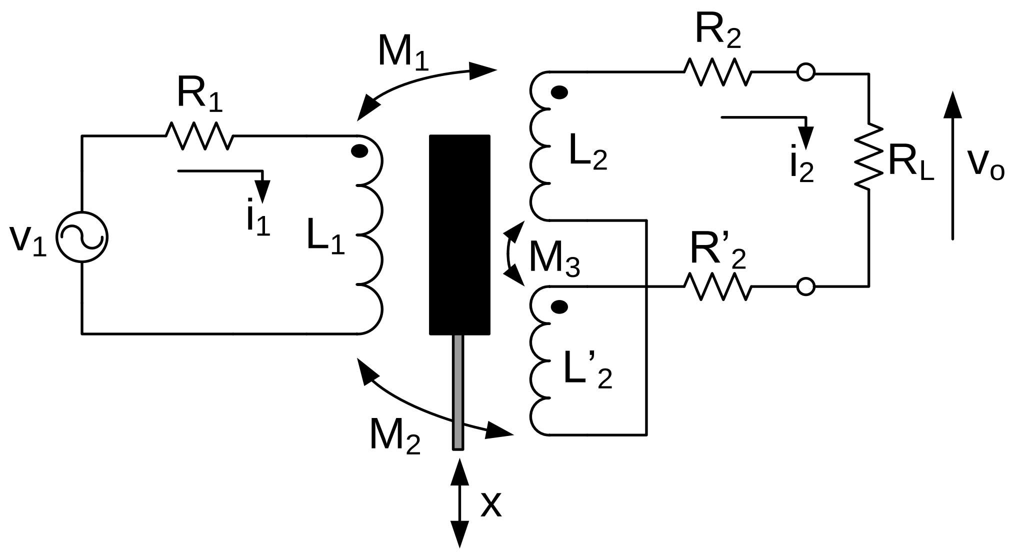
$$\phi = 90° - \arctan\frac{\omega L_1}{R_1}$$

  - consider phase shift when recovering position



Linear Variable Differential Transformer

- output voltage (load R_L connected to secondary winding)
  - output voltage
    $$V_o = I_2 R_L$$
  - current through secondary winding
    - define total resistance in secondary windings as
      $$R_{2c} = R_2 + R_2' + R_L$$
    - no voltage source in secondary winding, thus holds
      $$0 = I_1(-j\omega M_1 + j\omega M_2) + I_2(R_{2c} + j\omega L_2 + j\omega L_2' - j\omega 2M_3)$$
      $$\Leftrightarrow I_2 = \frac{-j\omega(M_2 - M_1)}{(R_{2c} + j\omega L_2 + j\omega L_2' - j\omega 2M_3)} I_1$$
  - I_2 depends on I_1
    $$\Leftrightarrow I_1 = \frac{(R_{2c} + j\omega L_2 + j\omega L_2' - j\omega 2M_3)}{-j\omega(M_2 - M_1)} I_2$$



Linear Variable Differential Transformer

- output voltage (load R_L connected to secondary winding)
  - output voltage

    $$V_o = I_2 R_L$$

  - I_2 depends on I_1

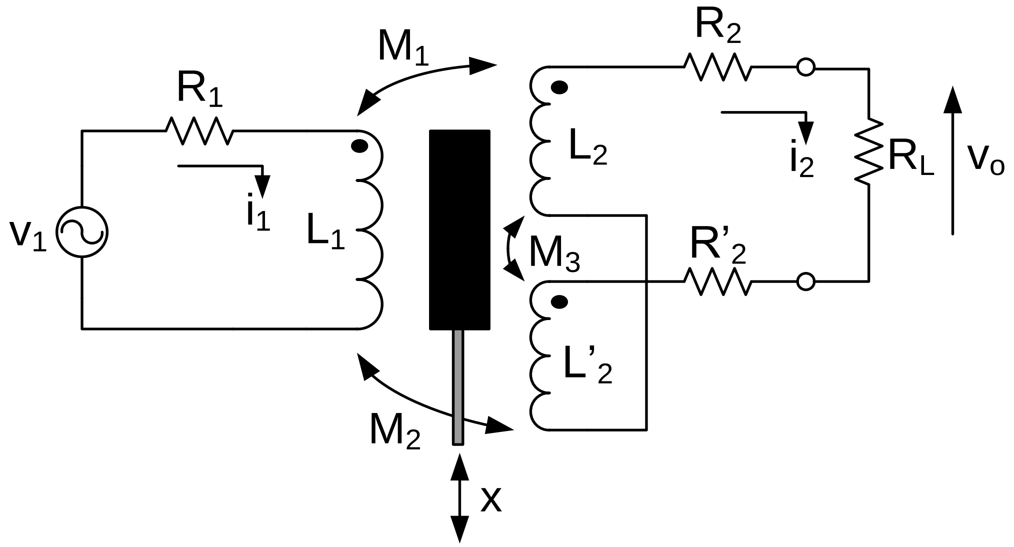
    $$I_1 = \frac{(R_{2c} + j\omega L_2 + j\omega L_2' - j\omega 2M_3)}{-j\omega(M_2 - M_1)} I_2$$

  - current (I_1) through primary winding

    $$V_1 = I_1(R_1 + j\omega L_1) + I_2(-j\omega M_1 + j\omega M_2)$$

    $$\Leftrightarrow I_1 = \frac{V_1 - j\omega(M_2 - M_1)I_2}{(R_1 + j\omega L_1)}$$

  - I_1 equal in both
  - expressions, hence …



Linear Variable Differential Transformer

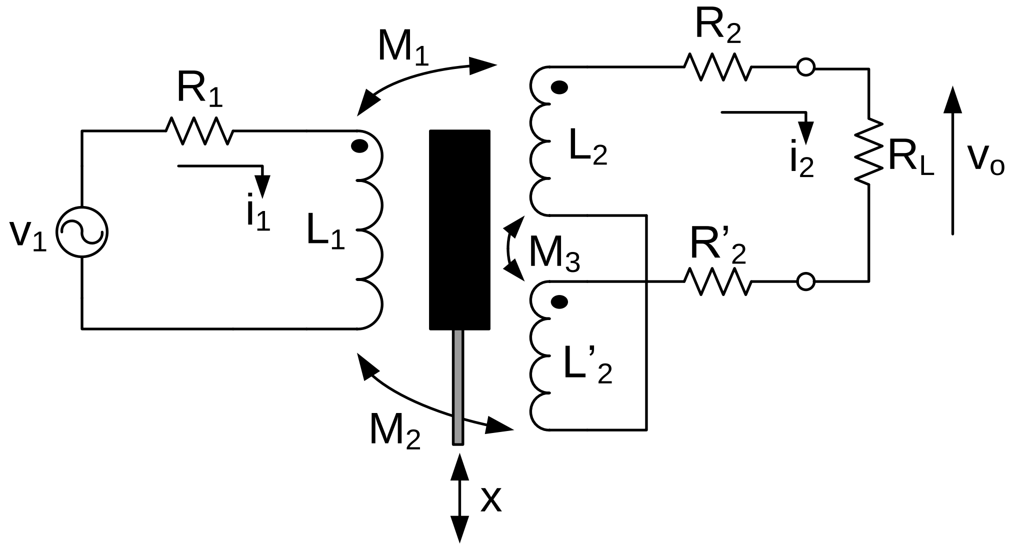
- output voltage (load R_L connected to secondary winding)
  - output voltage

    $$V_o = I_2 R_L$$

  - I_1 equal in both expressions, hence …

$$\frac{V_1 - j\omega(M_2 - M_1)I_2}{(R_1 + j\omega L_1)} = \frac{(R_{2c} + j\omega L_2 + j\omega L_2' - j\omega 2M_3)I_2}{-j\omega(M_2 - M_1)}$$

$$\Rightarrow I_2 = \frac{j\omega(M_2 - M_1)V_1}{(j\omega)^2\big((M_2 - M_1)^2 - L_1(L_2 + L_2' - 2M_3)\big) - j\omega\big(R_{2c}L_1 + R_1(L_2 + L_2' - 2M_3)\big) - R_1 R_{2c}}$$



Linear Variable Differential Transformer

- output voltage (load R_L connected to secondary winding)

$$V_o = \frac{j\omega(M_2 - M_1)R_L V_1}{(j\omega)^2\left((M_2 - M_1)^2 - L_1(L_2 + L_2' - 2M_3)\right) - j\omega\left(R_{2c}L_1 + R_1(L_2 + L_2' - 2M_3)\right) - R_1 R_{2c}}$$

  - difference in mutual inductance related to core position

$$(M_2 - M_1) = k_x x$$

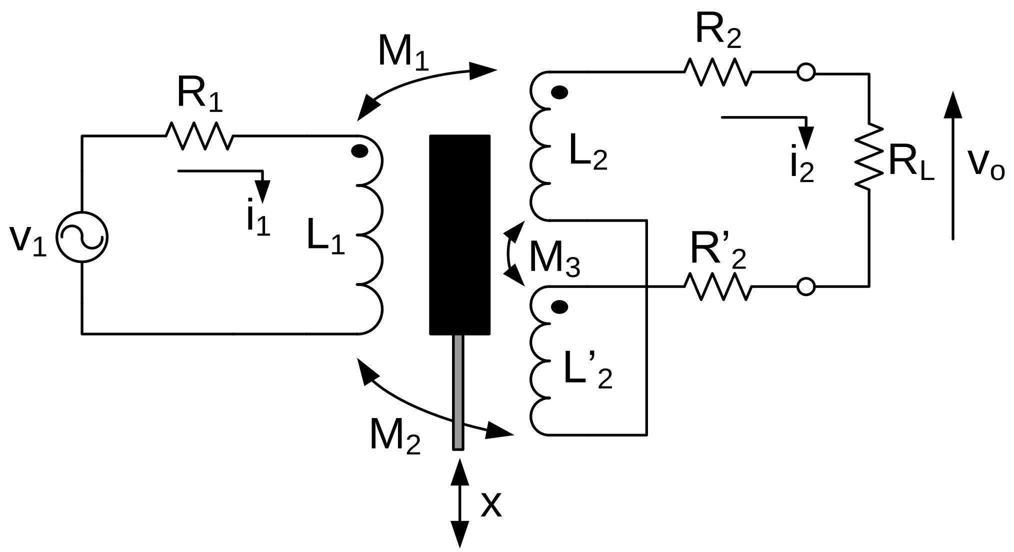
  - output voltage in terms of core position

$$V_o = \frac{j\omega k_x x R_L V_1}{(j\omega)^2\left((k_x x)^2 - L_1(L_2 + L_2' - 2M_3)\right) - j\omega\left(R_{2c}L_1 + R_1(L_2 + L_2' - 2M_3)\right) - R_1 R_{2c}}$$

  - $V_o = 0$ when $M_1 = M_2$ (core in center)
  - it holds that

$$L_1(L_2 + L_2' - 2M_3) >> (k_x x)^2$$

  - V_o changes linearly on both sides of center



Linear Variable Differential Transformer

- output voltage (load R_L connected to secondary winding)

$$V_o = \frac{j\omega k_x x R_L V_1}{(j\omega)^2\big((k_x x)^2 - L_1(L_2 + L_2' - 2M_3)\big) - j\omega\big(R_{2c}L_1 + R_1(L_2 + L_2' - 2M_3)\big) - R_1 R_{2c}}$$

  - it holds that

$$L_1(L_2 + L_2' - 2M_3) >> (k_x x)^2$$

$$L_2 + L_2' - 2M_3 \approx 2L_2$$

  - hence

$$V_o = \frac{j\omega k_x x V_1 R_L}{-(j\omega)^2 2L_1L_2 - j\omega(R_{2c}L_1 + 2R_1L_2) - R_1R_{2c}}$$

  - sensitivity

$$S = \frac{|V_o/V_1|}{x} = \frac{\omega k_x R_L}{\sqrt{(R_1R_{2c} - \omega^2 2L_1L_2)^2 + \omega^2(R_{2c}L_1 + 2R_1L_2)^2}}$$

    - sensitivity increases with R_L (up-to some frequency)
    - sensitivity depends on excitation frequency of primary circuit

Linear Variable Differential Transformer

- output voltage (load R_L connected to secondary winding)

$$V_o = \frac{j\omega k_x x V_1 R_L}{-(j\omega)^2 2L_1L_2 - j\omega(R_{2c}L_1 + 2R_1L_2) - R_1R_{2c}}$$

  - (in general) phase difference between V_1 and V_0
  - no phase shift when

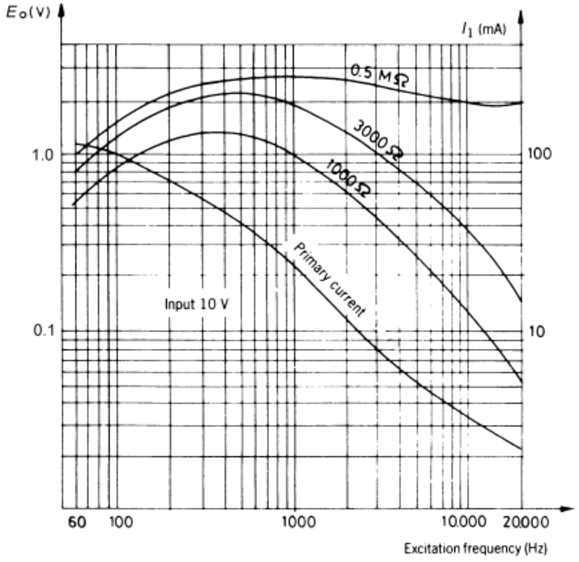
$$(j\omega)^2 2L_1L_2 + R_1R_{2c} = 0$$

$$\Leftrightarrow (j2\pi f_n)^2 2L_1L_2 + R_1R_{2c} = 0 \Leftrightarrow -(2\pi f_n)^2 2L_1L_2 + R_1R_{2c} = 0 \Rightarrow f_n = \frac{1}{2\pi}\sqrt{\frac{R_1R_{2c}}{2L_1L_2}}$$

- sensitivity at frequency f_n

$$S = \left.\frac{|V_o/V_1|}{x}\right|_{\omega=2\pi f_n} = \frac{k_x R_L}{\sqrt{(R_{2c}L_1 + 2R_1L_2)^2}} = \frac{k_x R_L}{R_{2c}L_1 + 2R_1L_2}$$

  - sensitivity increases with R_L
  - any other frequency has lower sensitivity for same R_L



Linear Variable Differential Transformer

example – sensitivity of an LVDT

- primary winding has a dc resistance of 67 Ω
- two series-opposition windings have a total dc resistance of 2800 Ω
- at 1kHz
  - primary winding has impedance of 290 Ω
  - secondary windings have impedance of 4800 Ω
  - sensitivity (normalized to excitation voltage) is 270 (μV/V)/μm
- **calculate the following**
  - inductance of primary and secondary winding
  - excitation frequency that yields zero phase shift between primary and secondary voltage with load resistance of 10 kΩ
  - normalized sensitivity when the LVDT is excited at 60 Hz and the load resistance is equal to 500 kΩ
  - normalized sensitivity when the LVDT is excited at 60 Hz and the load resistance is equal to 10 kΩ

Linear Variable Differential Transformer

example – sensitivity of an LVDT

calculate the inductance of primary and secondary winding

- primary winding

$$|Z_1| = \sqrt{R_1^2 + (2\pi f L_1)^2}$$

$$L_1 = \frac{\sqrt{|Z_1|^2 - R_1^2}}{2\pi f} = \frac{\sqrt{(290\Omega)^2 - (67\Omega)^2}}{2\pi(1000Hz)} = 45mH$$

- secondary winding

$$L_2 = \frac{1}{2}\frac{\sqrt{|Z_2|^2 - (R_2 + R_2')^2}}{2\pi f} = \frac{1}{2}\frac{\sqrt{(4800\Omega)^2 - (2800\Omega)^2}}{2\pi(1000Hz)} = 310mH$$

calculate the excitation frequency that yields zero phase shift with 10 kΩ load resistance

$$f_n = \frac{1}{2\pi}\sqrt{\frac{R_1 R_{2c}}{2 L_1 L_2}} = \frac{1}{2\pi}\sqrt{\frac{(67\Omega)(2800\Omega + 10k\Omega)}{2(45mH)(310mH)}} = 882Hz$$

Linear Variable Differential Transformer

example – sensitivity of an LVDT

calculate the normalized sensitivity when the LVDT is excited at 60 Hz and the load resistance is equal to 500 kΩ

- secondary circuit assumed open when load resistance is 500kΩ
- use $j\omega = j2\pi f, \quad |j\omega a + b| = \sqrt{\omega^2 a^2 + b^2}$
- sensitivity is equal to

$$S = \frac{|V_o/V_1|}{x} = \frac{2\pi f k_x}{\sqrt{R_1^2 + (2\pi f L_1)^2}}$$

- at 1kHz, S = 270μV/V/μm

$$k_x = \frac{270 \cdot 10^{-6}}{10^{-6} m} \frac{\sqrt{R_1^2 + (2\pi f L_1)^2}}{2\pi f} = \frac{270}{1m} \frac{\sqrt{(67\Omega)^2 + (2\pi \times 1kHz \times 45mH)^2}}{2\pi \times 1000 rad} = 12.5(\Omega/m)/(rad/s)$$

- sensitivity at 60Hz is thus equal to

$$S = \frac{(2\pi \times 60 rad) \times (12.5\Omega \cdot s/m \times rad)}{\sqrt{(67)^2 + (2\pi \times 60 rad \times 45mH)^2}} = 68.2(\mu V/V)/\mu m$$

Linear Variable Differential Transformer

example – sensitivity of an LVDT

calculate the normalized sensitivity when the LVDT is excited at 60 Hz when the load resistance is equal to 10 kΩ

- secondary is loaded (not open) when load resistance is 10 kΩ
- we computed earlier

$$S = \frac{|V_o/V_1|}{x} = \frac{\omega k_x R_L}{\sqrt{(R_1 R_{2c} - \omega^2 2 L_1 L_2)^2 + \omega^2 (R_{2c} L_1 + 2 R_1 L_2)^2}}$$

$$= \frac{(12.5\Omega \cdot s/m \times rad)(2\pi \times 60 rad)(10k\Omega)}{\sqrt{\big((67\Omega)(12800\Omega) - (2\pi \times 60Hz)^2 2(45mH)(310mH)\big)^2 + (2\pi \times 60Hz)^2 \big((12800\Omega)(45mH) + 2(67\Omega)(310mH)\big)^2}}$$

$$= 53.3(\mu V/V)/\mu m$$

Linear Variable Differential Transformer

- assume sinusoidal excitation of primary circuit

  $$v_1(t) = V_1 \sin(\omega t)$$

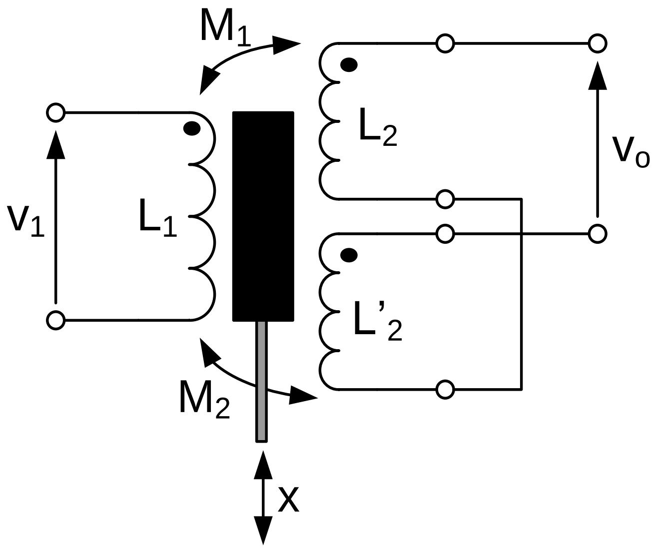
- output voltage of secondary circuit

  $$v_o(t) = S_\omega \cdot x \cdot V_1 \sin(\omega t + \phi)$$

  - S_ω – sensitivity at frequency ω
  - x – displacement of the core from center
  - ϕ – phase shift (in voltage) from primary to secondary circuit

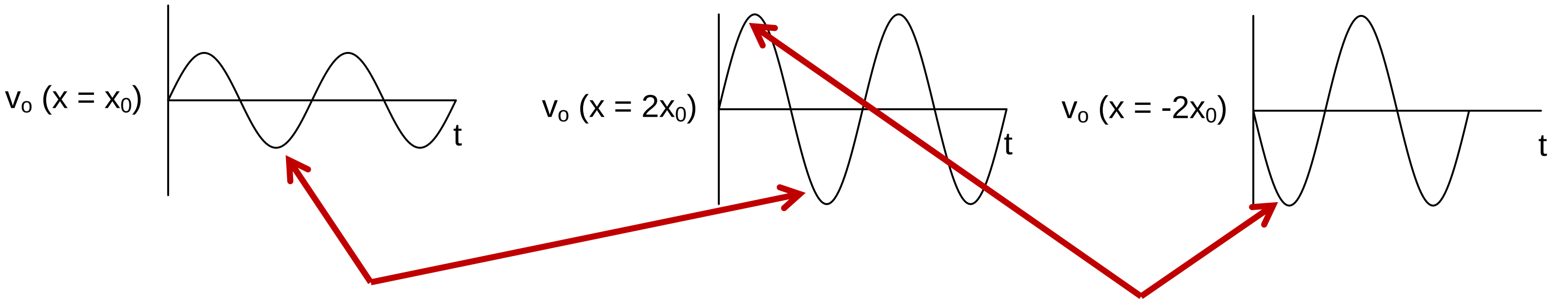
- S_ω and ϕ depend on
  - load R_L of measurement circuit
  - excitation frequency ω

- phase shift can be compensated



Signal conditioning for LVDT sensors

- output signal of LVDT is amplitude modulated ac signal



amplitude indicates magnitude of displacement

phase indicates direction of displacement