Sensing, Computing, Actuating
Lecture 7 - AD/DA conversion

Exercise 1: Electronic control unit

Electronic control units (ECUs) are commonly used in modern vehicles. The are used to control one or more of the electrical systems or subsystems inside a vehicle. The are often used to control for example the airbags, engine, and powertrain of a car. Other applications of ECUs are shown in Figure 1. Some modern cars contain up-to 80 ECUs and it is expected that this number will increase even further in the future. A micro-processor is often used at the heart of an ECU. Figure 2 shows the block diagram of the Freescale MPC5634M ECU. This ECU uses contains several micro-processors that can be used to run the digital signal processing and control algorithms that are needed to control the operation of a vehicle. These micro-processors process data that has been read by sensors. Typically, the sensors deliver an analog signal while the processor operates on a digital data stream. To connect these two components, the analog sensor value should first be digitized. For this purpose, the ECU contains an analog-to-digital converter (ADC). The ECU shown in Figure 2 contains two ADCs that are both able to perform an analog-to-digital conversion with a 12-bit resolution within 1μs. Hence, these ADCs can both output 1 million samples per second. They are based on the idea of successive approximation.

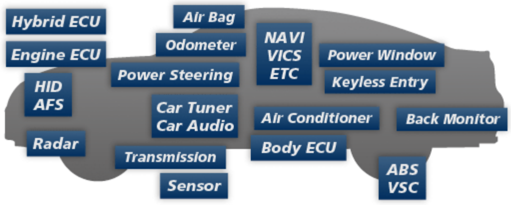


Figure 1: Applications of electronic control units (ECUs).

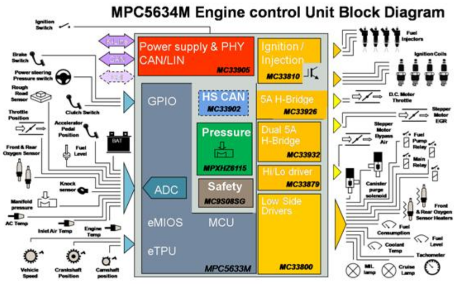


Figure 2: Electronic control unit (ECU) with ADC.

(a) A 5-bit DA converter has a voltage output. For a binary input of 10100, an output voltage of 12 mV is produced. What is the output voltage when the binary input is 11100?

Answer: The binary input 10100 is equivalent to the decimal number 20. The output voltage is 12 mV, hence

$$resolution = 12mv/20 = 0.6mV/digit$$

The binary input 11100 is equivalent to the decimal number 28. The output voltage is therefore equal to:

$$28 \cdot 0.6mV/digit = 16.8mV$$

(b) An 8-bit DA converter has a step size of 5 mV. What is the full-scale output voltage (i.e., maximal output voltage) of the DA converter?

Answer: An 8-bit system has $2^8 = 256$ levels. The lowest level has an output voltage of 0 V. The highest level is 255 ($2^8 - 1 = 255$) steps higher, hence:

$$full\ scale\ output = 255 \cdot 5mV = 1.275V$$

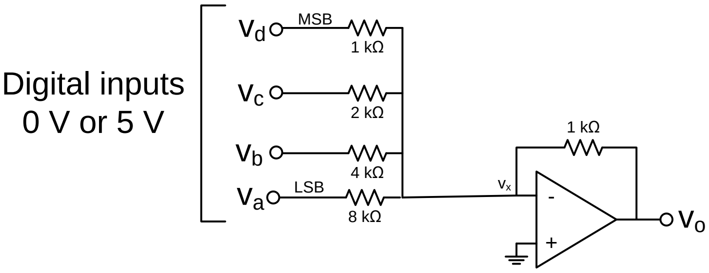


Figure 3: DA converter using summing op-amp.

(c) Show that the output voltage v_o of the DA converter shown in Figure 3 is equal to:

$$v_o = -(v_d + 0.5v_c + 0.25v_b + 0.125v_a)$$

Answer: Since the op-amp is used in feedback, it holds that $v_x = 0$ V. Using Kirchhoff current law at this junction yields:

$$\frac{v_d}{1k\Omega} + \frac{v_c}{2k\Omega} + \frac{v_b}{4k\Omega} + \frac{v_a}{8k\Omega} + \frac{v_o}{1k\Omega} = 0$$

Solving this equation yields:

$$v_o = -(v_d + 0.5v_c + 0.25v_b + 0.125v_a)$$

(d) What is the resolution of the DA converter shown in Figure 3?

Answer: The digital inputs accept a resolution of 0 V or 5V. The number 0001 (only LSB high) shows the smallest step the DA converter can make which is called the *resolution* of the DA converter. It is equal to:

$$resolution = |0.125 \cdot 5V| = 0.625V$$

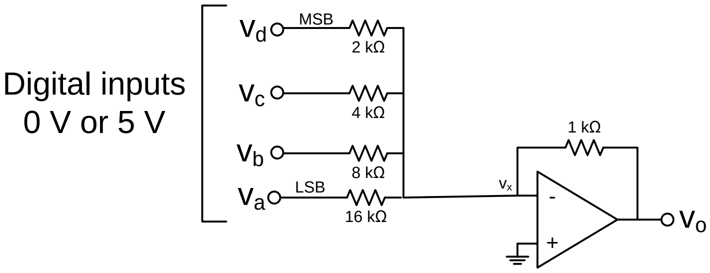


Figure 4: Alternative DA converter.

(e) What is the weight (contribution) of each input bit in the output voltage of the DA converter shown in Figure 4?

Answer: For input d:

$$v_d = -\frac{1 \cdot 10^3 \Omega}{2 \cdot 10^3 \Omega} \cdot 5V = -2.5V$$

For input c:

$$v_c = 0.5 \cdot v_d = -1.25V$$

For input b:

$$v_b = 0.5 \cdot v_c = -0.625V$$

For input a:

$$v_a = 0.5 \cdot v_b = -0.313V$$

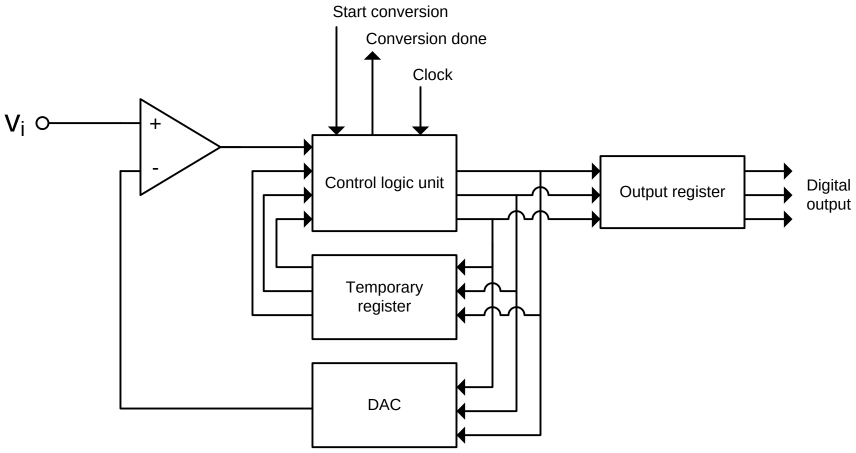


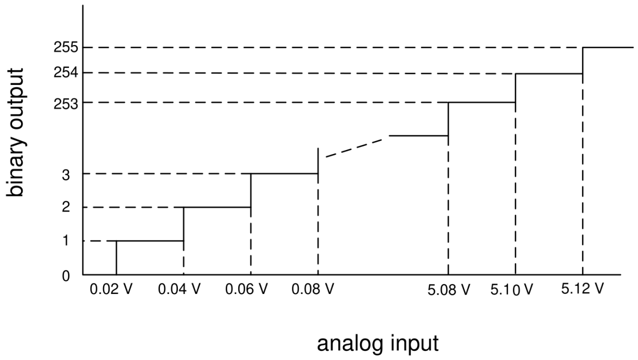
Figure 5: Successive approximation ADC.

(f) Figure 5 shows a successive approximation ADC that uses an 8-bit DAC which has a conversion range of 0 V to 5.12 V. Draw the ADC transfer curve (binary input versus v_i) showing all relevant values.

Answer: The resolution of the ADC is equal to:

$$\frac{range}{2^N - 1} = \frac{5.12V}{2^8 - 1} = 20mV$$

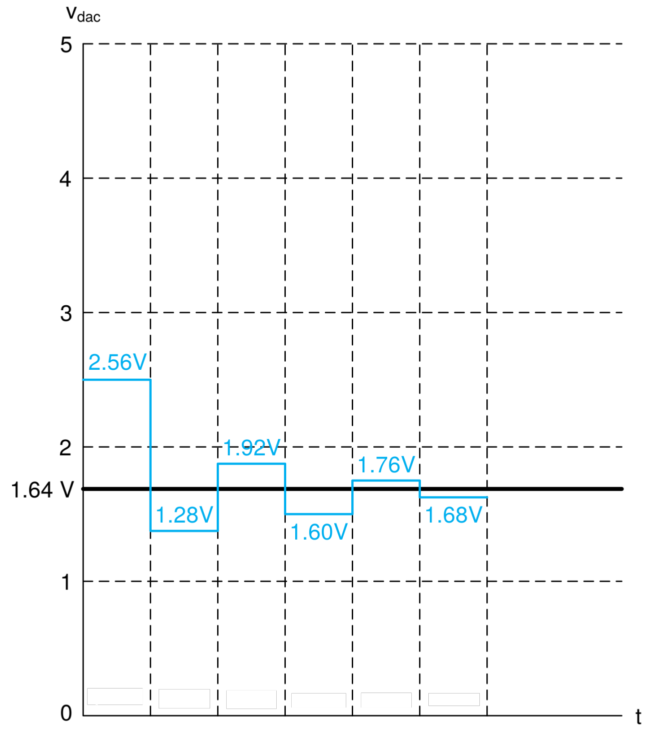
The ADC transfer function looks as follows:



(g) Assume that $v_i = 1.64$ V. Draw the DAC output (labels and levels) and its binary input for the first five bits tested. (*Hint:* calculate the weight of each bit.)

Answer: Weights of LSB bit: $b_0 = 5.1V/255steps = 20mV$. With each bit, the weight doubles and so does the output voltage. Hence, the weight of the other bits is equal to: $b_1 = 40mV,\ b_2 = 80mV,\ b_3 = 160mV,\ b_4 = 320mV,\ b_5 = 640mV,\ b_6 = 1.28V,\ b_7 = 2.56V$.

bit tested	binary input	output voltage DAC	comparator output	tested bit is
b_7	10000000	2.56 V	low	low
b_6	01000000	1.28 V	high	high
b_5	01100000	1.28 V + 0.64 V = 1.92 V	low	low
b_4	01010000	1.28 V + 0.32 V = 1.60 V	high	high
b_3	01011000	1.28 V + 0.32 V + 0.16 V = 1.76 V	low	low
b_2	01010100	1.28 V + 0.32 V + 0.08 V = 1.68 V	low	low



(h) What is the main advantage of a successive approximation ADC over a dual-slope ADC?

Answer: The conversion is performed much faster and intermediate (lower resolution) conversions are available during the conversion process.