Sensing, Computing, Actuating
Lecture 15 - Exam training

Disclamer: This document contains that exercises that cover part of the material that you need to study in preparation of the exam. The exam will also cover material not discussed in these exercises. Furthermore, the exam will contain more exercises.

Exercise 1: Electronic Stability Progam

ESP assists a driver to keep a vehicle on the road during dangerous driving conditions. For this purpose, the ESP system uses a large number of sensors in the vehicle. One of these sensors measures the angle of the steering-wheel and steering-column and the speed with which the driver changes this angle (note that one sensor measures both quantities). The RVDT (rotary variable differential transducer) from Figure 1 can be used to measure the angle (and its rate of change). When the driver moves the steer from the central position ($\Theta = 0°$) to the left or to the right, then this will lead to a change in the output voltage of the sensor. This electrical signal can then be send to the ESP computer. The primary winding of this RVDT is connected to a voltage supply that produces a sinusoidal voltage with an amplitude of 5V with a frequency of 10 Hz. The RVDT has a sensitivity S of 100 μV/(°/V). The output voltage of the RVDT is equal to:

$$v_2 - v_1 = S \cdot \Theta \cdot v_r$$

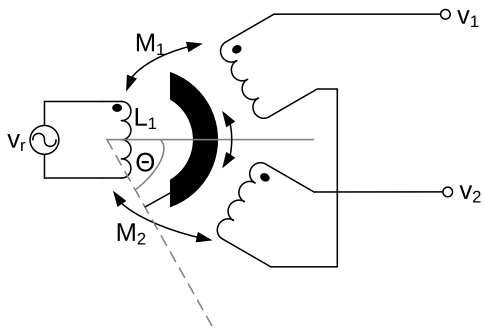
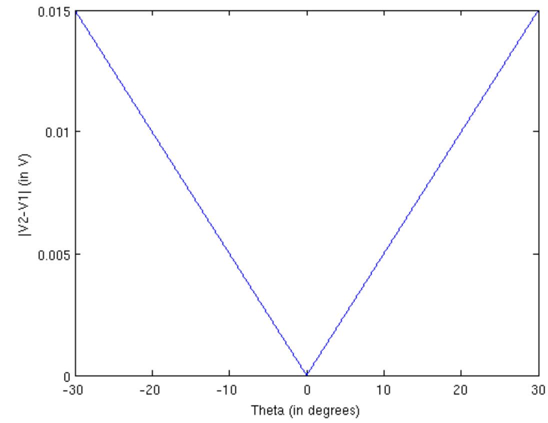


Figure 1: Measuring an angle using an RVDT.

(a) A driver moves the steer in 1 second from an angle $\Theta = -30°$ to an angle $\Theta = +30°$. Draw the amplitude of the output voltage $v_2 - v_1$ as a function of the angle Θ. (Clearly show the dimensions on both axis.)

Answer:



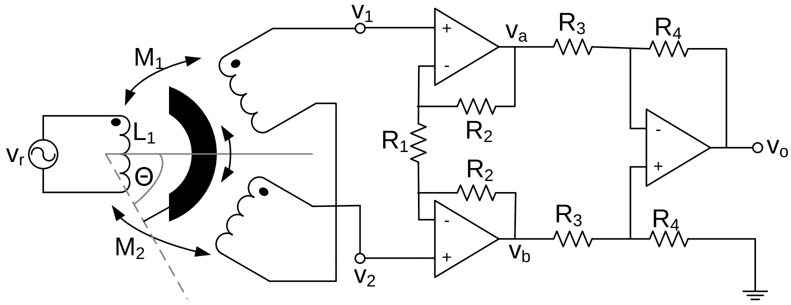


Figure 2: RVDT with instrumentation amplifier.

(b) The signal coming from the RVDT is too weak to be directly send to the ESP computer. The signal should first be amplified. For this purpose, the sensor from Figure 1 is connected to an instrumentation amplifier. The resulting circuit is shown in Figure 2. The instrumentation amplifier uses three operational amplifiers. You may assume that these op-amps show an ideal behaviour. Show that the output voltage v_o of the instrumentation amplifier in Figure 2 is equal to:

$$v_o = \left(1 + \frac{2R_2}{R_1}\right)\frac{R_4}{R_3}v_d$$

with $v_d = v_2 - v_1$.

Answer: It holds for the top op-amp in the first stage:

$$\frac{v_a - v_1}{R_2} = \frac{v_1 - v_2}{R_1} \Rightarrow v_a = \left(1 + \frac{R_2}{R_1}\right)v_1 - \frac{R_2}{R_1}v_2$$

It holds for the bottom op-amp in the first stage:

$$\frac{v_b - v_2}{R_2} = \frac{v_2 - v_1}{R_1} \Rightarrow v_b = \left(1 + \frac{R_2}{R_1}\right)v_2 - \frac{R_2}{R_1}v_1$$

The output voltage of the second stage is equal to:

$$\frac{v_o - v_n}{R_4} = \frac{v_n - v_a}{R_3} \wedge \frac{0 - v_p}{R_4} = \frac{v_p - v_b}{R_3}$$

It holds: $v_p = v_n$

$$\Rightarrow v_o = (v_b - v_a)\frac{R_4}{R_3}$$

Substituting v_a and v_b gives:

$$v_o = \left(1 + \frac{2R_2}{R_1}\right) \frac{R_4}{R_3} v_d$$

Exercise continues on the next page

(c) Assume that the resistors R_3 and R_4 in the instrumentation amplifier from Figure 2 both have a resistance of 100 kΩ. What ratio should the resistors R_1 and R_2 have such that the amplitude of the output voltage v_o is equal to 0 V when $\Theta = 0°$ and 5 V (peak) when $\Theta = 30°$?

Answer:
It holds for the voltage from the RVDT sensor that:

$$v_2 - v_1 = 100\mu V/°/V \cdot \Theta \cdot 5V sin\,(\omega t)$$

It must hold: $V_o = 0V$ als $\Theta = 0°$. This constraint is always met.

It must also hold: $V_o = 5V$ als $\Theta = 30°$. This constraint is met when:

$$5V = \left(1 + \frac{2R_2}{R_1}\right)\frac{R_4}{R_3} \cdot 15mV$$

$$\Leftrightarrow 333 = 1 + \frac{2R_2}{R_1}$$

$$\Leftrightarrow 332 = \frac{2R_2}{R_1}$$

$$\Leftrightarrow \frac{R_2}{R_1} = 166$$

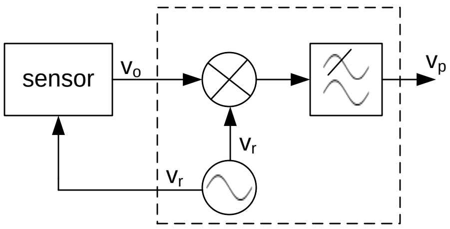


Figure 3: Sensor with phase sensitive detector.

(d) The rotation sensor from Figure 2 is connected to a phase sensitive detector. This detector consists of an analogue multiplier connected to a low-pass fulter. Using this processing circuit, it is possible to recover the magnitude and direction of the rotation from the output signal of the sensor. The block diagram of a phase sensitive detector is shown in Figure 3. The angle of the steering-wheel is given as a function $\Theta(t)$. Assume further that the reference voltage v_r is equal to:

$$v_r(t) = V_r cos\,(\omega_r \cdot t)$$

The output voltage of the sensor, v_o, is then equal to:

$$v_o(t) = S \cdot \Theta(t) \cdot v_r(t)$$

Show that the output voltage of the phase sensitive detector, v_p, is equal to:

$$v_p(t) = S \cdot \Theta(t) \cdot \frac{V_r^2}{2}$$

Hint: $cos(A)cos(B) = \frac{1}{2}\left(cos\,(A+B) + cos\,(A-B)\right)$.

Answer: Output voltage of the multiplier:

$$v_m(t) = v_o(t) \cdot v_r(t) = S \cdot \Theta(t) \cdot v_r(t) \cdot v_r(t)$$

$$\Leftrightarrow v_m(t) = S \cdot \Theta(t) \cdot \frac{V_r^2}{2} \left(cos\left(\omega_r + \omega_r\right) + cos\left(\omega_r - \omega_r\right)\right)$$

$$\Leftrightarrow v_m(t) = S \cdot \Theta(t) \cdot \frac{V_r^2}{2} \left(cos\left(2\omega_r\right) + cos\left(0\right)\right)$$

$$\Leftrightarrow v_m(t) = S \cdot \Theta(t) \cdot \frac{V_r^2}{2} \left(cos\left(2\omega_r\right) + 1\right)$$

The output voltage of the low-pass filter is then equal to:

$$v_p(t) = LPF\left(v_m(t)\right) = S \cdot \Theta(t) \cdot \frac{V_r^2}{2}$$

(e) In stead of a phase sensitive detector, it is also possible to connect the output signal of the RCDT to a double-sided rectifier with a low-pass filter as is shown in Figure 4. Is it possible to reconstruct from the output signal ($v_{o2} - v_{o1}$) the direction (positive or negative angle)? (Explain your answer.)

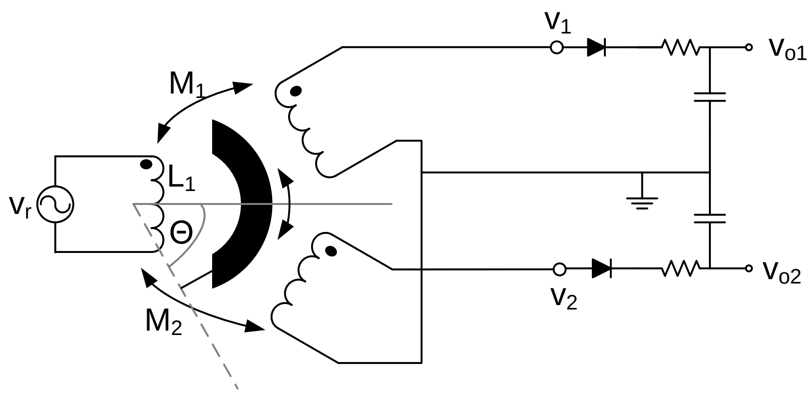


Figure 4: Double-sided rectifier with low-pass filter.

Answer: Yes. Both v_{o2} and v_{o1} will hold a DC voltage. Depending on the direction, the coupling between the primary and the top secondary winding will be larger/smaller then the coupling between the primary and the lower secondary winding. The difference between these two voltages will therefore have either a positive or a negative value depending on the direction in which the core moves.

(f) An RVDT measures the rotation of an object through the change in coupling between a primary and two secondary coils. Another way to measure a rotation would be to use a Hall effect sensor. Explain the operation of a Hall effect sensor. Clearly indicate how you can use the sensor to measure a rotation.

Answer: The Hall effect is based on the interaction between a moving electrical charge (electrons in metals and holes in semiconductors) and an external magnetic field. Without the magnetic field, the charge carriers move in a straight line between the two control electrodes. Under the influence of an external magnetic field, the charge carriers are bend. The magnitude and direction of this deflection depends on (1) the angle under which the external field hits the sensor and (2) the magnitude of the field.

Exercise 2: exhaust gas temperature measurement

It is necessary to change the ratio between fuel and air in a combustion engine to maximize the efficiency of the engine. The objective is to keep the temperature of the gasses going through the exhaust pipe within a certain temperature range. Because of the high temperature of these gases, it is necessary to use a thermocouple to measure this temperature. A thermocouple can only measure a temperature difference. Therefore it is necessary to also add a reference sensor to the circuit. This sensor is then used to measure the absolute temperature at the reference junction.

The thermocouple in Figure 5 contains a reference junction compensation based on a RTD temperature sensor. The output voltage of this sensor (v_{ref}) is given by the following transfer function: $v_{ref} = 10mV/^{\circ}C \cdot T_a$ with T_a the environmental temperature at the reference junction. The output of the reference temperature sensor is connected to the reference input of the instrumentation amplifier. It holds for this instrumentation amplifier:

$$v_o - v_{ref} = \left(1 + \frac{2R_2}{R_1}\right) \frac{R_4}{R_3} (v_2 - v_1) = (1 + G)\, k\, (v_2 - v_1)$$

The ratio of the resistors in the instrumentation amplifier is equal to: $k = R_4/R_3 = 1$. The thermocouple itself is a J-type (Cn/Fe) thermocouple with a sensitivity $S_J = 54\mu\text{V/K}$.

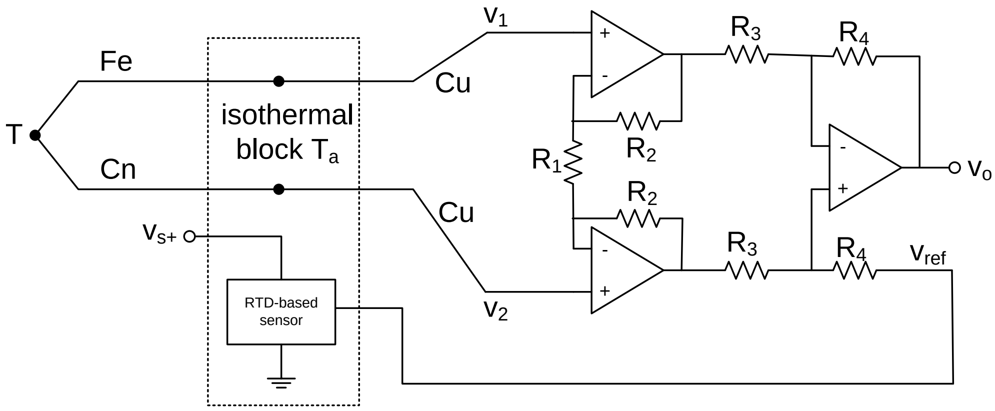


Figure 5: Compensation with a RTD-based sensor and instrumentation amplifier.

(a) There are three important law that you can use when analysing thermocouples, namely the law of the homogeneous circuits, the law of intermediate metals, and the law of intermediate temperatures. Show using these laws that the voltage across the thermocouple shown in Figure 5 is equal to:

$$v_2 - v_1 = V_T - V_{T_a} = S_J \cdot (T - T_a)$$

Hint: The Seebeck coefficient $\alpha_{Cn/Fe}$ of a J-type thermocouple is equal to: $\alpha_{Cn/Fe} = S_J$.

Answer: Analysis of the thermocouple circuit shows three junctions with respective temperatures T_a, T and T_a. The total output voltage generated by these three thermocouples is equal to:

$$v_2 - v_1 = \alpha_{Cu/Cn} T_a + \alpha_{Cn/Fe} T + \alpha_{Fe/Cu} T_a$$

, with $\alpha_{x/y}$ the Seebeck coefficient of the various thermocouples. According to the law of the intermediate metals it holds that:

$$\alpha_{Fe/Cu} + \alpha_{Cu/Cn} = \alpha_{Fe/Cn}$$

Using this law, we can rewrite the first equation as follows:

$$v_2 - v_1 = \alpha_{Cn/Fe} T + \alpha_{Fe/Cn} T_a$$

It holds for the Seebeck coefficient: $\alpha_{AB} = -\alpha_{BA}$. Applying this gives:

$$v_2 - v_1 = \alpha_{Cn/Fe}T - \alpha_{Cn/Fe}T_a = \alpha_{Cn/Fe}(T - T_a)$$

(b) What gain, G, should the instrumentation amplifier have to get an output voltage v_o that is independent of the environmental temperature?

Answer:
The output voltage of the instrumentation amplifier is equal to:

$$v_o = (1 + G)\,k\,(v_2 - v_1) + v_{ref} = (1 + G) \cdot k \cdot S_J \cdot (T - T_a) + v_{ref}$$

The reference voltage at the instrumentation amplifier is equal to:

$$v_{ref} = 10mV/^{\circ}C \cdot T_a$$

The output voltage of the instrumentation amplifier is thus equal to:

$$v_o = (1 + G) \cdot k \cdot S_J \cdot (T - T_a) + 10mV/^{\circ}C \cdot T_a$$

To ensure that the output voltage of the instrumentation amplifier is independent of the environmental temperature, it must now hold that:

$$(1 + G) \cdot k \cdot S_J \cdot T_a = 10mV/^{\circ}C \cdot T_a$$

Substituting $k = 1$ gives:

$$(1 + G) \cdot 54\mu/K = 10mV/^{\circ}C \Rightarrow G = 184$$

(c) A thermocouple uses three thermo-electrical effects. Name at least two of these effect and explain briefly (maximally 200 words) how these effects work.

Answer:
The thermoelectric effect occurs when there exists a temperature difference between a pair of (semi) conductors. Three thermoelectric effects are known:

- Seebeck effect: when two different conductors are joined together at one point and a temperature difference is maintained between the joined and non-joined parts, an open-circuit voltage will develop between the non-joined parts of this thermocouple.
- Peltier effect: when a current passes through the junction of two dissimilar conductors, heat is either released or absorbed at the junction depending on the direction of the current.
- Thomson effect: when a temperature gradient exists along a single conductor and at the same time a current is passed through trough this conductor, besides the release of normal Joule heat, this current also causes some extra heat to be produced or absorbed in the conductor.

(d) Instead of a thermocouple you can also use a thermistor to measure the temperature. The operation of a thermistor is based on the thermoresistive effect. Explain how this effect works in an NTC thermistor.

Answer: When the temperature raises, the number of free charge carriers will increase in an NTC thermistor. As a result, the specific resistance of the material will decrease and therefore the resistance will decrease.

(e) Give at least three reasons why we prefer a transducer who produces a signal in the electrical domain over a transducer who produces a signal in any of the other signal domains.

Answer:

- signal in any domain can be transduced to an electrical signal;
- hardly any energy needs to taken from the measured object since an electrical amplifier can be used;
- there are many different electrical signal processing circuits available;
- there are many ways to process, store and communicate electrical signals.

(f) Give a definition (maximally 100 words) for the following terms:

- Transducer
- Sensor
- Sensitivity of a sensor
- Transfer function of a sensor

Answer:

- **Transducer**: a transducer receives a signal from one signal domain and responds with a signal from a different signal domain.
- **Sensor**: a transducer that responds with a signal in the electrical domain.
- **Sensitivity of a sensor**: change in the electrical output per unit change in the physical input quantity that is being measured.
- **Transfer function of a sensor**: the relation between the physical input signal and the electrical output signal.