Informative Exam
5AIB0 Sensing, Computing, Actuating
26-5-2020, 13.30-14.30

Name: ______________________________

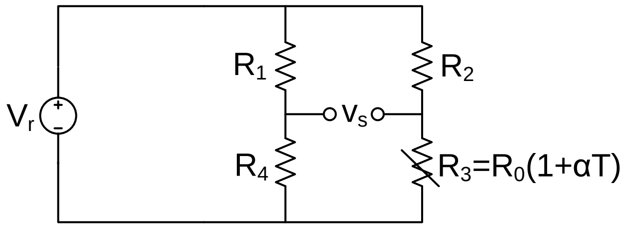
ID: ______________________________

- This interim exam consists of 1 question for which you can score at most 45 points. The final grade for this interim exam is determined by dividing the number of points you scored by 4.5.
- The solutions to the exercises should be clearly formulated and written down properly. Do not only provide the final answer. Explain your choices and show the results of intermediate steps in your computation.
- The use of a simple calculator is allowed. No graphical calculator or laptop may be used during the interim exam.

Exercise continues on next page

Exercise 1: temperature sensor **(45 points)**

A resistive temperature detector (RTD) can be used to measure the temperature of an object. Figure 1 shows a bridge circuit with an RTD which is exposed to a temperature T. This temperature will be in the range [-30°C, 80°C]. The RTD is a PT100 sensor with $R_0 = 100\ \Omega$ and $\alpha = 0.004/°\text{C}$ at 0°C.



Figuur 1: Bridge circuit with a RTD temperature sensor.

(5p) (a) Show that output voltage v_s of the sensor circuit is equal to:

$$v_s = -\frac{k\alpha T}{(k+1)(k+1+\alpha T)}V_r$$

, with $k = R_1/R_4 = R_2/R_0$.

Solution:

$$v_o = \left(\frac{R_4}{R_1+R_4} - \frac{R_3}{R_3+R_2}\right)V_r$$

$$\Leftrightarrow v_s = \left(\frac{R_4}{kR_4+R_4} - \frac{R_0(1+\alpha T)}{R_0(1+\alpha T)+kR_0}\right)V_r$$

$$\Leftrightarrow v_s = \left(\frac{1}{k+1} - \frac{1+\alpha T}{1+\alpha T+k}\right)V_r$$

$$\Leftrightarrow v_s = \left(\frac{-k\alpha T}{(k+1)(k+1+\alpha T)}\right)V_r = -\frac{k\alpha T}{(k+1)(k+1+\alpha T)}V_r$$

(5p) (b) Has the sensor output voltage v_s a non-linearity error in terms of the temperature T? (Explain your answer)

Solution: Yes. The output voltage of the circuit - see answer part a - has a temperature dependency in the denominator.

(5p) (c) Show that the relative non-linearity error in the output voltage v_s is equal to:

$$\epsilon = \left|\frac{-\alpha T}{k+1+\alpha T}\right|$$

Solution: The measured output voltage of the bridge is equal to (see part a):

$$v_s = \frac{k\alpha T}{(k+1)(k+1+\alpha T)}V_r$$

The ideal output voltage of the bridge is equal to

$$v_{s,ideaal} = \frac{k\alpha T}{(k+1)^2}V_r$$

You will find this ideal output voltage when ignoring the non-linearity (ignore αT in the denominator).

The relative error due to the non-linearity is thus equal to:

$$\epsilon = \left| \frac{v_s - v_{s,ideaal}}{v_{s,ideaal}} \right| = \left| \frac{v_s}{v_{s,ideaal}} - 1 \right| = \left| \frac{k+1}{k+1+\alpha T} - 1 \right| = \left| \frac{-\alpha T}{k+1+\alpha T} \right|$$

(5p) (d) What ratio k should the resistors R_2/R_0 have to ensure that the non-linearity error is less then 0.8% of the reading while maximizing the sensitivity?

Solution: Since $T \geq 0$, the relative error is equal to:

$$\epsilon = \frac{\alpha T}{k+1+\alpha T}$$

The non-linearity error reaches its maximum when T reaches its maximum temperature. Hence, non-linearity reaches its maximum at $80°C$. At that moment the following constraint must hold:

$$\epsilon = \frac{\alpha T}{k+1+\alpha T} < 0.008$$

$$\Rightarrow k \geq 38.7$$

Maximal sensitivity requires $k = 1$. Hence choosing k as small as possible while satisfying the non-linearity constraints maximizes the sensibility. Hence one should use $k = 38.7$.

(5p) (e) Explain how the self-heating effect causes an error in the resistance of the RTD.

Solution: The self-heating effect causes an increase in the temperature of the RTD which in turn increases its resistance. Hence the resistance of the RTD is different from the resistance that the device would have when exposed to ambient temperature (while assuming an absence of self-heating). This difference in resistance leads to a wrong (erroneous) conclusion on the ambient temperature.

(5p) (f) Assume that $k = 38.7$. Assume further that the dissipation constant of the environment $\delta = 1$ mW/K. What value should the supply voltage V_r have to keep the self-heating below 0.2% of the full-scale output (FSO)?

Solution: Error due to self-heating is equal to:

$$\Delta T = \frac{P_D}{\delta} < 0.2\% \cdot 80°C$$

$$\Rightarrow P_D = \left(\frac{V_r}{R_2 + R_3} \right)^2 R_3 < (0.002 \cdot 80°C) \cdot \delta$$

$$\Rightarrow V_r < \sqrt{\frac{(0.002 \cdot 80°C) \cdot \delta}{R_3}} \cdot (R_2 + R_3)$$

Maximal self-heating error occurs when $R_2 = R_3$. However, this is well beyond the measurement range since $R_2 = 38.7 \cdot 100\Omega = 3870\Omega$. Hence, self-heating reaches its maximum at the end of the measurement range, i.e, when $R_3 = 132\Omega$.

$$\Rightarrow V_r < \sqrt{\frac{(0.002 \cdot 80°C) \cdot 1mW/K}{132\Omega}} \cdot (38.7 \cdot 100\Omega + 132\Omega) = 4.4V$$

(5p) (g) The circuit in Figure 1 uses three fixed resistors and one RTD. What would be the advantage when the fixed resistor R_1 in Figure 1 is replaced with an RTD similar to the RTD used for R_3?

> **Solution:** The sensitivity is increased (doubled).

(5p) (h) The operation of a temperature dependent resistor (RTD) is based on the thermo-resistive effect. Explain briefly (maximal 200 words) how this effect works in metals.

> **Solution:** In metals, the average time between collisions depends on the temperature. This in turn influences the specific resistance of the material and as such the resistance of the sensor.

(5p) (i) What is the essential difference between an active and a passive sensor?

> **Solution:** An active sensor requires a voltage or current source to deliver an electrical signal. A passive sensor delivers an electrical signal directly (without voltage or current source) when an input signal is applied.

Formulae sheet

Characteristic temperature of material: $B_{T_1/T_2} = \frac{ln\left(\frac{R_2}{R_1}\right)}{\frac{1}{T_1}-\frac{1}{T_2}}$

Resistance: $R = \frac{m}{ne^2\tau}\frac{l}{A} = \rho\frac{l}{A}$

Strain: $\epsilon = \frac{dl}{l}$

Stress: $\sigma = \frac{F}{A} = E\frac{dl}{l}$

Poisson's ratio: $v = -\frac{dt/t}{dl/l}$

Change in specific resistance due to volume change (for metals): $\frac{d\rho}{\rho} = C\frac{dV}{V}$

Change in resistance due to strain: $\frac{dR}{R} = G\epsilon$

Capacitance of flat plate capacitor: $C = \frac{q}{V} = \epsilon_0\epsilon_r\frac{A}{d}$

Capacitance of cylindrical capacitor: $C = \frac{q}{V} = \epsilon_0\epsilon_r\frac{2\pi\cdot l}{ln(b/a)}$

Energy stored in capacitor: $E = \frac{C\cdot V^2}{2}$

Reluctance: $\Re = \frac{1}{\mu\mu_0}\frac{l}{A}$

Inductance: $L = \frac{N\cdot\Phi}{i} = \frac{N^2}{\Re}$

Flux: $\Phi = \mathbf{B}\times\mathbf{S}$

Magneto-motive force: $F_m = \Phi\cdot\Re = N\cdot i$

Amplitude response of Butterworth LPF: $|H\left(f\right)| = \frac{1}{\sqrt{1+\left(\frac{f}{f_n}\right)^{2n}}}$

Sideways force on electron moving through magnetic field: $\mathbf{F} = q\cdot\mathbf{v}\times\mathbf{B}$

Transverse Hall potential: $V_H = \frac{1}{N\cdot c\cdot q}\frac{i\cdot B}{d}sin(\alpha)$

Radius of warping of bimetal sensor: $r \approx \frac{2j}{3(\alpha_x-\alpha_y)(T_2-T_1)}$

Displacement of bimetal sensor: $\Delta = r(1-cos(\frac{180L}{\pi r}))$

Flow velocity and temperature difference: $v = \frac{K}{\rho}\left(\frac{e^2}{R_S}\frac{1}{T_s-T_0}\right)^{1.87}$

Voltage across P-N junction (quality factor 1): $V = \frac{kT}{q}ln\left(\frac{I}{I_0}\right)$

Saturation current through PN-junction (quality factor 1): $I_0 = BT^3e^{-E_g/kT}$

Thomson effect: $Q = I^2\cdot R - I\cdot\sigma\frac{dT}{dx}$

Peltier coefficient: $\pi_{AB}(T) = T\cdot(\alpha_A-\alpha_B) = -\pi_{BA}(T)$