Sensing, Computing, Actuating
Lecture 6 - Amplifiers

Exercise 1: Tire-pressure monitoring system

A tire-pressure monitoring system (TPMS) is an electronic system to measure the air pressure inside the tires of a vehicle. A TPMS sensor delivers real-time information on the tire pressure to the driver of the vehicle by displaying the pressure inside the tires or through a simple light that turns on when the pressure becomes too low. There exist two different types of TPMS sensors. The indirect type derives the tire pressure through external factors such at the rotation speed of the wheels. The direct type measures the pressure by placing a sensor inside the tire. This sensor measures the air pressure inside the tire and send the result through a wireless connection to the ECU who further processes the signal and controls the actuators (see Figure 1). The wireless sensor placed inside the tire (see Figure 2) measures the air pressure using a number of strain gauges. The voltage across these sensors is too low to immediately be transmitted wireless. For this purpose, the signal should first be amplified[1]. An instrumentation amplifier can be used for this purpose.

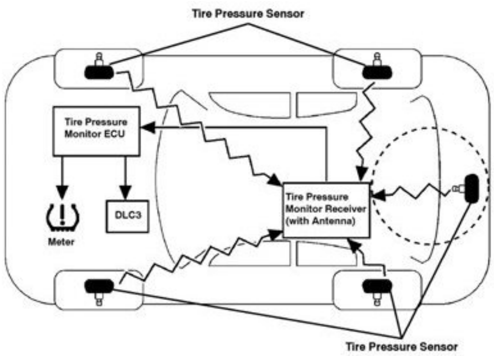

Figure 1: Tire-pressure monitoring system.

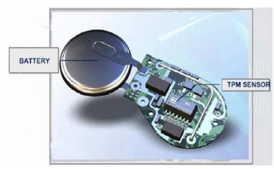

Figure 2: Tire-pressure sensor.

An instrumentation amplifier is a differential amplifier who at the same time has a high input-impedance, a low output-impedance and a high common-mode rejection-ratio (CMRR). The gain of an instrumentation amplifier can typically be changed with a single resistor. Figure 3 shows an instrumentation amplifier that consists of two operational amplifiers. (You may assume that these op-amps have an ideal behavior.)

[1] Additional signal processing is often applied in practice such that it becomes for example possible to detect errors in the wireless transmission.

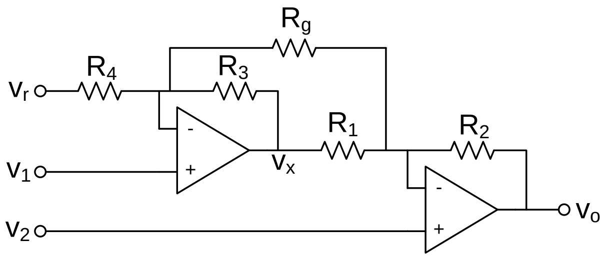

Figure 3: Instrumentation amplifier with two op-amps.

(a) Show that the output voltage v_o of the instrumentation amplifier shown in Figure 3 is equal to:

$$v_o = \left(1 + k + \frac{R_2 + R_4}{R_g}\right) v_d + v_r$$

with $v_d = v_2 - v_1$. Assume that $R_4/R_3 = R_2/R_1 = k$ (i.e., the CMRR is infinite).

Answer: It holds for the junction at v_- of the top op-amp:

$$\frac{v_r - v_1}{R_4} = \frac{v_1 - v_2}{R_g} + \frac{v_1 - v_x}{R_3}$$

and for the junction v_- on the bottom op-amp:

$$\frac{v_x - v_2}{R_1} + \frac{v_1 - v_2}{R_g} = \frac{v_2 - v_o}{R_2}$$

Step 1: solve v_x from top equation:

$$v_x = R_3 \left(-\frac{v_r - v_1}{R_4} + \frac{v_1 - v_2}{R_g}\right) + v_1$$

$$\Rightarrow v_x = -\frac{R_3}{R_g} v_d - \frac{R_3}{R_4} v_r + \left(1 + \frac{R_3}{R_4}\right) v_1$$

$$\Rightarrow v_x = -\frac{R_3}{R_g} v_d - \frac{1}{k} v_r + \left(1 + \frac{1}{k}\right) v_1$$

Step 2: solve v_o from second equation

$$v_o = -\frac{R_2}{R_1} v_x + \frac{R_2}{R_1} v_2 + \frac{R_2}{R_g} v_d + v_2$$

$$\Rightarrow v_o = -\frac{R_2}{R_1} v_x + \left(1 + \frac{R_2}{R_1}\right) v_2 + \frac{R_2}{R_g} v_d$$

$$\Rightarrow v_o = -k v_x + (1 + k)\, v_2 + \frac{R_2}{R_g} v_d$$

Step 3: substitute v_x in equation shown above

$$v_o = (1 + k)\, v_2 + \frac{R_2}{R_g} v_d + k \left(\frac{R_3}{R_g} v_d + \frac{1}{k} v_r - \left(1 + \frac{1}{k}\right) v_1\right)$$

$$\Rightarrow v_o = (1+k)\,v_2 + \frac{R_2}{R_g}v_d + k\frac{R_3}{R_g}v_d + v_r - (1+k)\,v_1$$

$$\Rightarrow v_o = (1+k)\,v_d + \frac{R_2}{R_g}v_d + k\frac{R_3}{R_g}v_d + v_r$$

$$\Rightarrow v_o = (1+k)\,v_d + \frac{R_2 + kR_3}{R_g}v_d + v_r$$

$$\Rightarrow v_o = \left(1 + k + \frac{R_2 + kR_3}{R_g}\right)v_d + v_r$$

Use: $k = R_4/R_3$

$$\Rightarrow v_o = \left(1 + k + \frac{R_2 + R_4}{R_g}\right)v_d + v_r$$

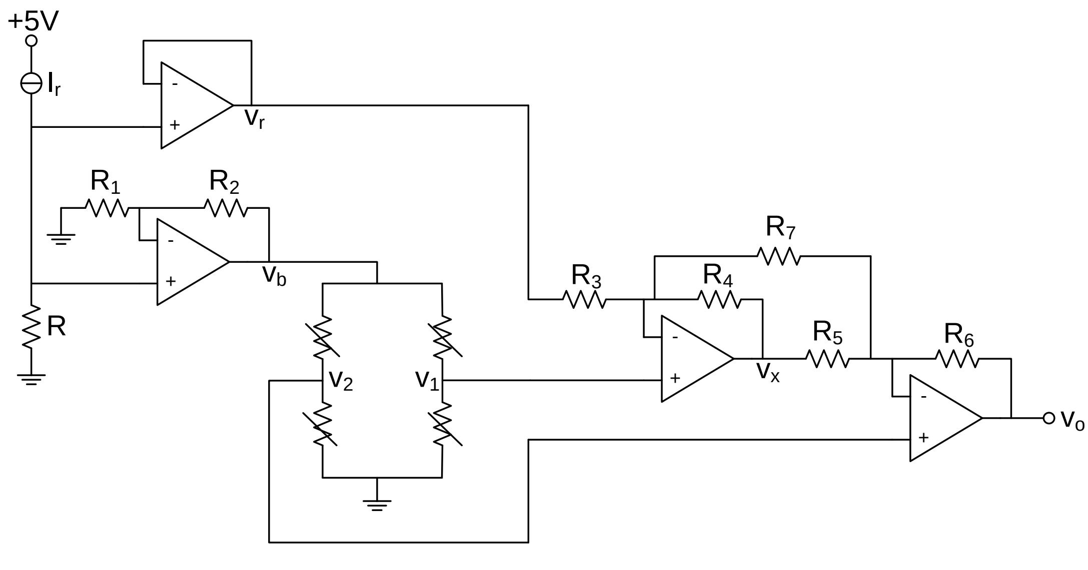

Figure 4: Signal processing circuit for a resistive pressure sensor.

The instrumentation amplifier from Figure 3 is used to amplify the output signal of a bridge circuit with four strain gauges (see Figure 4). The strain gauges each have a resistance R_0 of 4000 Ω and a sensitivity of 1 mV/psi when a supply voltage of 12 V is placed across the strain gauges. The current source delivers a stable current $I_r = 100\ \mu A$.

(b) Show that the output voltage v_o of this circuit is equal to:

$$v_o = (v_2 - v_1)\left(1 + \frac{R_6}{R_5} + \frac{R_3 + R_6}{R_7}\right) + V_r = (v_2 - v_1)\,G + v_r$$

assume that $R_3/R_4 = R_6/R_5 = k$. (Hint: use the result from the question 1(a).)

Answer: This result follows immediately from the answer to the previous question. The only difference are the change in the names of the resistors.

(c) The circuit should have an output voltage v_o of 0.5 V at a pressure of 0 psi. What value should R have to satisfy this constraint?

Answer: At a pressure of 0 psi it holds that $v_1 = v_2$. The output voltage is thus equal to:

$$v_o = v_r$$

It must hold in this situation that $v_o = 0.5$ V. It must therefore hold that $v_r = 0.5$ V. Therefore R must be equal to:

$$R = \frac{0.5V}{100\mu A} = 5k\Omega$$

(d) The bridge circuit is supplied with a voltage v_b. What ratio should R_2/R_1 have such that the supply voltage of the bridge is equal to 3.5 V?

Answer: The supply voltage of the bridge v_b is equal to:

$$v_b = (0.5V)\left(1 + \frac{R_2}{R_1}\right)$$

If this voltage needs to be equal to 3.5 V, then it must hold that:

$$3.5V = (0.5V)\left(1 + \frac{R_2}{R_1}\right)$$

$$\Rightarrow \frac{R_2}{R_1} = 6$$

(e) At a pressure of 100 psi, the circuit must have an output voltage v_o of 4 V. What gain G should the instrumentation amplifier have to satisfy this constraint?

Answer: At a pressure of 100 psi it holds:

$$4V = (v_2 - v_1)G + 0.5V = (100psi) \times (1mV/psi) \times \frac{v_b}{12V} \times G + 0.5V$$

It holds that $v_b = 3.5$ V. Therefore it must hold:

$$4V = 100mV \times \frac{3.5}{12} \times G + 0.5V$$

Solving this equation yields:

$$G = 120$$

(f) Assume that $R_3 = R_4 = R_5 = R_6 = 100$ kΩ. What value should R_7 have to realize the gain that you computed in the previous question?

Answer: As computed in the first question, the gain of the instrumentation amplifier is equal to:

$$G = 1 + k + \frac{R_6 + R_3}{R_7}$$

It holds that: $k = 1$. Therefore it must hold:

$$G = 1 + 1 + \frac{200k\Omega}{R_7} = 120$$

$$\Rightarrow R_7 = 1694.9\Omega$$