Sensing, Computing, Actuating
Lecture 10 - Inductive Sensors

Exercise 1: Active suspension

Most cars use a passive suspension system to regulate the vertical movement of the wheels. Some cars use an active suspension system to enhance the driving comfort and safety. The vertical movement in such a system is regulated using an on-board computer system which is more complex then the regular spring system used in most cars. The use of an active suspension system makes it possible to keep the flat surface of the wheels on the road during all possible circumstances. This results in an improved traction and therefore more control over the vehicle. The active suspension board computer measures using sensors on all wheels the movement of these wheels with respect to the chassis. The board computer uses an advanced control system to determine how the stiffness of the various shock absorbers should be changed to optimize the connection to the road. These settings are then transmitted to the actuators (active shock absorbers) which are mounted to the different wheels. There are two commonly used types of actuators for this purpose. The first type uses linear motors to literally raise or lower the chassis. The second type adjusts the stiffness of the shock absorber. Typically this is done through a magnetic fluid contained in the shock absorber. By changing the magnetic field around the shock absorber, the stiffness of the fluid can be changed. The impact of the actuator is then sensed again by the sensor who relays its reading to the board computer who uses this information in the next iteration of the control loop. Most of these active suspension systems uses a linear variable differential transformer (LVDT) as sensor.

Figure 1: Car with active suspension system.

A linear variable differential transformer (LVDT) can be used to measure a linear displacement. The sensor uses the magnetic coupling between a primary and two secondary coils to measure the displacement of a ferromagnetic core. Figure 2 shows a schematic overview of the sensor.

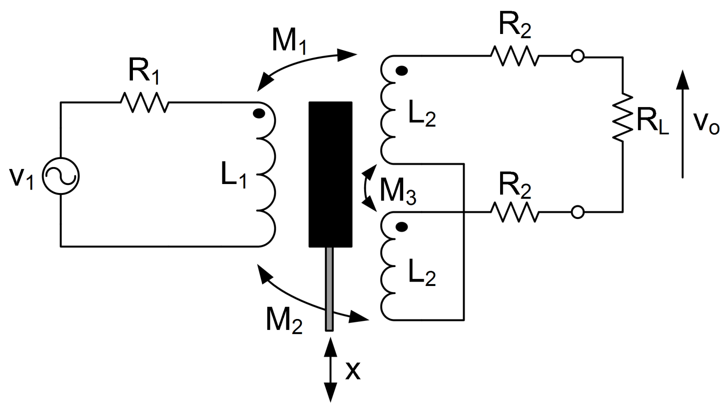


Figure 2: LVDT sensor.

The LVDT has a primary winding with a DC resistance of 76 Ω and two secondary windings who are placed opposed in series. Both windings have a DC resistance of 1600 Ω. The primary winding has an inductance of 45 mH. At 2 kHz the secundairy windings have a total impedance of 5600 Ω and the sensitivity - normalized to the execitation voltage - is equal to 320 $(\mu V/V)/\mu m$ when no loading resistance is connected to the sensor. The LVDT is connected to a measurement circuit with a purely resistive impedance of 10 kΩ.

(a) Which excitation frequency should be used to avoid a phase shift between the output voltage of the primary and secondary windings?

Answer: During the lecture we have derived that the execitation frequency at which there is no phase shift between the output voltage on the primary and secondary windings is equal to:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{R_1 R_{2c}}{2L_1 L_2}}$$

, with $R_{2c} = 2R_2 + R_L$. It is given: $R_1 = 76$ Ω, $L_1 = 45$ mH and $R_2 = 1600$ Ω. The inductance $|Z_2|$ from the secondary winding at 2 kHz is equal to 5600 Ω. It holds:

$$|Z_2| = 2 \cdot \sqrt{R_2^2 + (2\pi f L_2)^2} = 5600\Omega$$

$$\Rightarrow L_2 = \frac{1}{2}\frac{\sqrt{|Z_2|^2 - (R_2 + R_2)^2}}{2\pi f} = \frac{1}{2}\frac{\sqrt{(5600\Omega)^2 - (2 \cdot 1600\Omega)^2}}{2\pi 2000Hz} = 183mH$$

The excitation frequency that shows no phase shift is therefore equal to:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{(76\Omega)(2 \cdot 1600\Omega + 10k\Omega)}{2(45mH)(183mH)}} = 1242Hz$$

(b) How large is the phase shift between the input and output voltage when an excitation frequency of 50 Hz is used?

Answer: During the lecture we have derived that the phase shift ϕ between the voltage on the primary and secondary windings for a loaded LVDT is equal to:

$$\phi = 90^\circ - arctan\frac{\omega\left(R_{2c}L_1 + 2R_1 L_2\right)}{R_1 R_{2c} - 2L_1 L_2 \omega^2}$$

Substituting the know values gives:

$$\phi = 90^\circ - arctan\frac{(2\pi \cdot 50)\left((2 \cdot 1600\Omega + 10k\Omega)\left(45mH\right) + 2(76\Omega)(183mH)\right)}{(76\Omega)\left(2 \cdot 1600\Omega + 10k\Omega\right) - 2(45mH)(183mH)(2\pi \cdot 50)^2} = 90^\circ - 11^\circ = 79^\circ$$

Exercise continues on next page.

(c) What is the value of the normalized sensitivity of the LVDT shen the sensor is connected to a supply voltage with an excitation frequency of 50 Hz and the loading resistance R_L?

Answer: The sensitivity is equal to:

$$S = \frac{|V_o/V_1|}{x} = \frac{\omega k_x R_L}{\sqrt{(R_1R_{2c} - \omega^2 2L_1L_2)^2 + \omega^2 (R_{2c}L_1 + 2R_1L_2)^2}}$$

The first step is to compute the coupling-coefficient k_x. The normalized sensitivity is given for an unloaded situation. We can use this information to compute k_x. In this scenario, the sensitivity is equal to:

$$S = \frac{2\pi f k_x}{\sqrt{R_1^2 + (2\pi f L_1)^2}}$$

It holds: $S = 320\mu V/V/\mu m$ at 2 kHz, therefore it holds:

$$k_x = \frac{320 \times 10^{-6} V/V}{10^{-6} m} \frac{\sqrt{R_1^2 + (2\pi f L_1)^2}}{2\pi f} = \frac{320}{1m} \frac{\sqrt{(76\Omega)^2 + (2\pi \cdot 2kHz \cdot 45mH)^2}}{2\pi 2000 rad}$$

$$\Rightarrow k_x = 14.5(\Omega/m)/(rad/s)$$

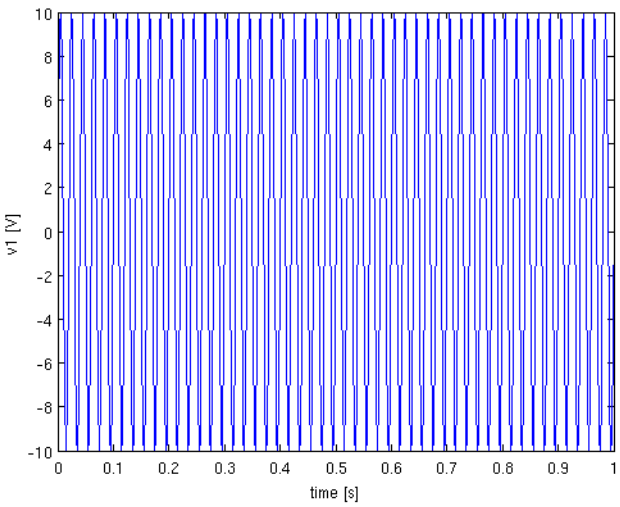
The sensitivity is thus equal to:

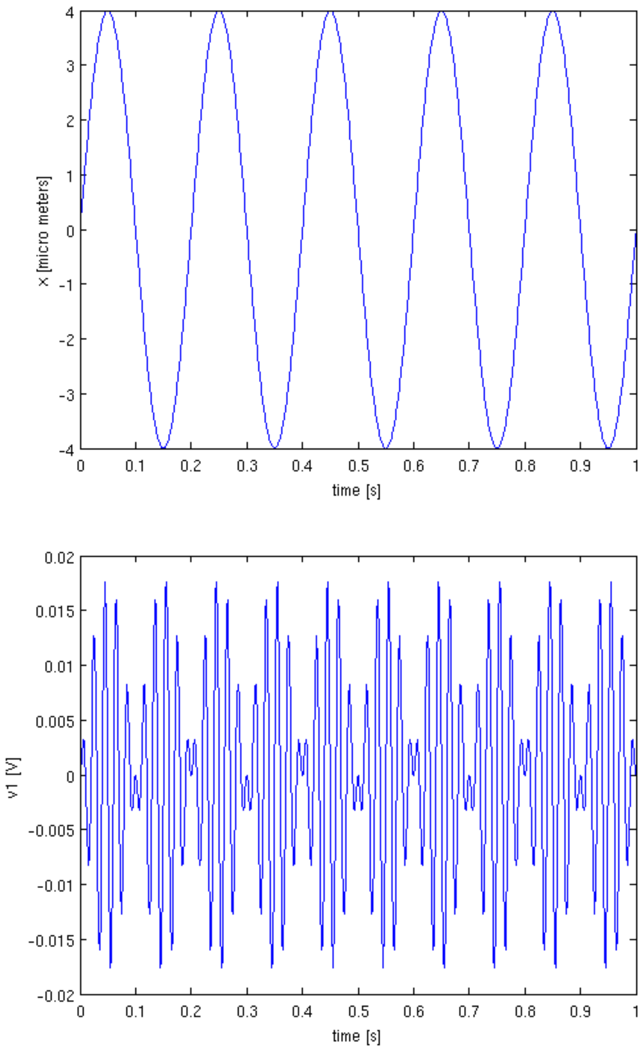
$$\frac{(2\pi 50 rad)(14.5(\Omega/m)/(rad/s))(10k\Omega)}{\sqrt{((76\Omega)(13200\Omega) - (2\pi 50Hz)^2 2(45mH)(183mH))^2 + (2\pi 50Hz)^2 ((13200\Omega)(45mH) + 2(76\Omega)(183mH))^2}}$$

$$\Rightarrow S = 44.7(\mu V/V)/\mu m$$

(d) Assume that the primary winding of the sensor is connected to a sinusoidal supply voltage v_1 with a frequency of 50 Hz and an amplitude (peak-too-peak) of 20 V. The object that is connected to the core of the sensor makes a sinusoidal movement with a frequency of 5 Hz. This causes a movement of the core (relative to the central position) between -4 μm and $+4$ μm. Draw (using Matlab) The primary excitation voltage ($v_1(t)$), the movement of the core ($x(t)$) and the output voltage of the sensor ($v_o(t)$). (Assume that the phase shift between the input and output voltage is compensated. I.e., you are allowed to ignore the phase shift between the input and output voltage in this plot.)

Answer:

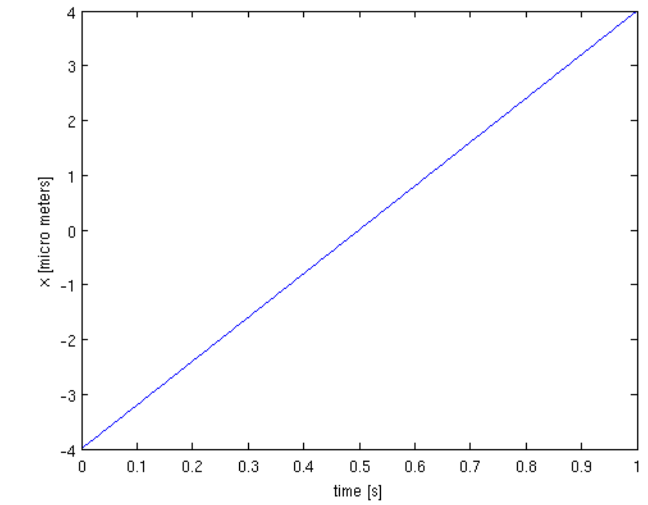


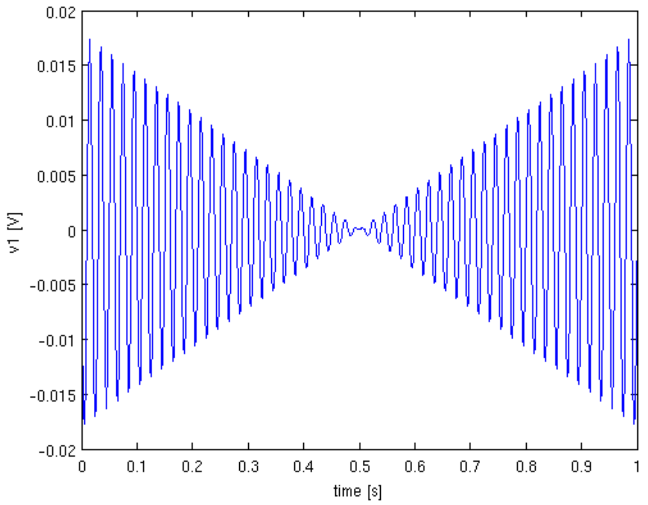


Exercise continues on next page.

(e) Assume that the primary winding of the sensor is connected to a sinusoidal supply voltage v_1 with a frequency of 50 Hz and an amplitude (piek-to-peak) of 20 V. The object that is connected to the core of the sensor makes a linear movement with a frequency of 1 Hz. This causes a displacement of the core (relative to the central position) between -4 μm (at $t = 0$ s) and $+4$ μm (at $t = 1$ s). Draw (using Matlab) the primary excitation voltage ($v_1(t)$), the movement of the core ($x(t)$) and the output voltage of the sensor ($v_o(t)$). (Assume that the phase shift between the input and output voltage is compensated. I.e., you are allowed to ignore the phase shift between the input and output voltage in this plot.)

Answer:





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(f) The frequency at who there is no phase shift between the voltage on the primary and secondary windings can be changed using an additional resistor R_1' which is placed in series or in parallel to the primary input of the sensor. Compute the value of the resistor R_1' to ensure that there is no phase shift when the excitation frequency is equal to 1500 Hz.

Answer: During the lecture we have derives that the excitation frequency at which there is no phase shift between the primary and secondary voltage is equal to:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{R_1 R_{2c}}{2L_1 L_2}}$$

We have computed before:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{(76\Omega)(2\cdot 1600\Omega + 10k\Omega)}{2(45mH)(183mH)}} = 1242Hz$$

Observation: R_1 is to small to have no phase shift at 1500 Hz. The idea is to place R_1' in series with R_1. The R_{1c} required to get no phase shift at 1500 Hz is equal to:

$$R_{1c} = \frac{(2\pi f_n)^2\, 2L_1 L_2}{R_{2c}} = \frac{(2\pi 500Hz)^2\, 2(45mH)(183mH)}{13200\Omega} = 111\Omega$$

We can then compute R_1' as follows:

$$R_1' = R_{1p} - R_1 = 111\Omega - 76\Omega = 35\Omega$$

(g) How does the sensitivity of the sensor change when the additional resistance R_1' is added?

Answer:
The sensitivity is then equal to:

$$\frac{(2\pi 1500rad)(14.5(\Omega/m)/(rad/s))(10k\Omega)}{\sqrt{((111\Omega)(13200\Omega) - (2\pi 1500Hz)^2 2(45mH)(183mH))^2 + (2\pi 1500Hz)^2\left((13200\Omega)(45mH) + 2(111\Omega)(183mH)\right)^2}}$$

$$\Rightarrow S = 228(\mu V/V)/\mu m$$

Without R_1' it is equal to:

$$\frac{(2\pi 1500rad)(14.5(\Omega/m)/(rad/s))(10k\Omega)}{\sqrt{((76\Omega)(13200\Omega) - (2\pi 1500Hz)^2 2(45mH)(183mH))^2 + (2\pi 1500Hz)^2\left((13200\Omega)(45mH) + 2(76\Omega)(183mH)\right)^2}}$$

$$\Rightarrow S = 232(\mu V/V)/\mu m$$

The sensitivity has been reduced. This is logical since the current I_1 has been decreased and the sensitivity is proportional to this current.