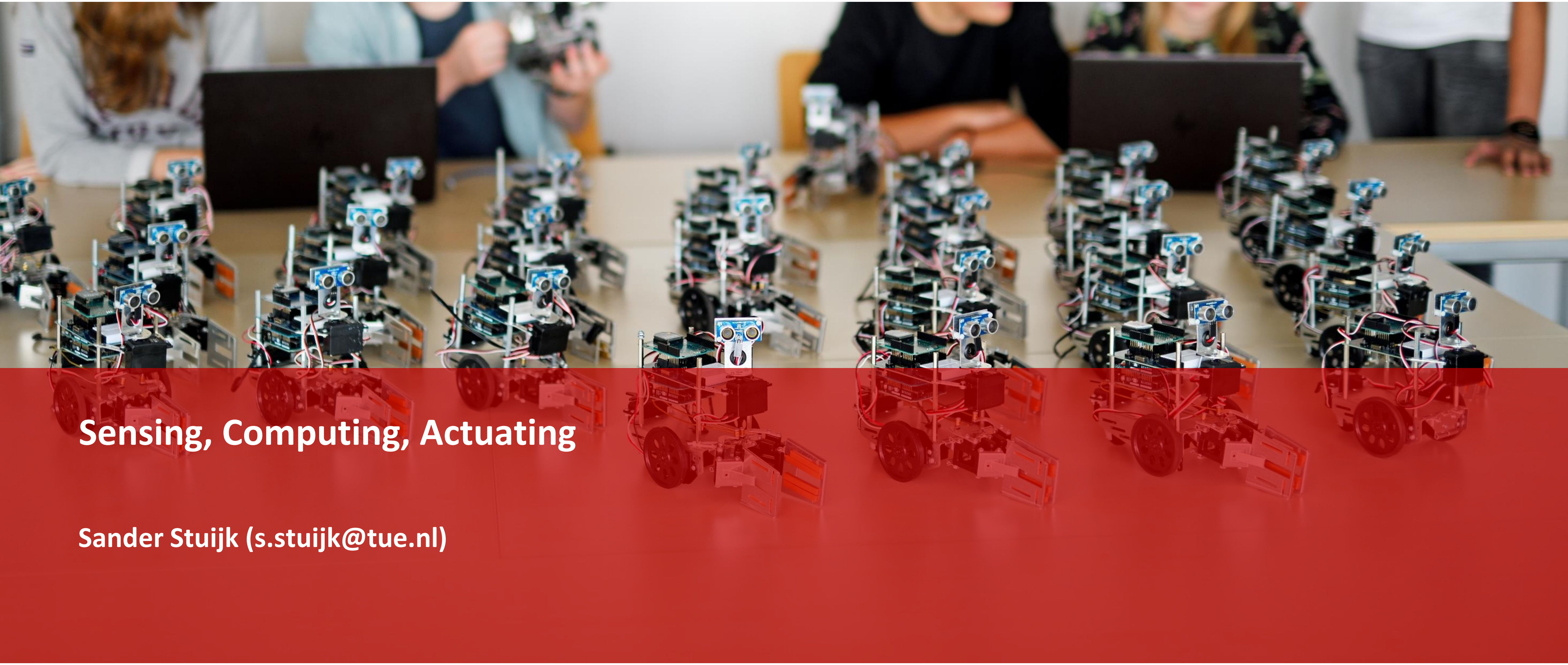


Sensing, Computing, Actuating

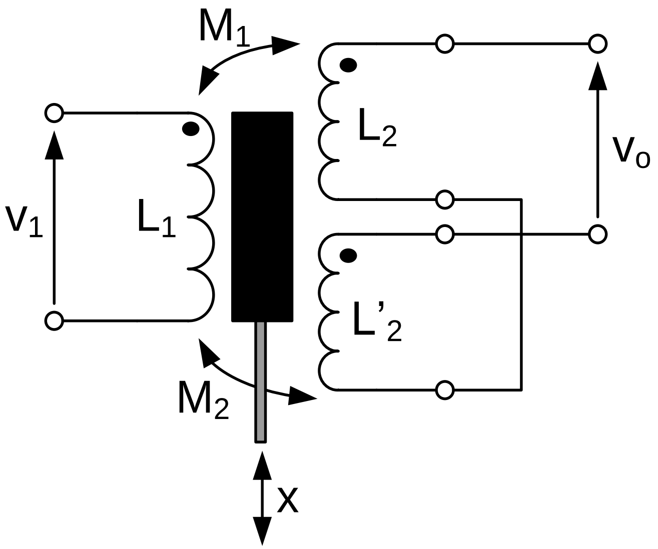
Sander Stuijk (s.stuijk@tue.nl)

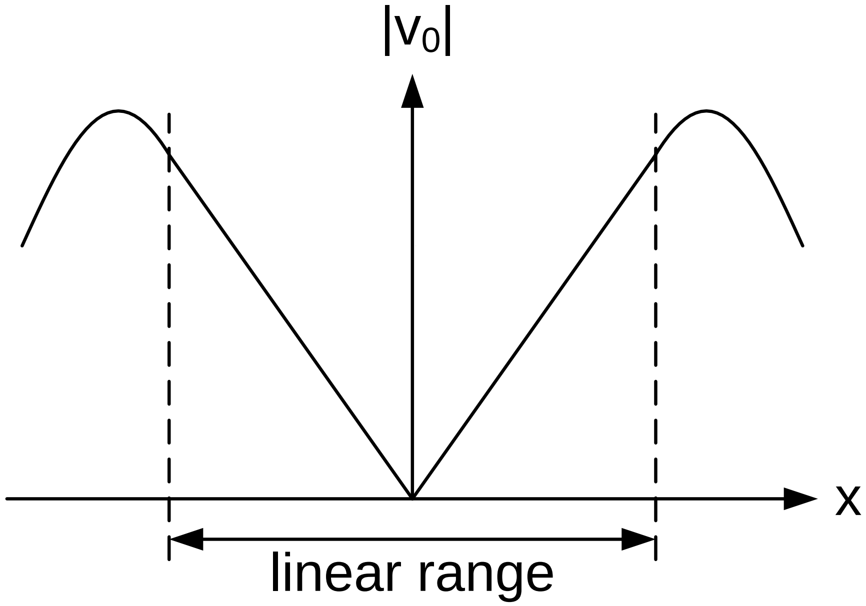
INDUCTIVE SENSORS / DEMODULATION

(Chapter 2.5, 2.6, 2.10, 5.4)

Linear Variable Differential Transformer

- Linear Variable Differential Transformer (LVDT)
  - two secondary coils in series-opposition
  - linear relation between output voltage and core displacement
  - operation based on mutual inductance





Linear Variable Differential Transformer

- assume sinusoidal excitation of primary circuit

  $$v_1(t) = V_1 \sin(\omega t)$$

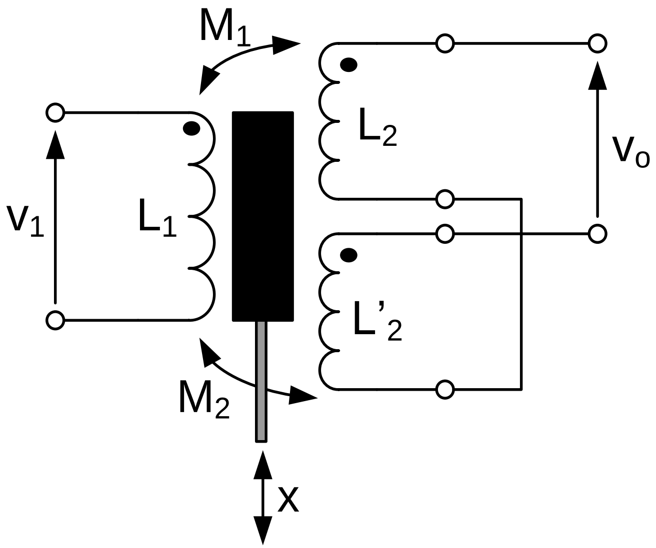
- output voltage of secondary circuit

  $$v_o(t) = S_\omega \cdot x \cdot V_1 \sin(\omega t + \phi)$$

  - S_ω – sensitivity at frequency ω
  - x – displacement of the core from center
  - ϕ – phase shift (in voltage) from primary to secondary circuit

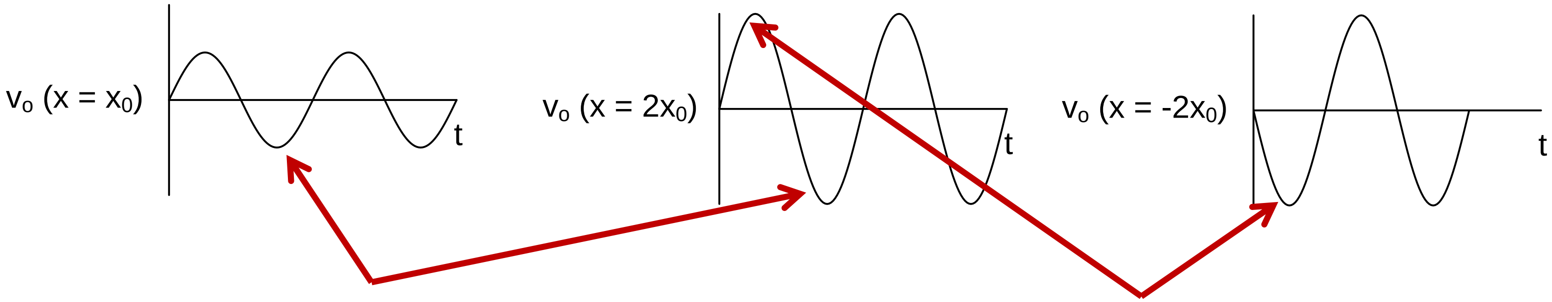
- S_ω and ϕ depend on
  - load R_L of measurement circuit
  - excitation frequency ω

- phase shift can be compensated



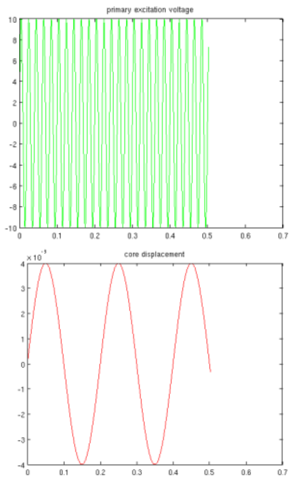
Signal conditioning for LVDT sensors

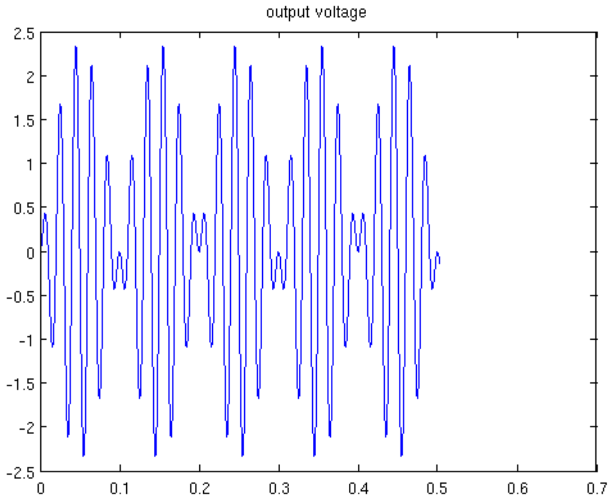
- output signal of LVDT is amplitude modulated ac signal

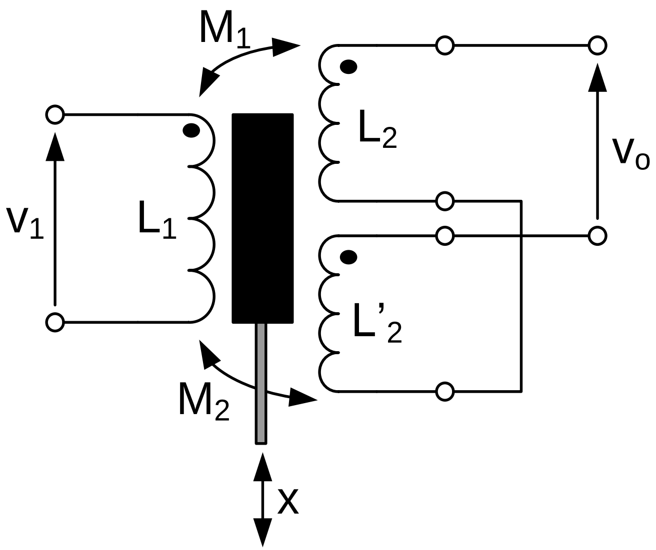


amplitude indicates magnitude of displacement

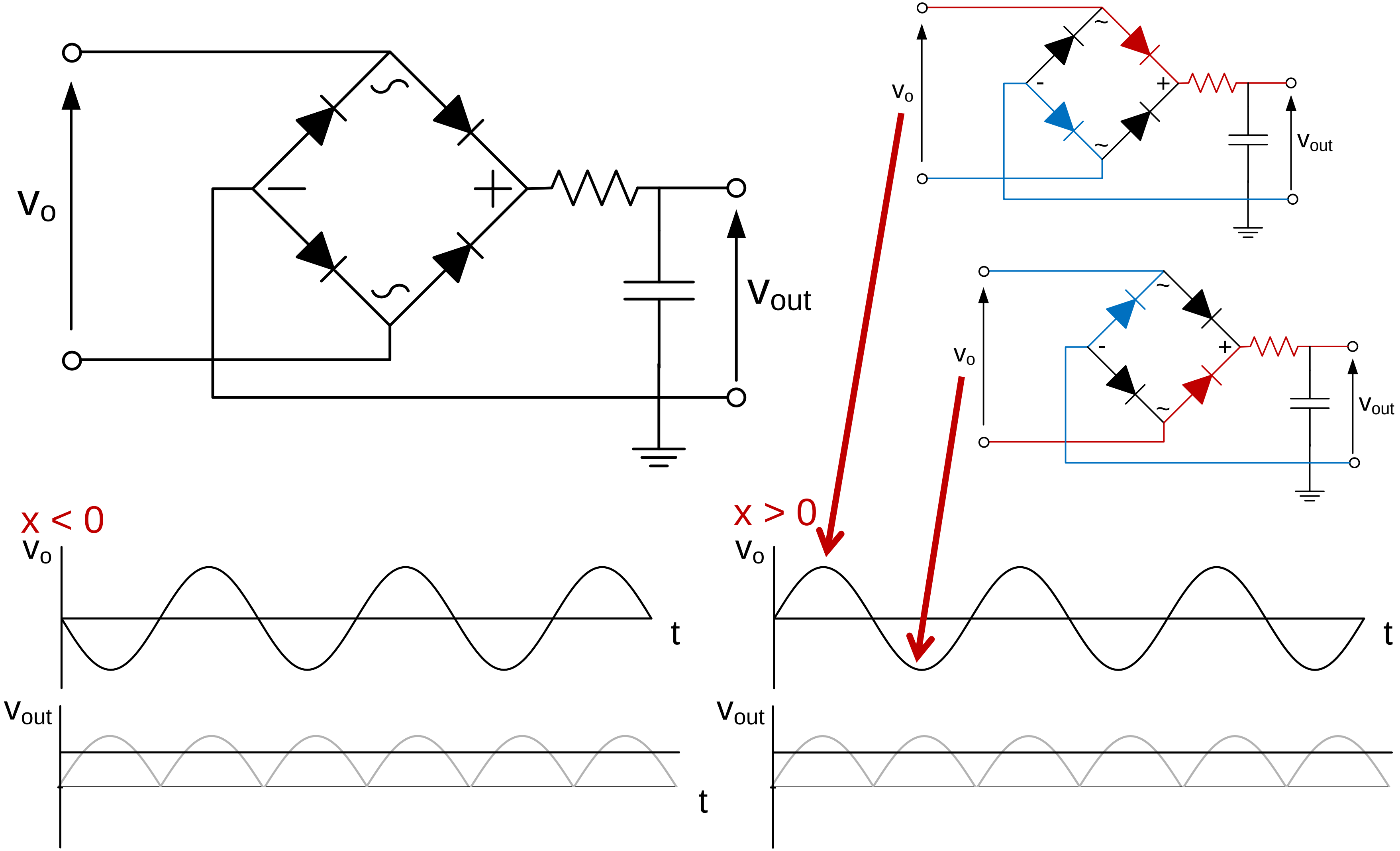
phase indicates direction of displacement







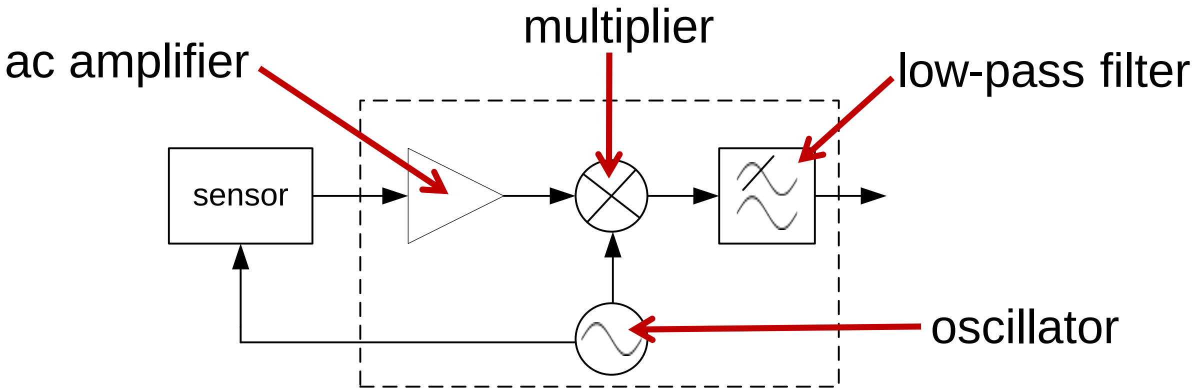
Bridge rectifier with low-pass filtering

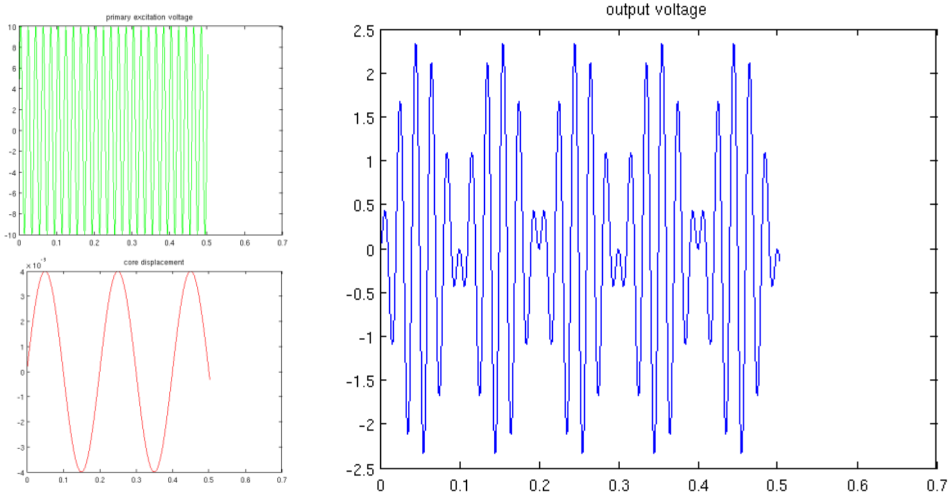


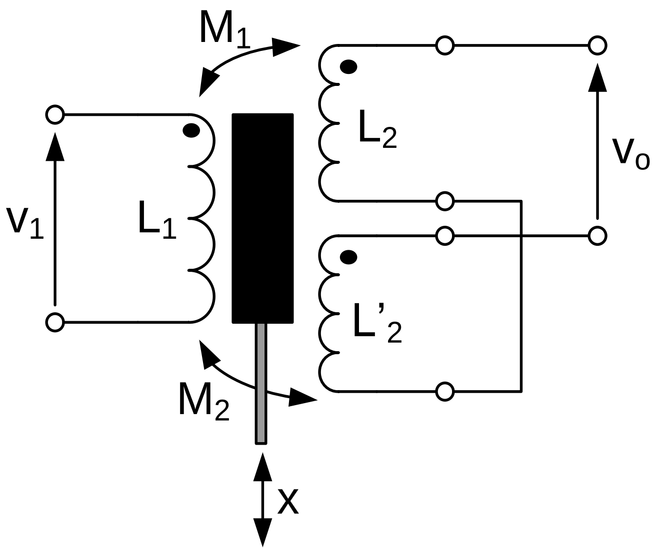
- amplitude of x recovered
- sign of x not recovered

Signal conditioning for LVDT sensors

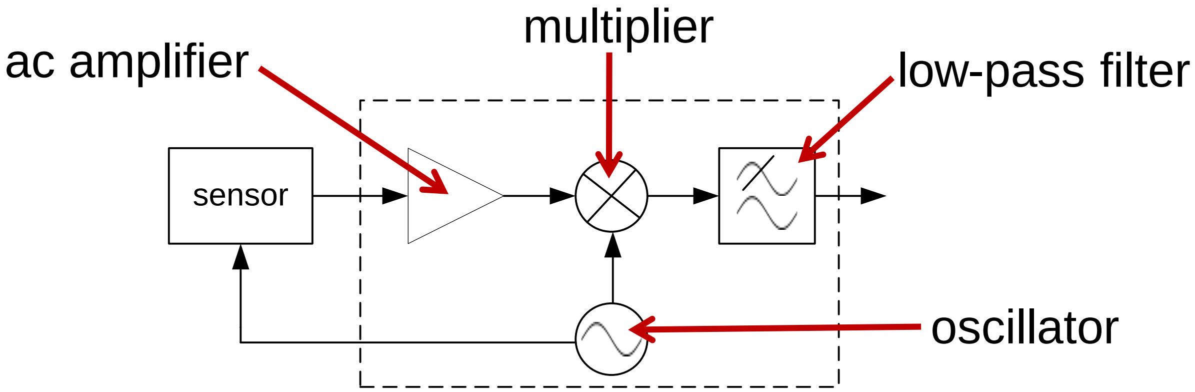
- output signal of LVDT is amplitude modulated ac signal
- carrier amplifier and coherent detector







Phase-sensitive (synchronous) demodulation



- excitation voltage of the sensor

$$v_e(t) = V_e \cos(2\pi \cdot f_e \cdot t)$$

- output of the sensor (assume no phase shift input/output voltage)

$$v_o(t) = S_\omega \cdot x(t) \cdot v_e(t)$$

- assume measured object is moving

$$x(t) = X \cos(2\pi \cdot f_x \cdot t + \phi_x)$$

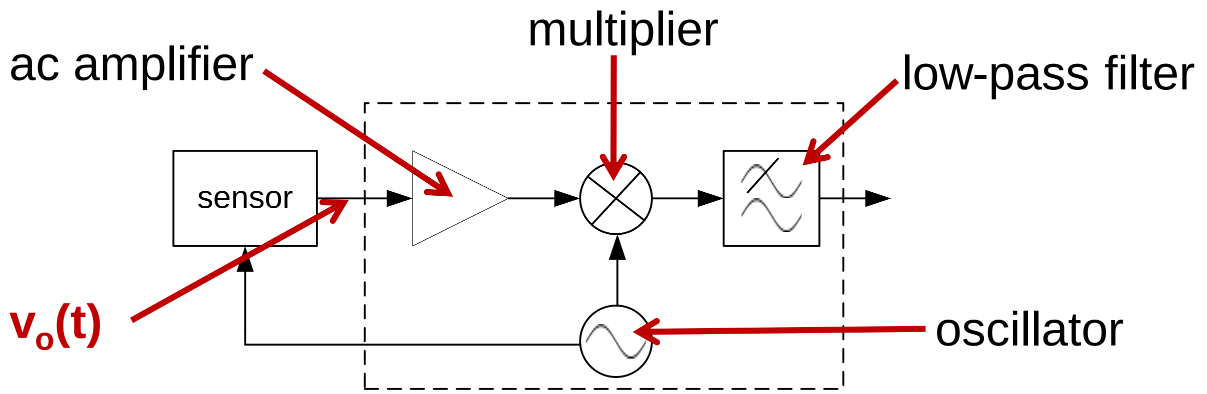
- output voltage then equal to

$$v_o(t) = S_\omega X V_e \cos(2\pi \cdot f_x \cdot t + \phi_x) \cos(2\pi \cdot f_e \cdot t)$$

$$= \frac{S_\omega V_e X}{2}\left(\cos(2\pi(f_e - f_x)t - \phi_x) + \cos(2\pi(f_e + f_x)t + \phi_x)\right)$$

$$\boxed{\cos(A)\cos(B) = \frac{1}{2}(\cos(A+B) + \cos(A-B))}$$

Phase-sensitive (synchronous) demodulation


multiplier

ac amplifier

low-pass filter

sensor

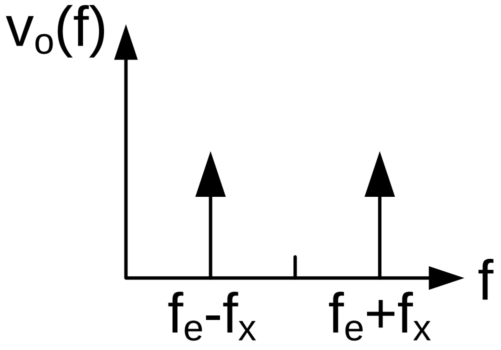
$v_o(t)$

oscillator

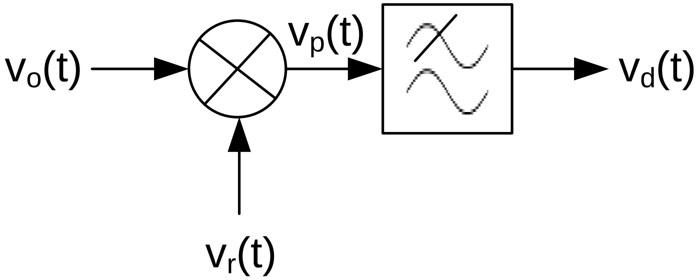
- output of the sensor (=input to detector)

$$v_o(t) = \frac{S_\omega V_e X}{2}\left(\cos(2\pi(f_e - f_x)t - \phi_x) + \cos(2\pi(f_e + f_x)t + \phi_x)\right)$$

- frequency spectrum (double-sideband signal)
- detector must recover $x(t) = X\cos(2\pi \cdot f_x \cdot t + \phi_x)$
  - maximal displacement of object (X)
  - frequency with which object changes direction (f_x)
  - phase shift of moving object (ϕ_x)



Phase-sensitive (synchronous) demodulation



- multiplier inputs

$$v_r(t) = V_r \cos(\omega_r t + \phi_r)$$
$$v_o(t) = S_\omega \cdot x(t) \cdot v_e(t) = S_\omega x(t) \cdot V_e \cos(\omega_e t + \phi_e)$$

- signals have same phase ($\phi_r = \phi_e$)
- output of the multiplier

$$v_p(t) = v_r(t) \cdot v_o(t) = \frac{V_r V_e}{2} S_\omega x(t)\left[\cos\big((\omega_e - \omega_r) \cdot t\big) + \cos\big((\omega_e + \omega_r) \cdot t\big)\right]$$

- frequency of the signals are equal ($\omega_r = \omega_e$)

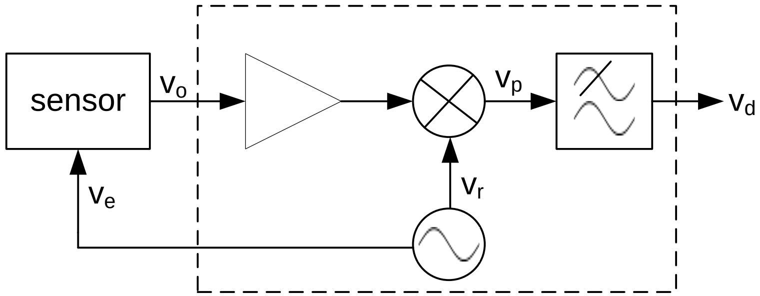
$$v_p(t) = \frac{V_r V_e}{2} S_\omega x(t)[1 + \cos(2\omega_e t)]$$

- output of low-pass filter

$$v_d(t) = LPF\{v_p(t)\} = \frac{V_r V_e}{2} S_\omega x(t)$$

- output of demodulator equal to x(t) (except for scaling factor)

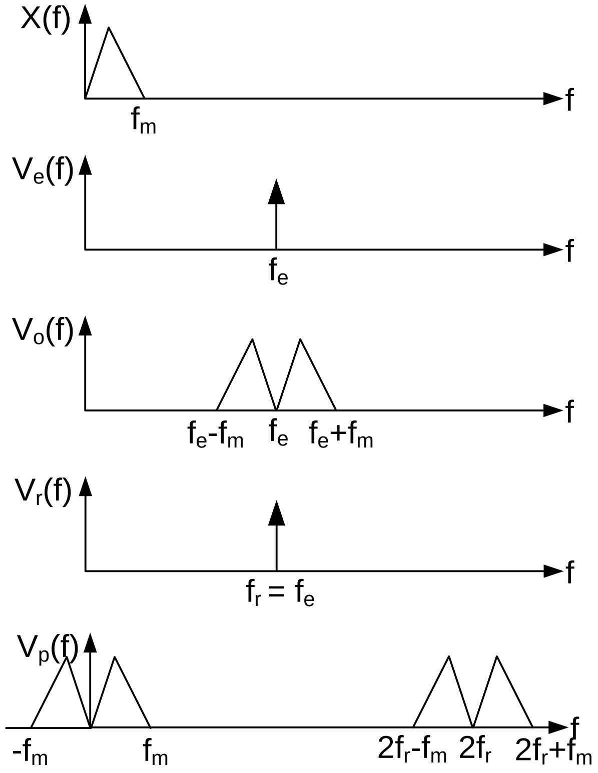
Phase-sensitive (synchronous) demodulation



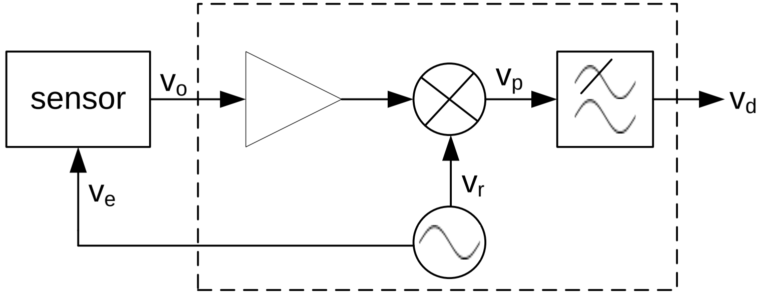
- coherent detector output

$$v_d(t) = \frac{V_r V_e}{2} S_\omega x(t)$$

- signal x(t) does not have to be a sinusoid
    - band-limited input signal
    - sensor excitation signal
    - sensor output signal
    - reference signal
    - multiplier output signal



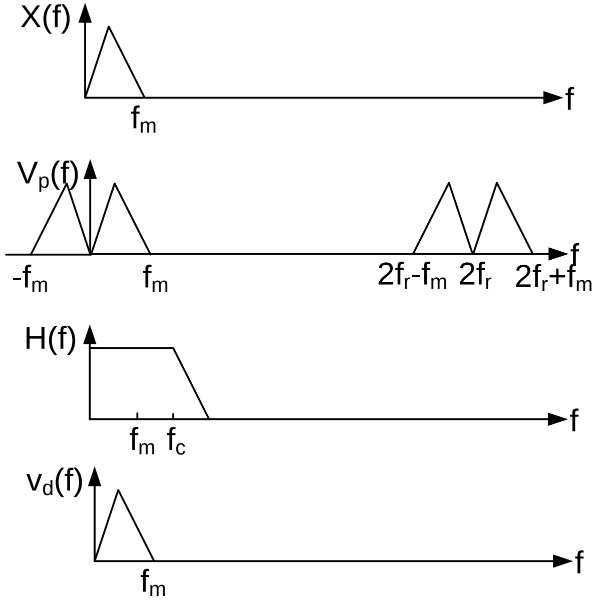
Phase-sensitive (synchronous) demodulation



- coherent detector output

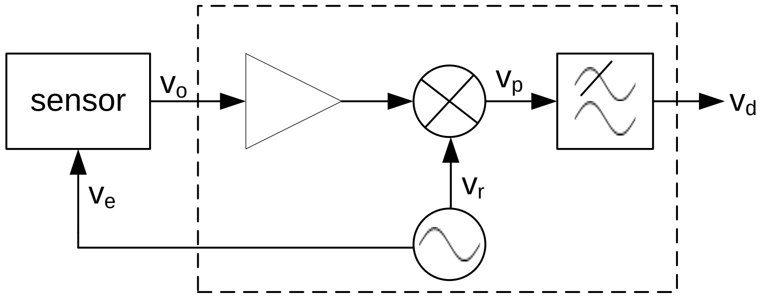
$$v_d(t) = \frac{V_r V_e}{2} S_\omega x(t)$$

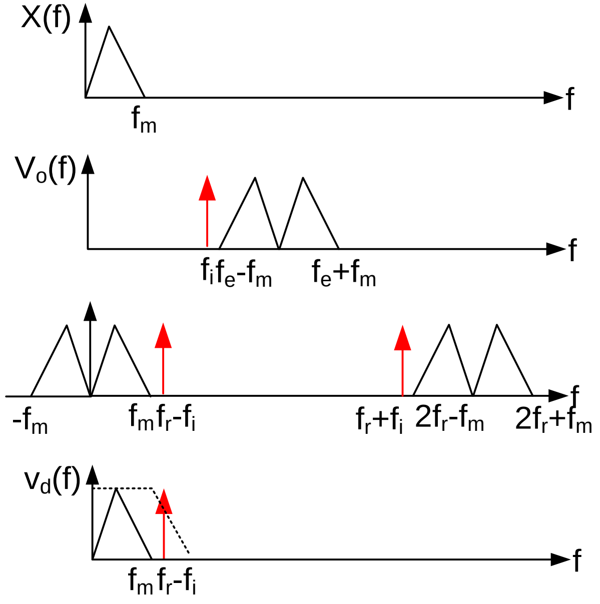
- signal x(t) does not have to be a sinusoid
  - band-limited input signal
  - multiplier output signal
  - LPF frequency response
  - detector output



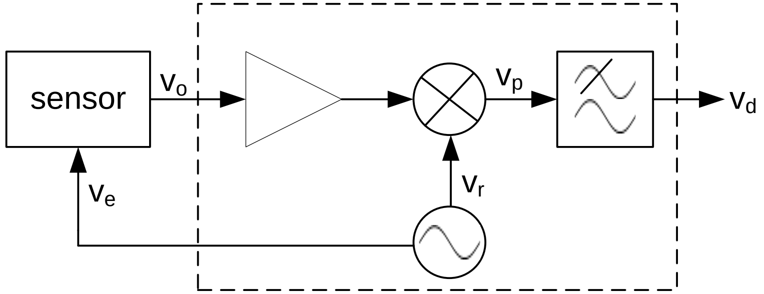
Interference

- output of sensor may contain interference (e.g. from power line)
- signal x(t) and interference signal
  - band-limited input signal
  - sensor output signal
  - multiplier output signal
  - detector output
- interference may be part of output signal
- attenuation of interference may be limited by LPF response





Interference



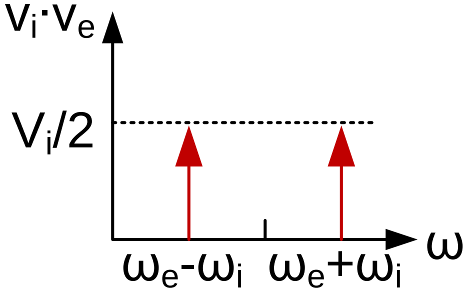
- amplitude response LPF

$$\left|\frac{V_d}{V_p}\right| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$$

  - ω$_c$ – corner frequency
- output voltage due to interference

$$|v_d|_i = \frac{V_r V_i}{2\sqrt{1+\left(\frac{\omega_e-\omega_i}{\omega_c}\right)^2}}$$

$$v_i(t) = V_i\cos(\omega_i t)$$



- normal mode rejection ratio (NMRR)

$$NMRR = 20\log\left|\frac{v_d(\omega_e)}{v_d(\omega_i)}\right| = 10\log\left[1+\left(\frac{\omega_e-\omega_i}{\omega_c}\right)^2\right] \approx 20\log\left[\frac{|\omega_e-\omega_i|}{\omega_c}\right]$$

  - reflects capability of filter to reject interference
  - approximation valid when $\omega_i \ll \omega_e$

Interference

- **example – select frequencies for coherent detector**
- measure 5 Hz signal with amplitude error < 1 LSB for 8 bit ADC
- 40dB attenuation for 50 Hz interference at input of demodulator
- **which excitation and corner frequencies should be used?**
- amplitude error should be less than 1/2^8
- corner frequency should be

$$\frac{1}{\sqrt{1+\left(\frac{2\pi\cdot 5Hz}{2\pi\cdot f_c}\right)^2}} > 1-\frac{1}{2^8} \quad \Rightarrow f_c > \frac{5Hz}{\sqrt{\left(\frac{2^8}{2^8-1}\right)^2-1}} = 56.4Hz$$

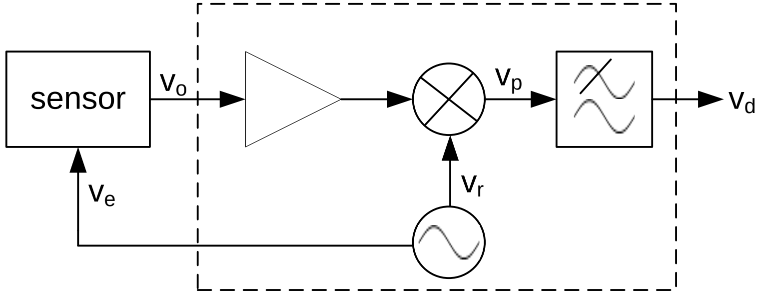
- NMMR(50Hz) = 40dB, hence

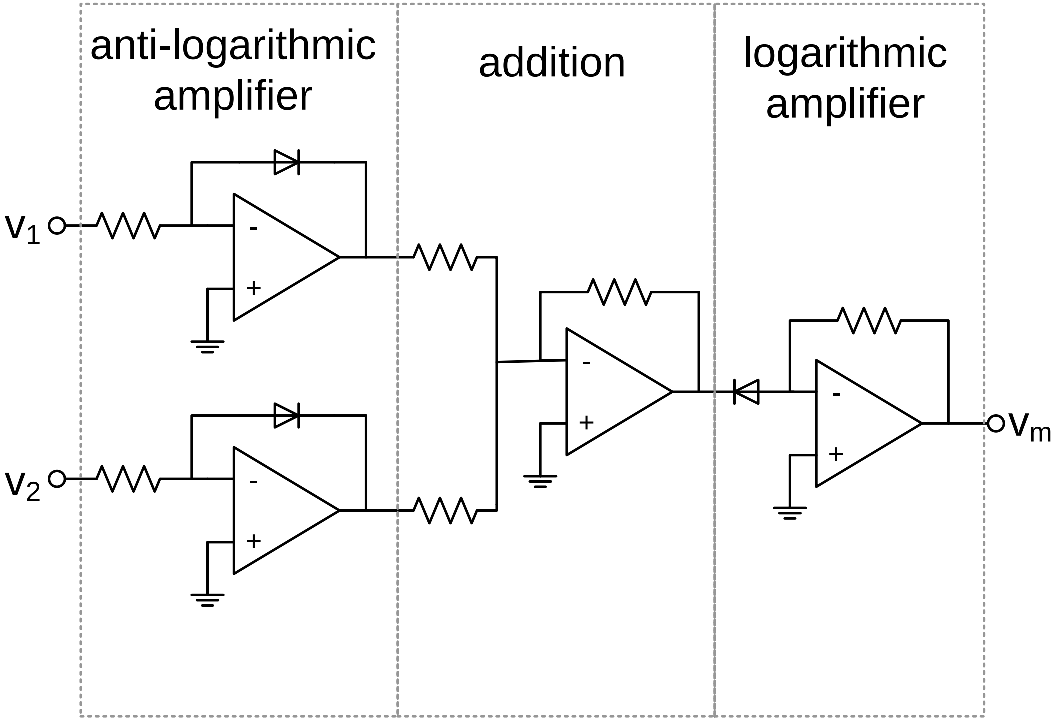
$$40dB = 20\cdot\log\left[\frac{f_e-50Hz}{56.4Hz}\right] \Rightarrow f_e = 5.69kHz$$

- excitation frequency may be too high for practical circuit
  - use bandwidth filter in front of coherent detector
  - use high-order LPF filter

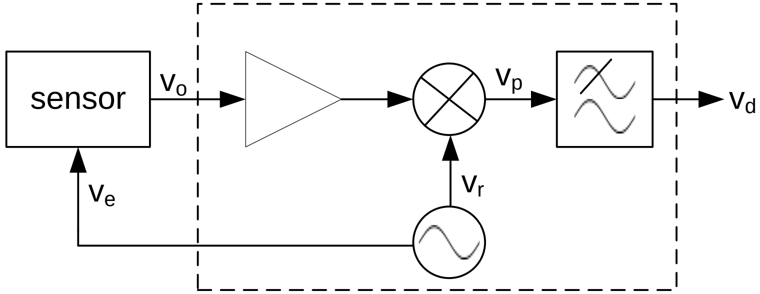
Phase-sensitive (synchronous) demodulation

- multiplier at the center of the PSD
- analog multipliers are expensive
- two solutions for multiplier
  - use (anti-)logarithmic amplifiers
  - use symmetrical square wave as reference





Phase-sensitive (synchronous) demodulation



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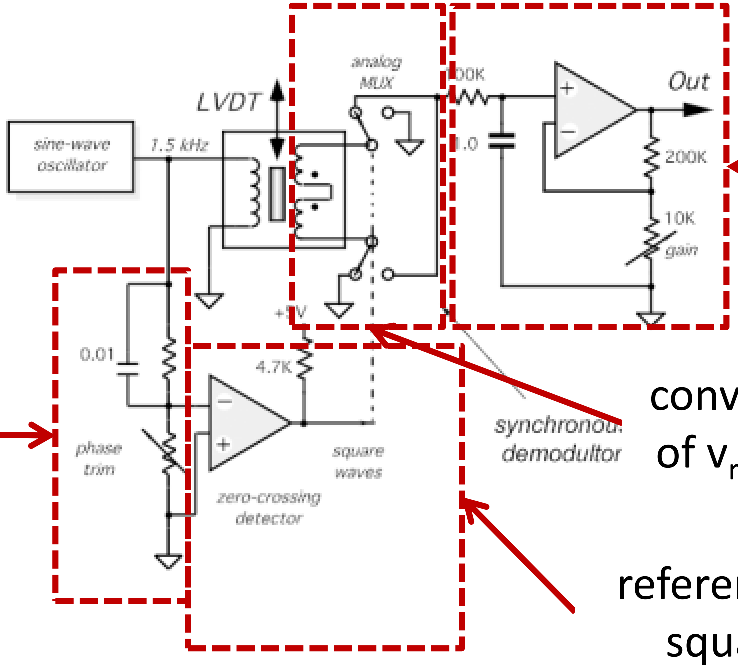


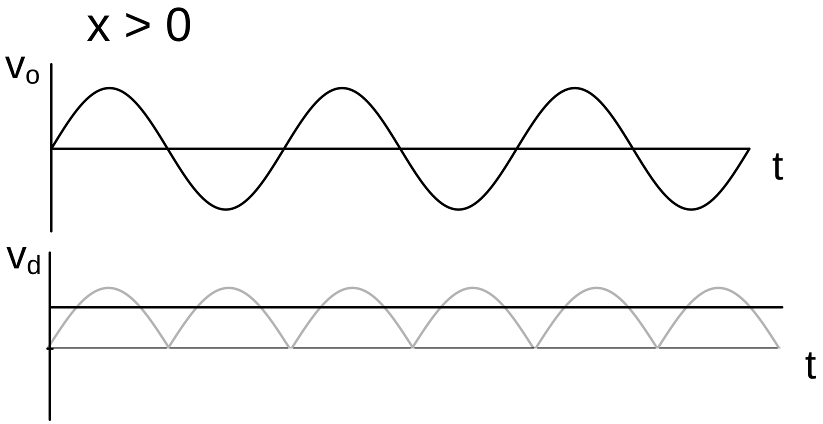
LPF with gain amplifier

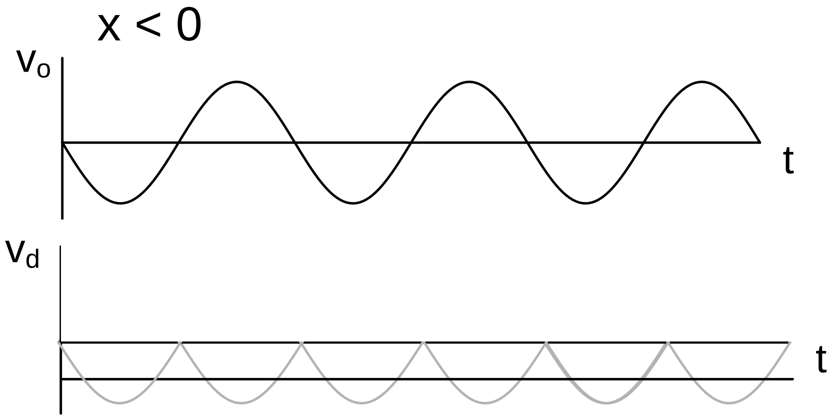
compensate phase difference

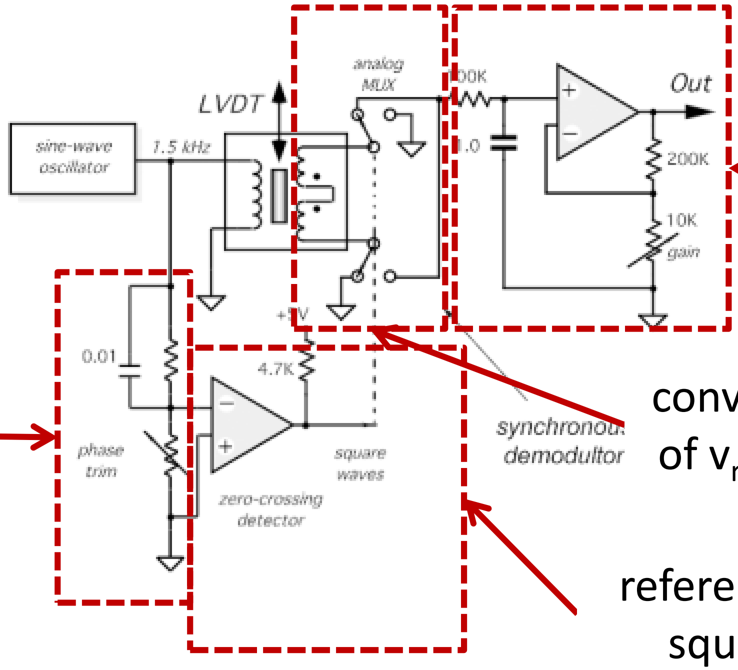
convolution of v_r and v_o

reference signal to square waves

Phase-sensitive (synchronous) demodulation







LPF with gain amplifier

convolution of v_r and v_o

compensate phase difference

reference signal to square waves

LVDT

pro’s and con’s of LVDTs

(+) non-contact sensor (no friction)

(+) infinitesimal resolution

(+) solid and robust construction

(+) no hysteresis (mechanical and magnetic)

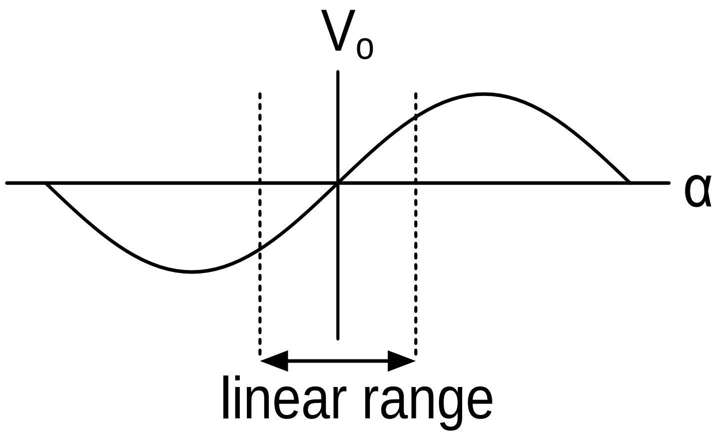
(+) output impedance is very low

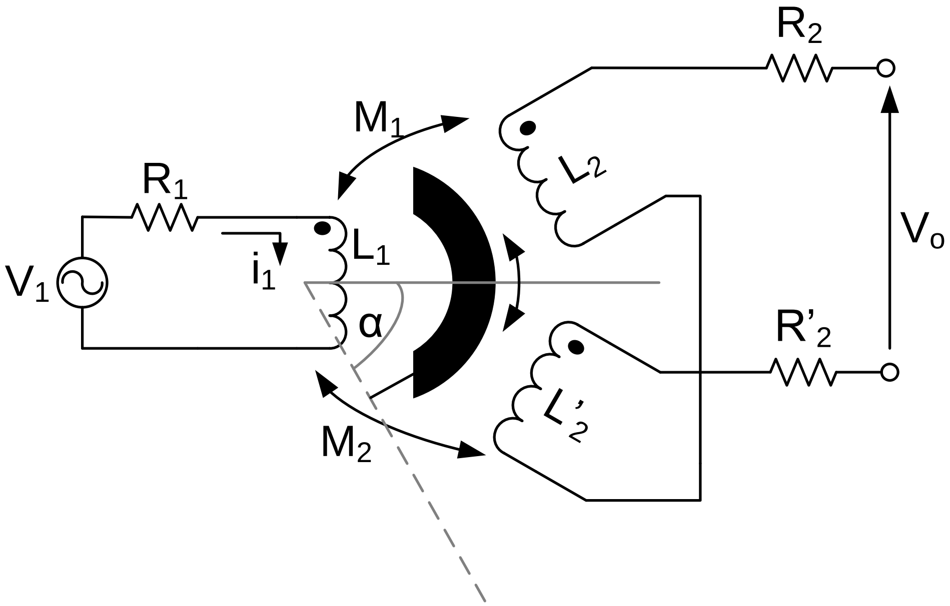
(-) sensitive to stray magnetic fields (interference)

(-) complex signal processing required

Rotary Variable Differential Transformer

- similar construction and operation as LVDT



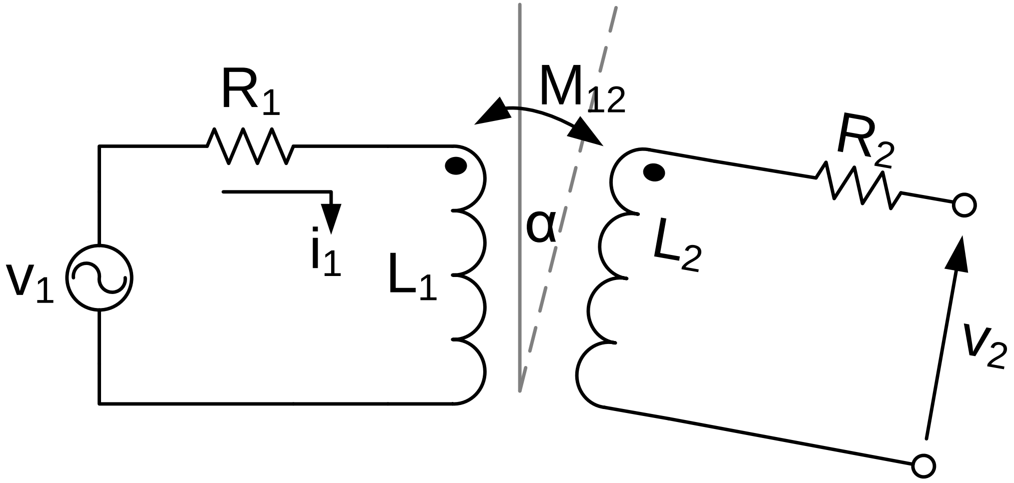


- output of (unloaded) RVDT

$$V_o = S_\omega \alpha V_1$$

  - linear relation between core rotation and output voltage
  - linear measurement range limited to ±20°
  - measuring full rotation is not possible

Variable transformer



- mutual inductance

$$M_{12} = N_2 \frac{\Phi_2}{i_1}$$

- flux linked by secondary winding

$$\Phi_2 = \mathbf{B} \cdot \mathbf{S} = BS \cos\alpha = \mu HS \cos\alpha = \mu \frac{N_1 i_1}{l} S \cos\alpha$$

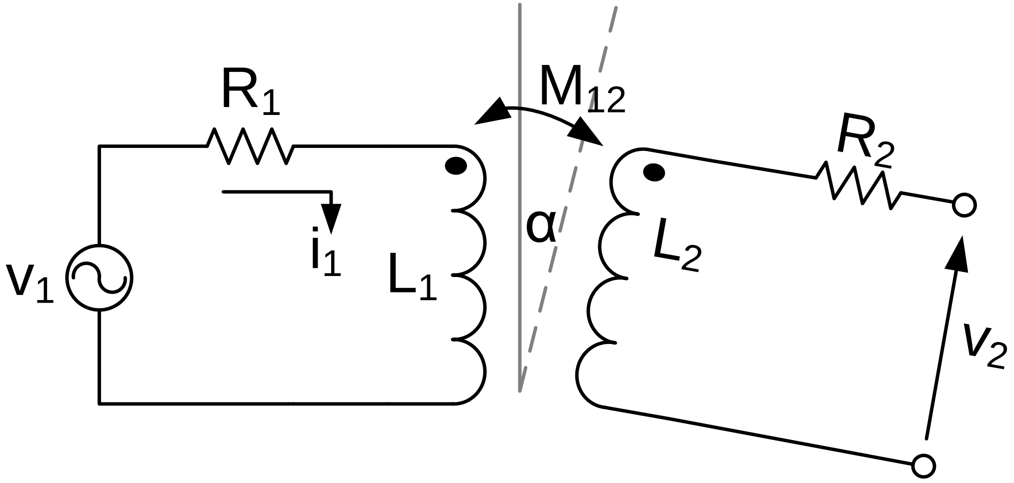
  - B – magnetic flux density
  - S – secondary cross section
  - H – magnetic field strength
  - µ – magnetic permeability of the core
  - l – length of primary winding

- mutual inductance is equal to

$$M_{12} = N_2 N_1 \frac{\mu}{l} S \cos\alpha = M \cos\alpha$$

- mutual inductance has relation with angle

Variable transformer



- mutual inductance

$$M_{12} = N_2 N_1 \frac{\mu}{l} S \cos\alpha = M \cos\alpha$$

- **what is the voltage on the secondary winding?**
  - consider open-circuit situation
  - voltage on primary winding

$$v_1 = V_p \sin(\omega t)$$

  - current through primary winding

$$i_1 = I_p \cos(\omega t)$$

  - voltage on secondary winding

$$v_2 = M_{12} \frac{di_1}{dt}$$

$$\Rightarrow V_2 = sI_1 M_{12} = sI_p M \cos(\alpha)\cos(\omega t) = k\cos(\alpha)\cos(\omega t)$$

- amplitude of output voltage depends on angle between windings

Resolver

- rotor winding acts as primary winding
- two stator windings at 90° act as secondary windings

- voltage on primary winding

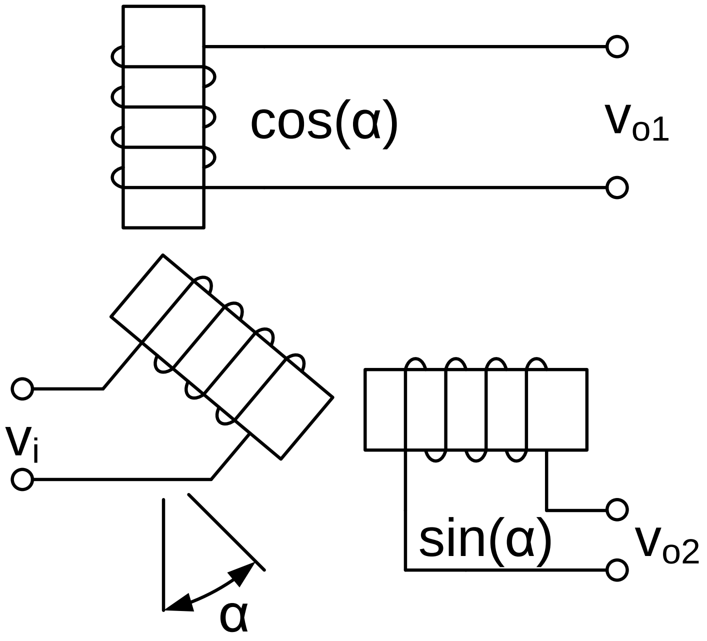
$$v_i = V_p \sin(\omega t) \Rightarrow i_i = I_p \cos(\omega t)$$

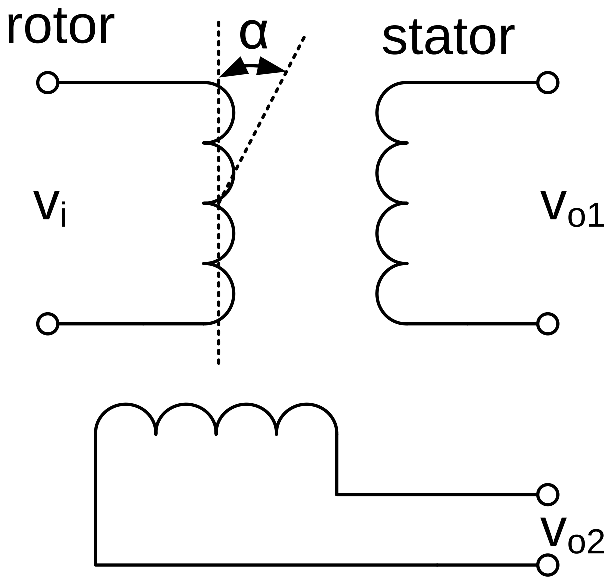
- induced voltages on secondary windings

$$v_{o1} = K \cos(\omega t) \cos(\alpha)$$

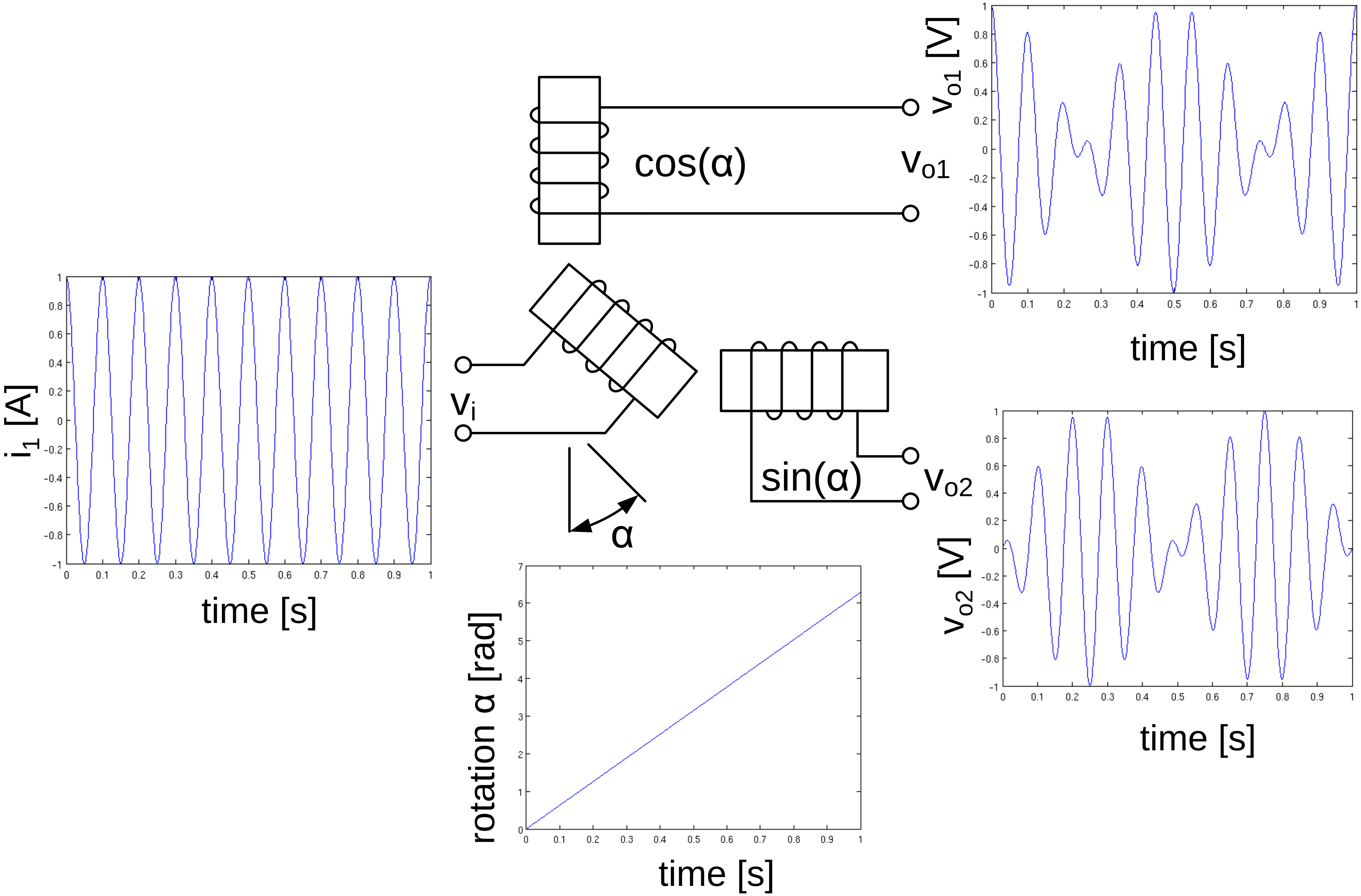
$$v_{o2} = K \cos(\omega t) \sin(\alpha)$$

  - output voltage is product of
    - measured object (α)
    - excitation voltage (v_i)
  - outputs differ by 90° phase difference

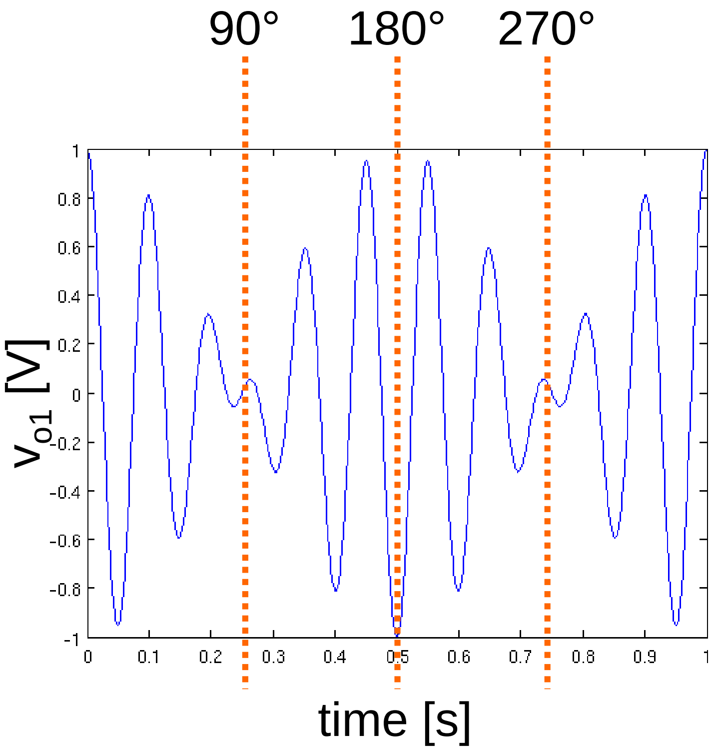


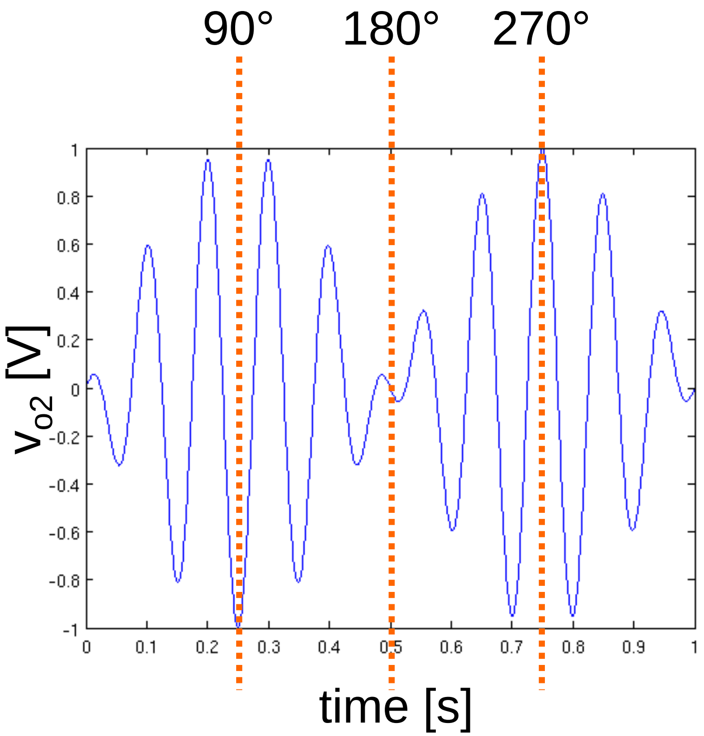


Resolver


cos(α)
v_{o1}
v_{o1} [V]
time [s]
v_i
α
sin(α)
v_{o2}
v_{o2} [V]
time [s]
i_1 [A]
time [s]
rotation α [rad]
time [s]
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
2
3
4
5
6
7

Resolver



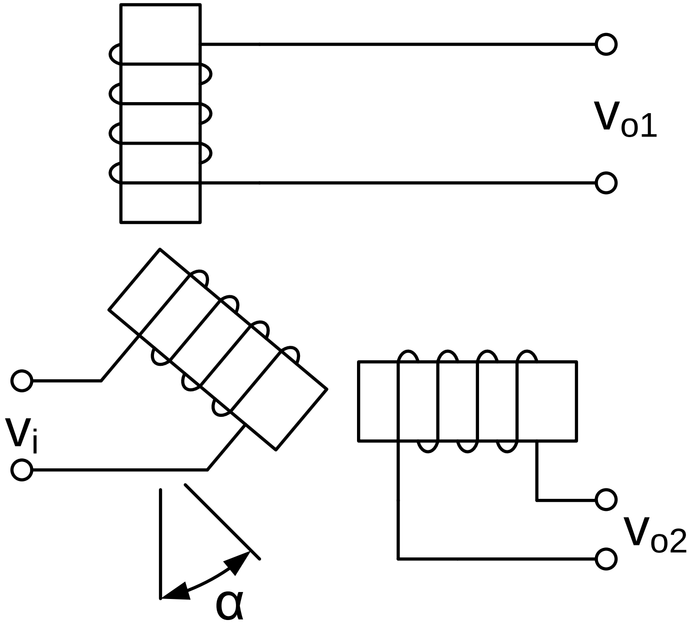


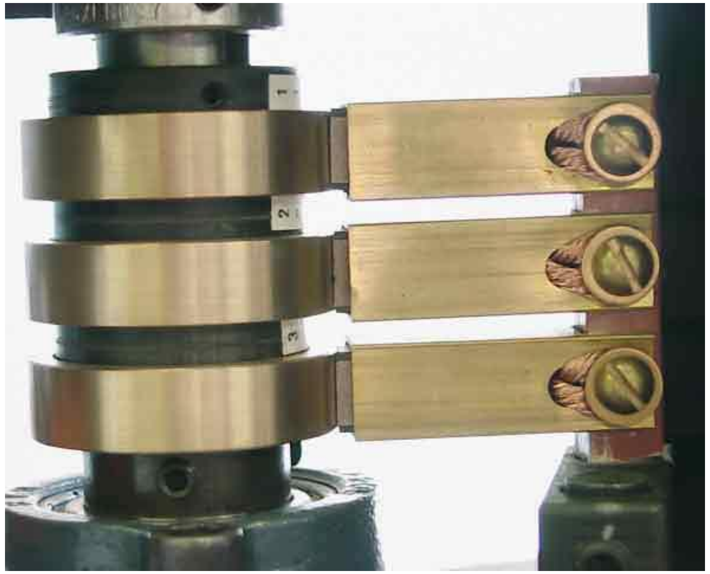
	0°	90°	180°	270°
v_{o1} [V]	1	0	-1	0
v_{o2} [V]	0	-1	0	1

- bridge rectifier can be used to recover angle between 0° ≤ α ≤ 90°
- phase sensitive detector needed to recover angle in all quadrants

Resolver

- **how to supply v_i to the rotor?**
  - use brushes or slips (friction, wear)
  - brushless transformer (preferred)





Resolver

- **how to supply v_i to the rotor?**
  - use brushes or slips (friction, wear)
  - brushless transformer (preferred)