Sensing, Computing, Actuating
Lecture 2 - Thermo-resistive sensor

Exercise 1: measuring temperature in a climate control system

Automatic climate control systems are found in many cars that are sold nowadays. The system allows the driver or its passengers to set the desired in-door temperature. The climate control system will cool or heat the in-door air till the desired temperature is reached. The temperature inside the car is an important factor in the operation of the control system. Since this temperature is not known at the time the system is designed, it must be measured using a sensor. This sensor reading can then be processed by the control algorithm to compute the required actuation action (i.e., heat or cool the in-door environment).

Figure 1: Climate control.

The circuit in Figure 2 can be used to measure the in-car temperature. This circuit is designed to operate between -40°C and +40°C. The resistor R_2 is a temperature dependent resistor (RTD) of type PT100. The relation between temperature and resistance (transfer function) can be approximated with the following linear equation: $R_2(T) = R_0(1 + \alpha T)$, with R_0 equal to 100Ω and $\alpha = 0.004/°\text{C}$. The resistor R_1 has a fixed value ($R_1 = R_0 \cdot k$).

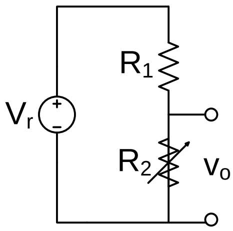

Figure 2: Temperature sensor.

(a) Show that the output voltage v_o of the sensor is equal to:

$$v_o = \frac{1 + \alpha T}{1 + \alpha T + k} V_r$$

Answer:

$$v_o = \frac{R_2}{R_1 + R_2} V_r = \frac{R_0(1 + \alpha T)}{R_0(1 + \alpha T) + kR_0} V_r = \frac{1 + \alpha T}{1 + \alpha T + k} V_r$$

(b) Show that the sensitivity of the sensor for a change in the temperature is equal to:

$$S = \frac{dv_o}{dT} = \frac{\alpha k}{(1 + \alpha T + k)^2} V_r$$

Hint: $\frac{d}{d\alpha}\left(\frac{u}{v}\right) = \frac{v\frac{du}{d\alpha} - u\frac{dv}{d\alpha}}{v^2}$

Answer: The sensitivity is equal to:

$$S = \frac{dv_o}{dT} = \frac{d}{dT}\left(\frac{1 + \alpha T}{1 + \alpha T + k} V_r\right)$$

Use:

$$\frac{d}{d\alpha}\left(\frac{u}{v}\right) = \frac{v\frac{du}{d\alpha} - u\frac{dv}{d\alpha}}{v^2}$$

This results in:

$$S = \frac{\alpha(1 + \alpha T + k) - (1 + \alpha T)\alpha}{(1 + \alpha T + k)^2} V_r$$

$$= \frac{\alpha k}{(1 + \alpha T + k)^2} V_r$$

(c) Show that the maximal sensitivity is reached when $k = 1 + \alpha T$.

Answer: The maximal sensitivity is reached when:

$$\frac{dS}{dk} = 0 \Rightarrow \frac{d}{dk}\left(\frac{\alpha k}{(1 + \alpha T + k)^2} V_r\right) = 0$$

$$\Rightarrow \frac{\alpha(1 + \alpha T - k)}{(1 + \alpha T + k)^3} V_r = 0$$

$$\Rightarrow k = 1 + \alpha T$$

(d) The resistor R_1 does not only influence the sensitivity of the sensor. It also has an impact on the self-heating error of the sensor. Show that the error due to the self-heating effect is equal to:

$$\Delta T = \frac{V_r^2}{\delta R_0} \frac{1 + \alpha T}{(1 + \alpha T + k)^2}$$

with δ the dissipation constant of the environment. *Hint:* $\Delta T = \left(I^2 R\right)/\delta$.

Answer:

$$\Delta T = \frac{\frac{V_r^2}{(R_1 + R_2)^2} R_2}{\delta} = \frac{V_r^2}{\delta} \frac{R_2}{(R_1 + R_2)^2} = \frac{V_r^2}{\delta} \frac{R_0(1 + \alpha T)}{(kR_0 + R_0(1 + \alpha T))^2} = \frac{V_r^2}{\delta R_0} \frac{1 + \alpha T}{(k + 1 + \alpha T)^2}$$

(e) Show that the temperature T at which the maximal self-heating error ΔT is reached is equal to $T = (k-1)/\alpha$?

Answer:

$$\frac{d\Delta T}{dT} = 0 \Rightarrow \frac{d}{dT}\left(\frac{V_r^2}{\delta R_0}\frac{1+\alpha T}{(1+\alpha T+k)^2}\right) = 0$$

$$\Leftrightarrow \frac{V_r^2}{\delta R_0}\frac{\alpha(-1-\alpha T+k)}{(1+\alpha T+k)^3} = 0$$

$$\Rightarrow -1-\alpha T+k = 0 \Leftrightarrow T = \frac{k-1}{\alpha}$$

(f) Assume that $\delta = 6$ mW/K and $V_r = 5$ V. What value should the resistor R_1 have such that the error due to the self-heating effect is less then 0.5°C within the whole range of the sensor while at the same time the sensitivity is maximized?

Answer: The value of k is limited by the self-heating. To achieve maximal sensitivity (see part C), it must hold that $k = 1 + \alpha T$. Hence k should be as close as possible to 1. It follows from part E, that the maximal self-heating effect will appear at the end of the range ($T = 40$°C). Using this temperature we find two values for k:

$$0.5 = \frac{(5V)^2}{(6mW/K)(100\Omega)}\frac{1+(0.004/°C)(40°C)}{(1+(0.004/°C)(40°C)+k)^2}$$

Solving this equation yields: $k = 8.7$ of $k = -11.0$. Negative resistive values are not meaningful. Since k needs to be as close as possible to 1, but it may not be smaller then 8.7 (otherwise the self heating error becomes too big) we choose $k = 8.7$.

Therefore it must hold that:

$$R_1 = kR_0 = 870\Omega$$

(g) The operation of a temperature dependent resistor (RTD) is based on the thermo-resistive effect. Explain briefly (maximal 200 words) how this effect works in metals.

Answer: In metals, the average time between collisions depends on the temperature. This in turn influences the specific resistance of the material and as such the resistance of the sensor.