Sensing, Computing, Actuating
Lecture 14 - Thermocouple

Exercise 1: Gasoline exhaust gas temperature measurement

To maximize the fuel efficiency of a gasoline engine and to minimize the toxic gases exhausted by the engine, the engine control unit will regulate the fuel/air mixture to keep the temperature of the exhaust gas within certain limits. Exhaust gases in a gasoline engine are lower then in a diesel engine, but they may still reach temperatures between 700°C and 1200°C. A K-type thermocouple can be used to measure the temperature of this gas. Figure 1 shows a circuit to measure a temperature T by means of such a K-type thermocouple. The ambient temperature T_a at the reference junction is compensated using a NTC thermistor. The thermocouple has a sensitivity $k = 41\ \mu V/K$. The NTC thermistor has $B = 3546$ K and resistance $R_0 = 10\ k\Omega$ at 25°C. The voltage source $V_R = 1.35V$ and $R_2 = 100\ \Omega$. The output voltage of the circuit $v_o = k \cdot T$ (with T in °C).

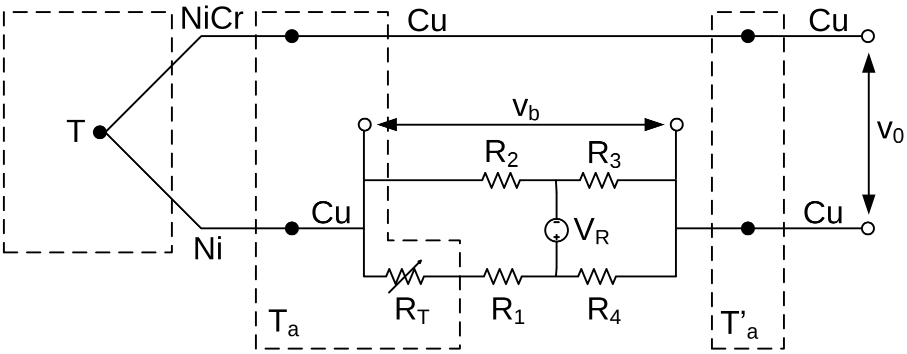
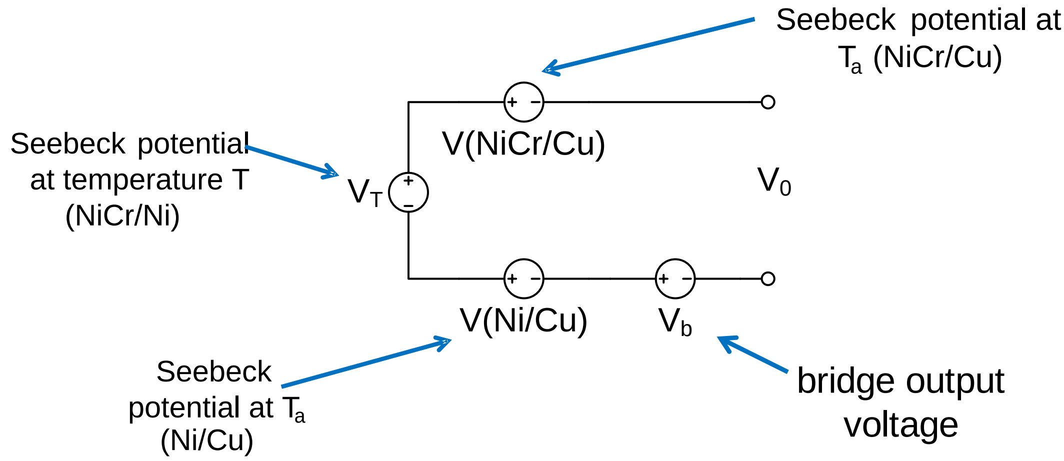


Figure 1: Circuit for cold junction compensation.

(a) Draw an equivalent circuit that represents the three thermocouple junctions and the bridge as voltage sources. (Note your circuit should contain in total four voltage sources, i.e., V_T, $V(NiCr/Cu)$, $V(Ni/Cu)$, and V_b.)

> **Answer:** Assumption: volt meter has high impedance. As a result, the current in thermocouple wires is low.



(b) Show that the bridge output voltage V_b should be equal to $k \cdot T_a$ to compensate for the ambient temperature T_a. (Hint: use law of intermediate metals.)

Answer: It follows from the circuit that

$$V_o = V_T - V(NiCr/Cu)\,|_{T_a} + V(Ni/Cu)\,|_{T_a} + V_b$$

The ambient temperature should not affect measurement. It must therefore hold that

$$V_o = V_T$$

Applying law of intermediate metals gives

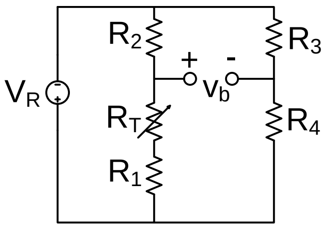
$$-V(NiCr/Cu)\,|_{T_a} + V(Ni/Cu)\,|_{T_a} = -V(NiCr/Ni)\,|_{T_a} \approx -kT_a$$

We need $V_b = kT_a$ to compensate for the ambient tempature at the reference junction.

(c) Show that the bridge sensitivity at the reference junction is equal to:

$$\frac{dV_b}{dT} = \left(\frac{\frac{BR_2}{T^2} R_0 e^{B(1/T - 1/T_0)}}{\left(R_1 + R_0 e^{B(1/T-1/T_0)} + R_2\right)^2} \right) V_R$$

Answer: The bridge circuit is shown (rearranged) below. The + and - sign on V_b follow from the choice made in assignment (a). Note that the top of V_r is negative. This follows from Figure 1.



The bridge output is equal to:

$$V_b = -V_R \cdot \left(\frac{R_1'}{R_1' + R_2} - \frac{R_4}{R_3 + R_4} \right) = V_R \cdot \left(\frac{R_4}{R_3 + R_4} - \frac{R_1'}{R_1' + R_2} \right)$$

with $R_1' = R_1 + R_T = R_1 + R_0 e^{B(1/T - 1/T_0)}$.

Note that you can also compute the bridge output voltage as:

$$V_b = V_R \cdot \left(\frac{R_2}{R_1' + R_2} - \frac{R_3}{R_3 + R_4} \right)$$

The sign on V_r changes in this case. The output voltage V_b will not change. Hence you will find the same sensitivity as when you continue with the earlier expression.

Use quotient rule to compute derivative:

$$\frac{d}{dx}\frac{g(x)}{h(x)} = \frac{g'(x)h(x) - g(x)h'(x)}{(h(x))^2}$$

The derivative of R_T is given by:

$$\frac{dR_T}{dT} = \frac{d}{dT} R_0 e^{B(1/T - 1/T_0)} = -\frac{B}{T^2} R_T$$

Hence, the derivative of the bridge voltage (sensitivity) is equal to:

$$\frac{dV_b}{dT} = \left(\frac{-\frac{B}{T^2} R_T \left(R_1 + R_T + R_2\right) + \left(R_1 + R_T\right) \frac{B}{T^2} R_T}{\left(R_1 + R_T + R_2\right)^2} \right) V_r$$

$$= \left(\frac{\frac{B}{T^2} R_2 R_T}{\left(R_1 + R_T + R_2\right)^2} \right) V_r$$

$$= \left(\frac{\frac{BR_2}{T^2} R_0 e^{B(1/T - 1/T_0)}}{\left(R_1 + R_0 e^{B(1/T - 1/T_0)} + R_2\right)^2} \right) V_R$$

(d) In question 1(b), you showed that the output voltage V_b should be equal to $k \cdot T_a$ to compensate for the ambient temperature T_a. Hence, the bridge should have a sensitivity k. What value for R_1 should be used to ensure that the bridge sensitivity is equal to k?

Answer: We derived earlier that it must hold that $V_b = kT_a$. It follows from this requirement that the bridge sensitivity should be equal to k. In the previous part we computed the sensitvity of the bridge. This bridge sensivitity is not constant (k) as required. Instead, the bridge sensitivity depends on the temperature. As a result, the bridge output is non-linear. We can however choice a linearization criterion by choosing the desired slope k at 25 °C.

It must hold:

$$\frac{dV_b}{dT} = \left(\frac{\frac{BR_2}{T^2} R_0 e^{B(1/T - 1/T_0)}}{\left(R_1 + R_0 e^{B(1/T - 1/T_0)} + R_2\right)^2} \right) V_R = k$$

It is given that: $R_2 = 100\ \Omega$, $R_0 = 10\ \text{k}\Omega$, $B = 3546$ K, $T_0 = 25$°C, $T = 25$°C, $V_r = 1.35$ V, $k = 41\ \mu$V/K.

Solving this equation yields $R_1 = 26160\ \Omega$.

(e) What ratio should R_3/R_4 have to ensure that the circuit shown in Figure 1 compensates the ambient temperature at the reference junction?

Answer: We computed before that it must hold that $V_b = kT_a$ to compensate for the ambient temperature at the reference junction. We can choose **any** value for T_a, but the easiest is to use $T_a = 25$°C. (Note that any other value for T_a will yield the same ratio R_3/R_4.)

It must thus hold that

$$V_b(25°C) = 41\mu V \cdot 25°C = 1.0mV$$

The bridge output is equal to

$$V_b = \left(\frac{R_4}{R_3 + R_4} - \frac{R_1'}{R_1' + R_2} \right) V_R$$

Using $R_2 = 100\ \Omega$, $R_1 = 22097\ \Omega$, $T = 25°\text{C}$, and $V_R = 1.35$ V, we find $R_3/R_4 = 2.0 \cdot 10^{-3}$.