Final Exam
5AIB0 Sensing, Computing, Actuating
24-6-2014, 9.00-10.00
Location MA atelier 2

- This final exam consists of 3 question for which you can score at most 100 points. The final grade for this exam is determined by dividing the number of points you scored by 10.
- The solutions to the exercises should be clearly formulated and written down properly. Do not only provide the final answer. Explain your choices and show the results of intermediate steps in your computation.
- The use of a simple calculator is allowed. No graphical calculator or laptop may be used during the interim exam.

Formulae sheet

Characteristic temperature of material: $B_{T_1/T_2} = \frac{ln\left(\frac{R_2}{R_1}\right)}{\frac{1}{T_1}-\frac{1}{T_2}}$

Resistance: $R = \frac{m}{ne^2\tau}\frac{l}{A} = \rho\frac{l}{A}$

Strain: $\epsilon = \frac{dl}{l}$

Stress: $\sigma = \frac{F}{A} = E\frac{dl}{l}$

Poisson's ratio: $v = -\frac{dt/t}{dl/l}$

Change in specific resistance due to volume change (for metals): $\frac{d\rho}{\rho} = C\frac{dV}{V}$

Change in resistance due to strain: $\frac{dR}{R} = G\epsilon$

Capacitance of flat plate capacitor: $C = \frac{q}{V} = \epsilon_0\epsilon_r\frac{A}{d}$

Capacitance of cylindrical capacitor: $C = \frac{q}{V} = \epsilon_0\epsilon_r\frac{2\pi\cdot l}{ln(b/a)}$

Energy stored in capacitor: $E = \frac{C\cdot V^2}{2}$

Reluctance: $\Re = \frac{1}{\mu\mu_0}\frac{l}{A}$

Inductance: $L = \frac{N\cdot\Phi}{i} = \frac{N^2}{\Re}$

Flux: $\Phi = \mathbf{B}\times\mathbf{S}$

Magneto-motive force: $F_m = \Phi\cdot\Re = N\cdot i$

Amplitude response of Butterworth LPF: $|H(f)| = \frac{1}{\sqrt{1+\left(\frac{f}{f_n}\right)^{2n}}}$

Sideways force on electron moving through magnetic field: $\mathbf{F} = q\cdot\mathbf{v}\times\mathbf{B}$

Transverse Hall potential: $V_H = \frac{1}{N\cdot c\cdot q}\frac{i\cdot B}{d}sin(\alpha)$

Radius of warping of bimetal sensor: $r \approx \frac{2j}{3(\alpha_x-\alpha_y)(T_2-T_1)}$

Displacement of bimetal sensor: $\Delta = r(1-cos(\frac{180L}{\pi r}))$

Flow velocity and temperature difference: $v = \frac{K}{\rho}\left(\frac{e^2}{R_S}\frac{1}{T_s-T_0}\right)^{1.87}$

Voltage across P-N junction (quality factor 1): $V = \frac{kT}{q}ln\left(\frac{I}{I_0}\right)$

Saturation current through PN-junction (quality factor 1): $I_0 = BT^3e^{-E_g/kT}$

Thomson effect: $Q = I^2\cdot R - I\cdot\sigma\frac{dT}{dx}$

Peltier coefficient: $\pi_{AB}(T) = T\cdot(\alpha_A-\alpha_B) = -\pi_{BA}(T)$

Exercise 1: measuring temperature in a climate control system (30 points)

Automatic climate control systems are found in many cars that are sold nowadays. The system allows the driver or its passengers to set the desired in-door temperature. The climate control system will cool or heat the in-door air till the desired temperature is reached. The temperature inside the car is an important factor in the operation of the control system. Since this temperature is not known at the time the system is designed, it must be measured using a sensor. This sensor reading can then be processed by the control algorithm to compute the required actuation action (i.e., heat or cool the in-door environment).

Figure 1: Climate control.

The circuit in Figure 2 can be used to measure the in-car temperature. This circuit is designed to operate between -40°C and +40°C. The resistor R_2 is a temperature dependent resistor (RTD) of type PT100. The relation between temperature and resistance (transfer function) can be approximated with the following linear equation: $R_2(T) = R_0(1 + \alpha T)$, with R_0 equal to 100Ω and $\alpha = 0.004/°\text{C}$. The resistor R_1 has a fixed value ($R_1 = R_0 \cdot k$).

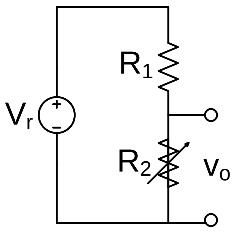

Figure 2: Temperature sensor.

(2p) (a) Show that the output voltage v_o of the sensor is equal to:

$$v_o = \frac{1 + \alpha T}{1 + \alpha T + k} V_r$$

(5p) (b) Show that the maximal sensitivity is reached when $k = 1 + \alpha T$.

(3p) (c) The resistor R_1 influence the sensitivity of the sensor as well as the self-heating error of the sensor. Show that the error due to the self-heating effect is equal to:

$$\Delta T = \frac{V_r^2}{\delta R_0} \frac{1 + \alpha T}{(1 + \alpha T + k)^2}$$

with δ the dissipation constant of the environment. *Hint:* $\Delta T = \left(I^2 R\right) / \delta$.

(5p) (d) Assume that $\delta = 6$ mW/K and $V_r = 5$ V. What value should the resistor R_1 have such that the error due to the self-heating effect is less then 0.5°C within the whole range of the sensor while at the same time the sensitivity is maximized?

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(5p) (e) The operation of a temperature dependent resistor (RTD) is based on the thermo-resistive effect. Explain briefly (maximal 200 words) how this effect works in metals.

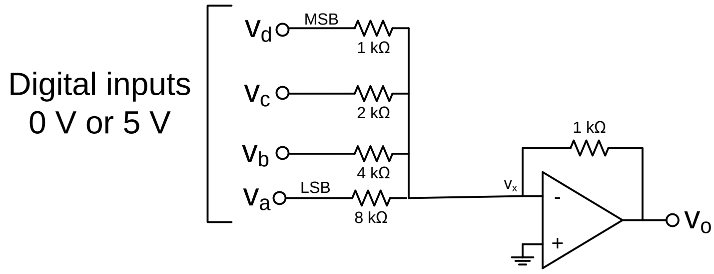

Figure 3: DA converter using summing op-amp.

(5p) (f) A summing DA converter is shown in Figure 3. What is the output voltage of this DA converter when the binary input 0010 is applied to it?

(5p) (g) Give at least three reasons why we prefer transducers who produce a signal in the electrical domain over transducers that produce a signal in any of the other signal domains.

Exercise 2: thermocouple **(30 points)**

Systems with a thermal capacity such as a thermocouple require a transfer of heat, Q, from the environment to the sensor in order to show a change in temperature. This change in energy, E, as a function of time is described by the following first-order differential equation:

$$Q = \frac{dE}{dt} = mC_V \frac{dT_s(t)}{dt} = hA_s \left(T_o(t) - T_s(t)\right)$$

, with m the weight of the sensor, C_v the specific heat of the sensor, h the heat transfer coefficient, A_s the contact surface (area) of the sensor, T_o the environmental temperature, en T_s the sensor temperature.

(5p) (a) Show that the transfer function of the sensor $T_s(s)/T_o(s)$ is equal to:

$$\frac{T_s(s)}{T_o(s)} = \frac{k}{\tau s + 1}$$

, with $k = 1$ and $\tau = \frac{mC_v}{hA_s}$.

(5p) (b) The response of the sensor to a step function on its input is given by:

$$T_s(t) = k\left(1 - e^{-t/\tau}\right)$$

Assyme that the sensor has an initial temperature $T_s(0) = T_i$ when the sensor is suddenly exposed to a constant environmental temperature T_o. Show that the response of the sensor is equal to:

$$T_s(t) = T_o + (T_i - T_o)\, e^{-t/\tau}$$

(5p) (c) To determine the time constant τ the sensor is exposed from $t = 0$ to a (constant) environmental temperature. The temperature is measured every 4 seconds. This results in the following series of readings:

Time (s)	0	4	8	12	16	20	24	28
Temperature (°C)	5.00	56.55	70.14	73.72	74.66	74.91	74.98	74.99

What is the time constant τ from this sensor?

(5p) (d) Because of temperature fluctuations in the environment, the environmental temperature T_o changes according to: $T_o(t) = 0.50°\text{C}\cdot sin(0.01t) + 74.99°\text{C}$. Assume that the time constant τ is equal to 3.00 s. What is the steady-state output of this sensor $T_s(t)$?

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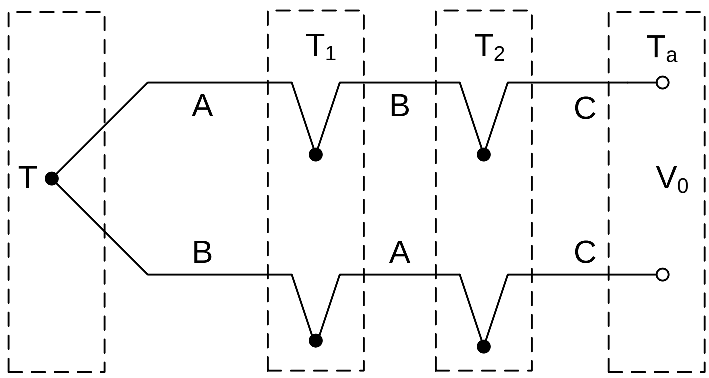

Figure 4: Simulate reference junction at 0°C.

(5p) (e) Figure 4 shows a circuit in which three types of wires (A, B, C) are combined into several thermocouples with two intermediate temperatures T_1 and T_2 and a temperature T at the measurement junction. What relation must T_1 and T_2 have such that the output voltage only depends on T?

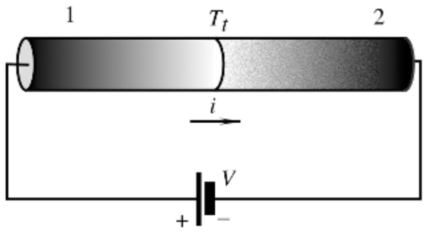

Figure 5: Peltier effect in single junction.

(5p) (f) Figure 5 shows a single junction of two different materials. Explain (in maximally 200 words) why the Peltier effect results in the production or liberation of energy at a junction when a current is passed through this junction.

Exercise 3: Electronic Stability Progam **(40 points)**

ESP assists a driver to keep a vehicle on the road during dangerous driving conditions. For this purpose, the ESP system uses a large number of sensors in the vehicle. One of these sensors measures the angle of the steering-wheel and steering-column and the speed with which the driver changes this angle (note that one sensor measures both quantities). The RVDT (rotary variable differential transducer) from Figure 6 can be used to measure the angle (and its rate of change). When the driver moves the steer from the central position ($\Theta = 0°$) to the left or to the right, then this will lead to a change in the output voltage of the sensor. This electrical signal can then be send to the ESP computer. The primary winding of this RVDT is connected to a voltage supply that produces a sinusoidal voltage with an amplitude of 5V with a frequency of 10 Hz. The RVDT has a sensitivity S of 100 μV/(°/V). The output voltage of the RVDT is equal to:

$$v_s = v_2 - v_1 = S \cdot \Theta \cdot v_r$$

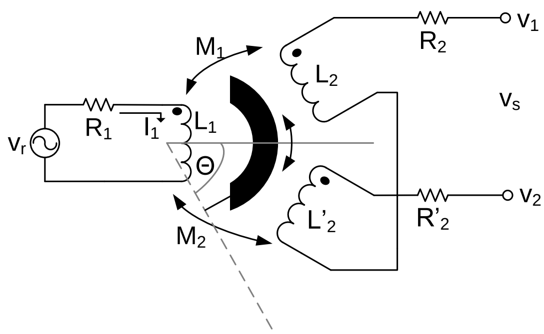

Figure 6: Measuring an angle using an RVDT.

(5p) (a) Show that the output voltage of the sensor, v_s, is equal to:

$$v_s = \frac{sk_\Theta \Theta v_r}{sL_1 + R_1}$$

with $(M_2 - M_1) = k_\Theta \Theta$.

(5p) (b) Assume that the resistor R_1 has a resistance of 250 Ω and the inductor L_1 has an inductance of 40 mH. What is the value of the coupling coefficient k_Θ?

(5p) (c) A driver moves the steer in 1 second from an angle $\Theta = -20°$ to an angle $\Theta = +20°$. Draw the output voltage v_s as a function of time t. (Clearly show the dimensions on both axis.)

Exercise continues on next page

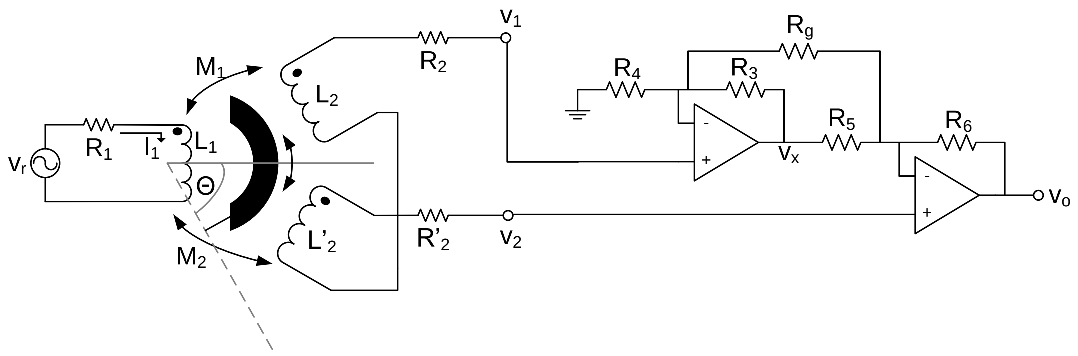

Figure 7: RVDT with instrumentation amplifier.

(5p) (d) The signal coming from the RVDT is too weak to be directly send to the ESP computer. The signal should first be amplified. For this purpose, the sensor from Figure 6 is connected to an instrumentation amplifier. The resulting circuit is shown in Figure 7. The instrumentation amplifier uses two operational amplifiers. You may assume that these op-amps show an ideal behaviour. Show that the output voltage v_o of the instrumentation amplifier in Figure 7 is equal to:

$$v_o = \left(1 + k + \frac{R_6 + R_4}{R_g}\right) v_d$$

with $v_d = v_2 - v_1$. Assume that $R_4/R_3 = R_6/R_5 = k$ (i.e., the CMRR is infinite).

(5p) (e) Assume that the resistors R_3, R_4, and R_6 in the instrumentation amplifier from Figure 7 have a resistance of 10 Ω. What resistance should the resistors R_g have such that the amplitude of the output voltage v_o is equal to 0 V when $\Theta = 0°$ and 5 V (peak) when $\Theta = 20°$?

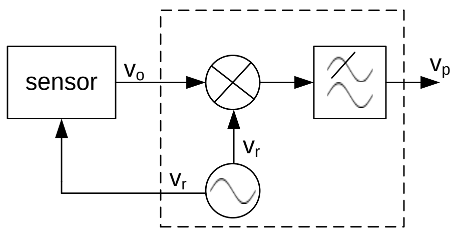

Figure 8: Sensor with phase sensitive detector.

(5p) (f) The rotation sensor from Figure 7 is connected to a phase sensitive detector. This detector consists of an analogue multiplier connected to a low-pass fulter. Using this processing circuit, it is possible to recover the magnitude and direction of the rotation from the output signal of the sensor. The block diagram of a phase sensitive detector is shown in Figure 8. The angle of the steering-wheel is given as a function $\Theta(t)$. Assume further that the reference voltage v_r is equal to:

$$v_r(t) = V_r cos\left(\omega_r \cdot t\right)$$

The output voltage v_o of the sensor circuit and instrumentation amplifier is then equal to:

$$v_o(t) = G \cdot S \cdot \Theta(t) \cdot v_r(t)$$

, with G the gain of the instrumentation amplifier.

Show that the output voltage of the phase sensitive detector, v_p, is equal to:

$$v_p(t) = G \cdot S \cdot \Theta(t) \cdot \frac{V_r^2}{2}$$

Hint: $cos(A)cos(B) = \frac{1}{2}\left(cos\left(A + B\right) + cos\left(A - B\right)\right)$.

Exercise continues on next page

(5p) (g) In stead of a phase sensitive detector, it is also possible to connect the output signal of the RVDT to a double-sided rectifier with a low-pass filter as is shown in Figure 9. Is it possible to reconstruct from the output signal ($v_{o2} - v_{o1}$) the direction (positive or negative angle)? (Explain your answer.)

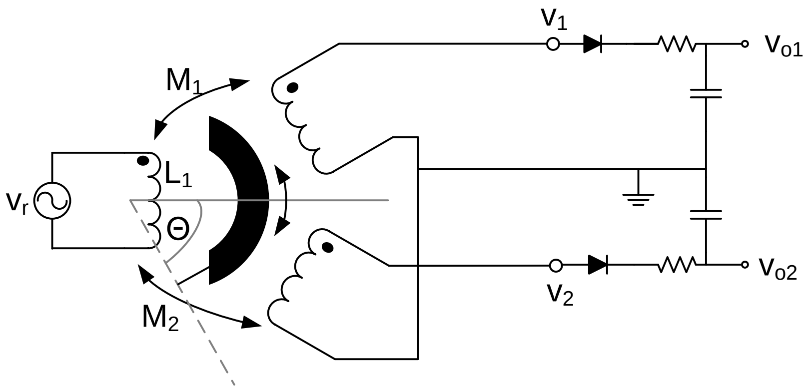

Figure 9: Double-sided rectifier with low-pass filter.

(5p) (h) An RVDT measures the rotation of an object through the change in coupling between a primary and two secondary coils. Another way to measure a rotation would be to use a Hall effect sensor. Explain the operation of a Hall effect sensor. Clearly indicate how you can use the sensor to measure a rotation.